



# Resilient design and operation of urban water infrastructure networks

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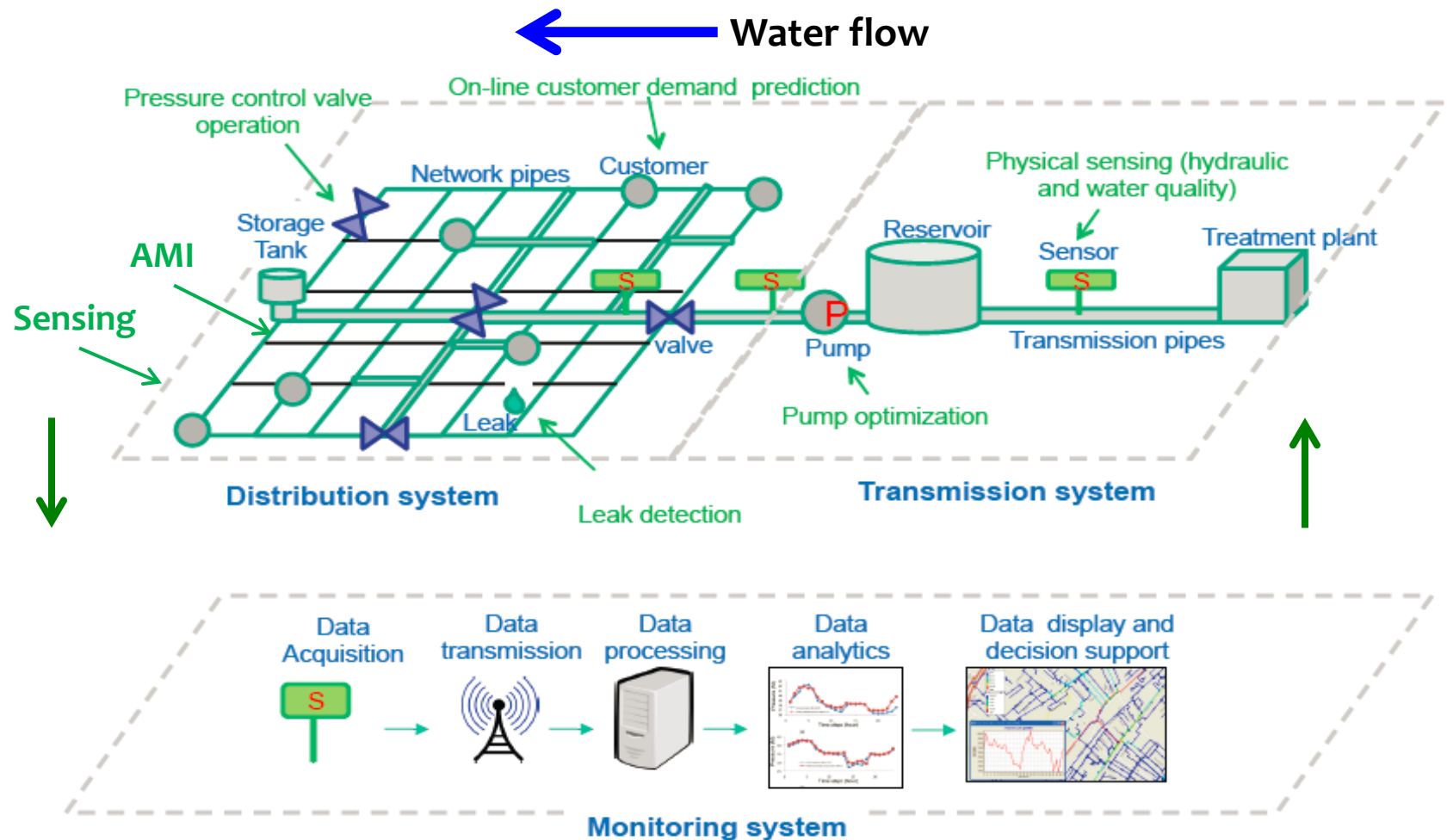
Challenge: How to ensure water security in the urban sector through resilient water networks aided by sensing and real-time data analytics?

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# Motivation: Smart water networks

- Sensing Modeling Analytics and Real-time Technology (SMART)



Data flow 

Adapted from Whittle A. 2012

# Smart water networks

## Objective

- How to ensure water security in the urban sector through resilient water networks?

## Challenges

- Infrastructure deterioration and risk of disruptions
- Demand-supply uncertainty
- Cyber-physical systems interdependency

## Approach

- Strategic design of network of sensors
- Real-time data acquisition and analytics for fault diagnosis
- Active network control and demand management

# Smart water networks

## Objective

- How to ensure water security in the urban sector through resilient water networks?

## Challenges

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## Approach

- Strategic design of network of sensors

Joint work with Waseem Abbas and Xenofon Koutsoukos  
Vanderbilt University

# Water losses

## Challenges

- Aging infrastructure
- Leaks & bursts

## Impacts

- Service disruption
- Public health risk
- Waste of water and energy resources

## Active leakage control

### Network of sensor nodes

- What to sense?
- When to sense?
- Where to place the sensors?



### US:

- No regulations for auditing & reporting water losses from public water systems
- ~250K reported;  
> 500K estimated breaks/year

# Where to place the sensors?

## Objective

- Sensor placement for detection and **location identification** of bursts

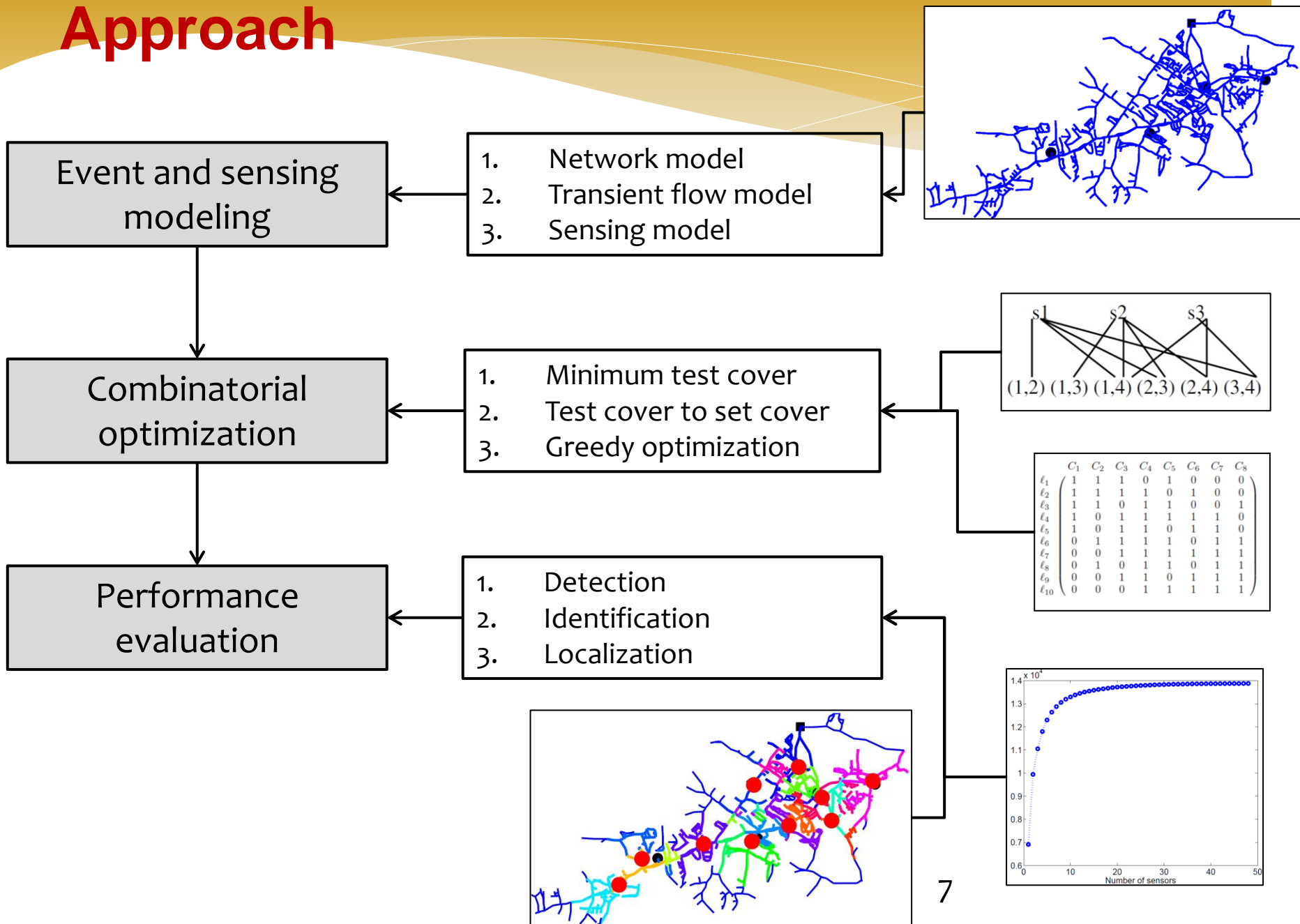
## Challenges

- Uncertainty in pipe failure events
- Uncertainty in sensing quality
- Budget constraints

## Impacts

- Early detection of reported losses (visible)
- Detection of unreported losses (not visible)
- Improved localization

# Approach



# Problem formulation

Find the subset of sensor locations  $S \subseteq \mathcal{S}$  such that sensor network performance function,  $f$ , is maximized:

$$\max_{S \subseteq \mathcal{S}} \left\{ f(S; \mathcal{L}), |S| \leq M \right\}$$

$\mathcal{S} = \{S_1, \dots, S_m\}$  : set of sensors     $\mathcal{L} = \{\ell_1, \dots, \ell_n\}$  : set of failure events

- **Detection:**

$f_D(S; \mathcal{L})$  - the number of events  $\ell$  that are detected by the set of sensors  $S$

- **Identification:**

$f_I(S; \mathcal{L})$  - the number of pair-wise events  $(\ell_i, \ell_j)$  that are distinguishable by the set of sensors  $S$



# Network dynamics

- Influence matrix represents events and sensors states:

$$\mathcal{M}(\mathcal{L}, \mathcal{S}) = \begin{bmatrix} \mathbf{y}_s(\ell_1) \\ \vdots \\ \mathbf{y}_s(\ell_n) \end{bmatrix}$$

$\mathcal{S} = \{S_1, \dots, S_m\}$  - set of sensors

$\mathcal{L} = \{\ell_1, \dots, \ell_n\}$  - set of failure events

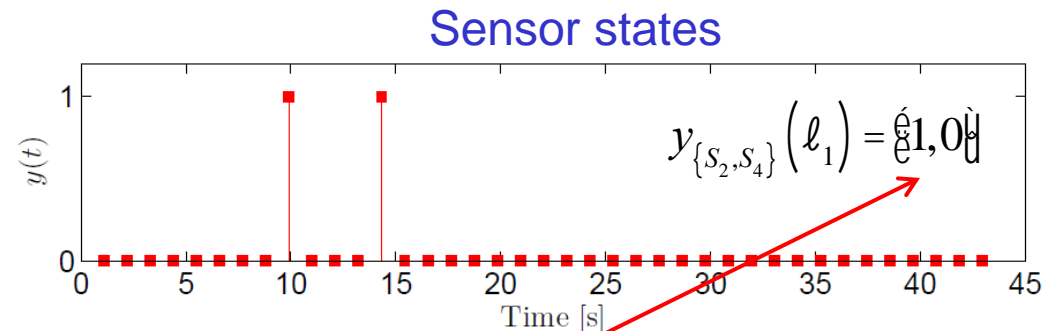
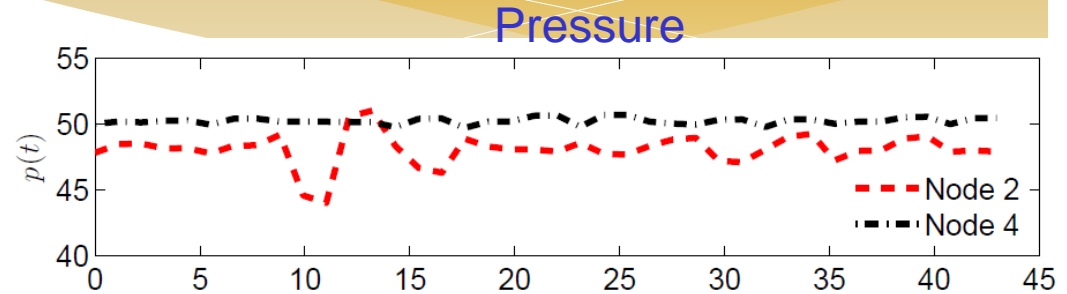
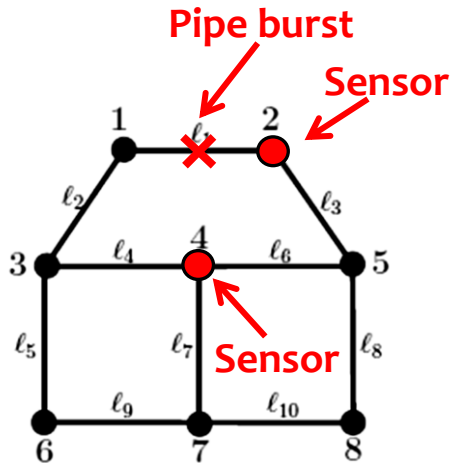
$\mathbf{y}_s(\ell_j)$  - output of sensors in response to event  $\ell_j$

$C_i \subseteq \mathcal{L}$  - is a set of link failures detected by  $S_i$

$\mathcal{M}_{ij} = 1$  - if sensor  $S_i$  detects event  $\ell_j$ ; 0 otherwise

# Network dynamics

- Example: event  $\ell_1$



- Consider:

$y_s(\ell_j)$ : sensors' response to event  $\ell_j$

$C_i$ : events detected by sensor  $i$

$\mathcal{M}(\mathcal{L}, \mathcal{S}) =$

$\ell_1$	1	1	1	0	1	0	0	0
$\ell_2$	1	1	1	1	0	1	0	0
$\ell_3$	1	1	0	1	1	0	0	1
$\ell_4$	1	0	1	1	1	1	1	0
$\ell_5$	1	0	1	1	0	1	1	0
$\ell_6$	0	1	1	1	1	0	1	1
$\ell_7$	0	0	1	1	1	1	1	1
$\ell_8$	0	1	0	1	1	0	1	1
$\ell_9$	0	0	1	1	0	1	1	1
$\ell_{10}$	0	0	0	1	1	1	1	1

# Detection as minimum set cover

## Detection:

- Find the minimum number of sensors and their locations such that every link failure can be detected by at least one sensor

## Minimum set cover (MSC) problem:

- Find the smallest number of sets in a family of sets that cover the family, i.e., their union is equal to the union of all sets in the family
- Submodular:  $f(A \cup \{C\}) - f(A) \geq f(B \cup \{C\}) - f(B)$   $A \subseteq B \subseteq \mathcal{C}$  and  $C \in \mathcal{C} \setminus B$
- Greedy solution with the best approximation ratio:  $O(\ln k)$

**Proposition 1:** Detection of failures in the network is comparable to the set cover problem

# Identification as minimum test cover

## Identification:

- Find the minimum number of sensors and their locations so that every link failure can be uniquely identified, i.e., distinguished from any other link failure

## Minimum test cover (MTC) problem:

- Unknown a fault must be classified in one of the given categories based on the outcome of the set of tests

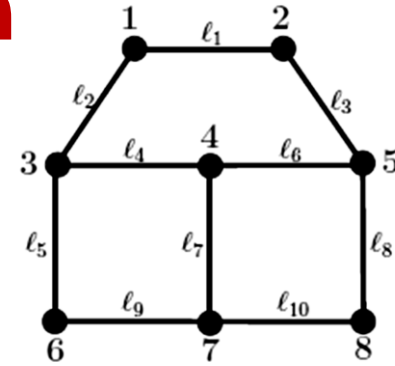
**Proposition 2:** Identification of failures in the network is comparable to the test cover problem

Pipe burst  
Sensors' states  
Location identification



Fault  
Set of tests  
Classification

# Detection and identification



## Example (cont.):

	Output	Localization-set
$S_A = \{S_1, S_7\}$	1   0	$\{l_1, l_2, l_3\}$
	0   1	$\{l_6, \dots, l_{10}\}$
	1   1	$\{l_4, l_5\}$

$$S_B = \{S_1, S_2, S_3, S_5\}$$

$$\mathcal{M} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \\ l_7 \\ l_8 \\ l_9 \\ l_{10} \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{matrix}$$

$$\mathcal{M} = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \\ \begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \\ l_7 \\ l_8 \\ l_9 \\ l_{10} \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{matrix}$$

- **All** events are detected
- Only **three** sets of events are identified

- **All** events are detected
- **All** events are uniquely identified

# Solving the MTC problem

1. **Input:** given a set of sensors and a set of events

$$\mathcal{S} = \{S_1, \dots, S_m\}, \mathcal{L} = \{\ell_1, \dots, \ell_n\}$$

2. **Transform:** MTC to MSC  $\mathcal{L}, \mathcal{C} \rightarrow \mathcal{L}^t, \mathcal{C}^t$

- obtain a new matrix  $\mathcal{M}^t(\mathcal{L}^t, \mathcal{S})$  of dimension  $\binom{n}{2} \times m$  such that  $\mathcal{M}^t(e_{ij}, k) = 1$  if sensor  $k$  detects and distinguishes between events  $\{\ell_i, \ell_j\}$ ; 0 otherwise

4. **Solve:** the counterpart MSC using greedy algorithm

1. Start with an empty set:  $S^* \leftarrow \emptyset$

2. Find the sensor that covers the most uncovered elements:

$$S_i = \arg \max_{S_i \subseteq \mathcal{S}} \left\{ f_D \left( S^* \cup S_i; \mathcal{L}^t \right) \right\}$$

3. Add to current set:  $S^* \leftarrow S^* \cup S_i$

$$f_I(S; \mathcal{L}) = f_D(S; \mathcal{L}^t)$$

4. Repeat steps 2-3 until no more elements are covered

5. **Output:**  $S^* \subseteq \mathcal{S}$

# Performance measures

## Detection score

- The number of events detected by the sensor set

$$I_D(S; \mathcal{L}) = \left| \bigcup_{C_j \in \mathcal{C}_S} C_j \right|$$

## Identification score

- The number of uniquely identified pairs of failure events

$$I_I(S; \mathcal{L}) = \left| \bigcup_{C_j^t \in \mathcal{C}_S^t} C_j^t \right| = I_D(S; \mathcal{L}^t)$$

## Localization score

- The number of unique sensors' states  $I_S$  or the number of localization sets, i.e. unique rows in  $\mathcal{M}(\mathcal{L}, S)$

$$I_L(S; \mathcal{L}) = |I_S|$$

# Application

$|\mathcal{S}| = 959$  - number of potential sensor locations

$|\mathcal{L}| = 1156$  - number of failure events

Example: Consider

$$\mathcal{S} = \{S_1, S_2, S_3\}$$

No. of detected events

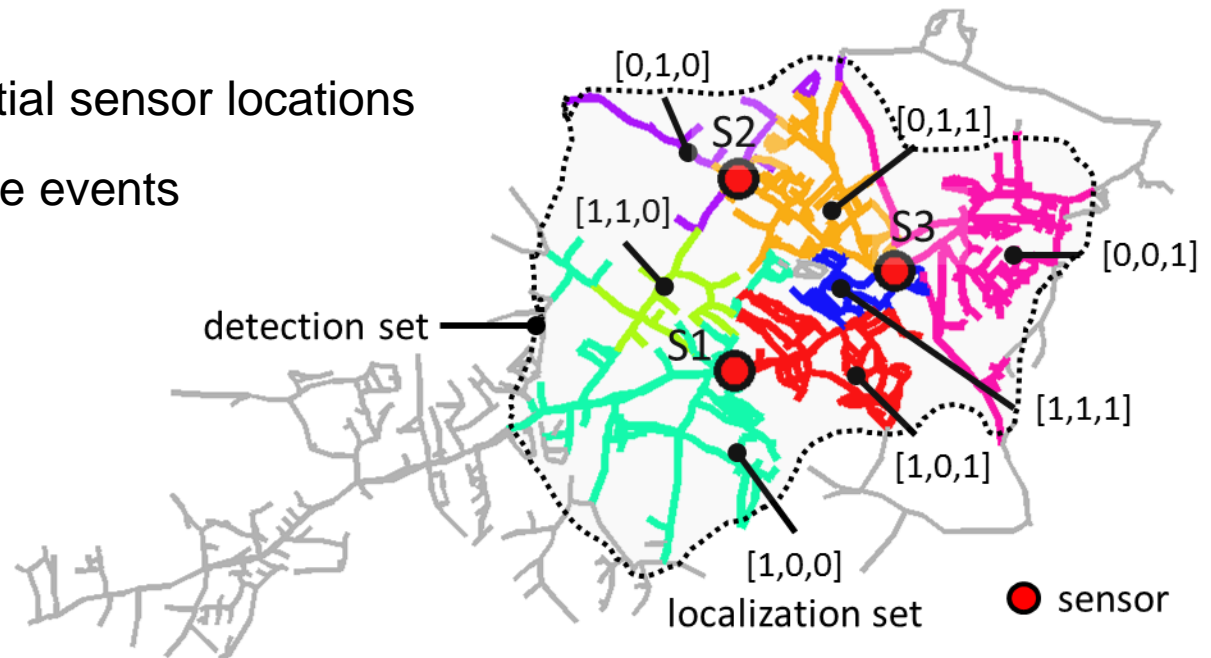
$$I_D(\mathcal{S}; \mathcal{L}) = 586$$

No. of unique pair-wise events

$$I_I(\mathcal{S}; \mathcal{L}) = 474,581 \quad \text{out of: } |\mathcal{L}^t| = \binom{1156}{2}$$

No. of localization sets

$$I_L(\mathcal{S}; \mathcal{L}) = |I_S| = 7 \quad \text{out of: } |\mathcal{L}| = 1156$$



Kentucky network

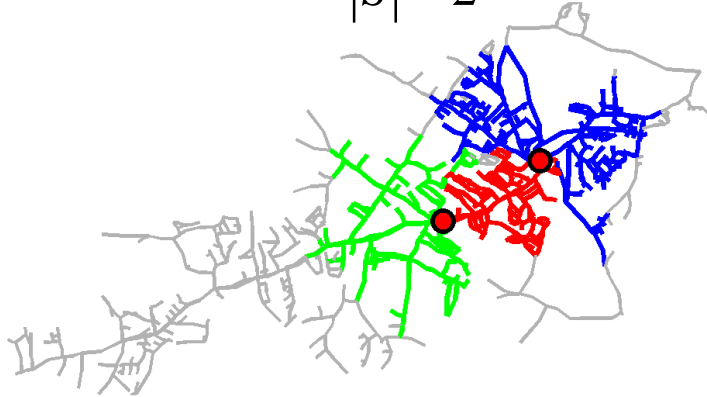
Adopted from Jolly et al 2014

- 260 km of total pipe length
- Daily supply ~ 1.5M gal/day
- 1 reservoirs; 4 storage tanks
- 959 nodes; 1156 pipes;



# Application

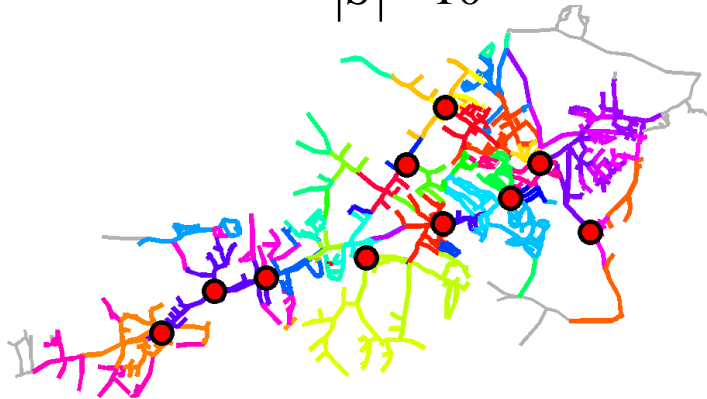
$|S| = 2$



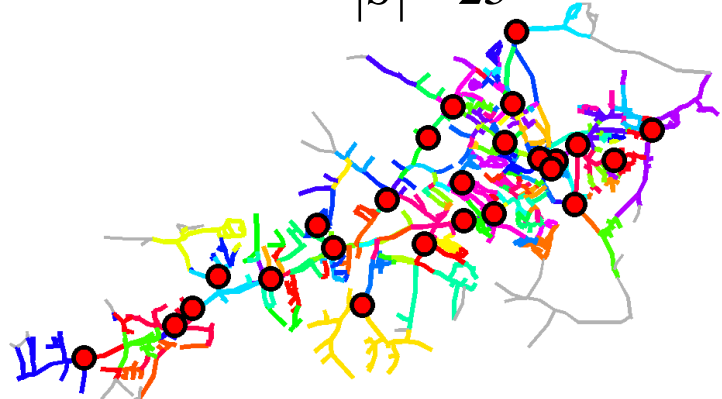
$|S| = 5$



$|S| = 10$



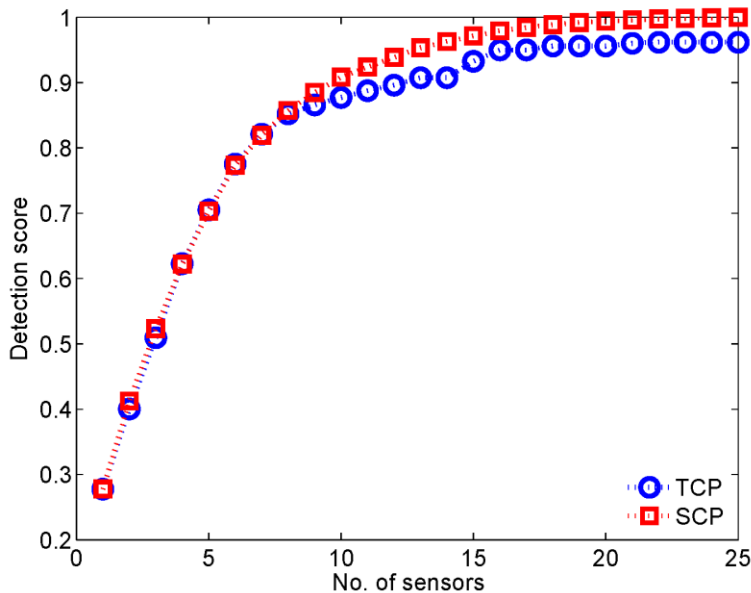
$|S| = 25$



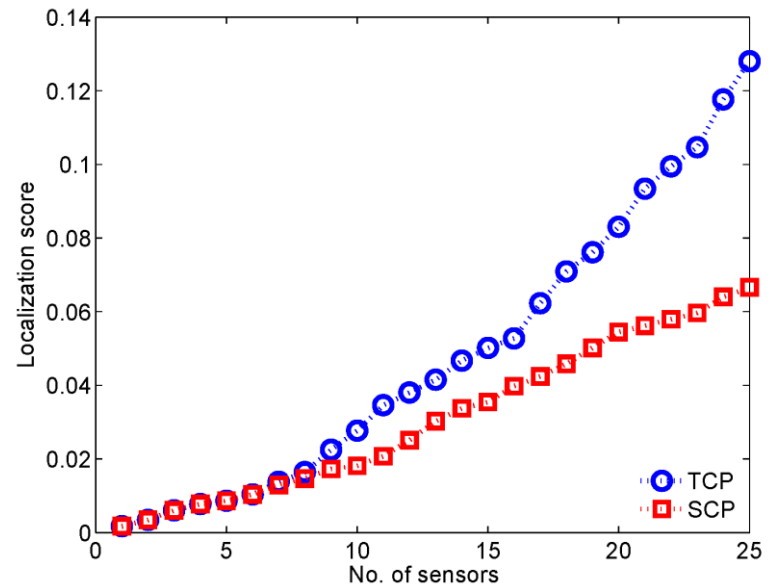
- Everything that is colored is detected
- Different colors represent unique localization sets, i.e. we can distinguish between events in different colored sets and cannot distinguish within same color set

# Detection vs. Identification

Detection score



Localization score



- **TCP** – solution obtained solving the MTC problem
- **SCP** – solution obtained solving the MSC problem

# Smart water networks

## Objective

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## Challenges

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- Cyber-physical systems interdependency

## Approach

- Strategic design of network of sensors
- Real-time data acquisition and analytics for fault diagnosis
- **Active network supply-demand control**

# Control of water networks

## Objective

- Strategic supply-demand control

## Challenges

- Nonlinear network flow
- Collective vs. individual demand shedding

## Approach

- Nonlinear network flow and demand modeling
- Convex approximation using geometric programming (GP)
- Standard convex solver (CVX + Mosek)

# Geometric programming (GP) approach

## Geometric programming

A class of structured convex optimization problems with special form objective and constraints:

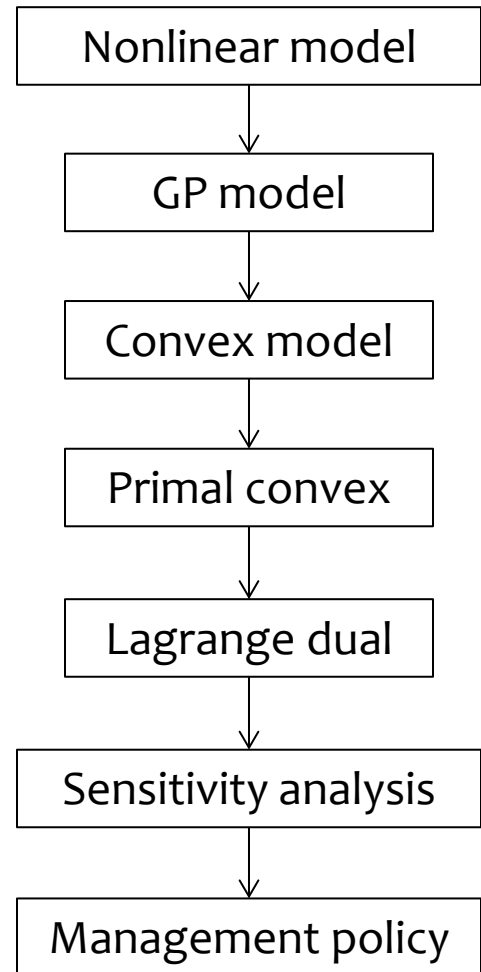
$$\begin{array}{ll} \underset{x}{\text{minimize}} & f_0(x) \\ \text{subject to} & f_i(x) < 1 \quad i = 1, \dots, m \\ & h_i(x) = 1 \quad i = 1, \dots, l \\ & x > 0 \end{array}$$

Where:

monomials:  $h(x) = cx_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$

posynomials:  $f(x) = \sum_{j=1}^k c_j x_1^{a_{1j}} x_2^{a_{2j}} \cdots x_n^{a_{nj}}$

## Approach



# Network flow

- Flow conservation at nodes:  $\forall i \in N$

$k$  - link  
 $i$  - start node  
 $j$  - end node

$$\sum_{k \in E_{out}(i)} q_k + d_i = \sum_{k \in E_{in}(i)} q_k \quad \longrightarrow$$

$$\sum_{k \in E_{out}(i)} q_k + s_i d_i \leq \sum_{k \in E_{in}(i)} q_k \quad (1)$$

demand shedding

inequality

- Energy conservation over links:  $\forall k \in E$

$R$  - resistance  
 $\alpha$  - power

$$R_k q_k^\alpha + H_j = H_i \quad \longrightarrow$$

$$R_k q_k^\alpha + H_j \leq H_i \quad (2)$$

inequality

- Operating pumps:  $H_j = \beta_k H_i$  (3)  $1 \leq \beta_k \leq \bar{\beta}_k$  (4) - adding head

- Control valves:  $H_j = \gamma_k H_i$  (5)  $0 \leq \gamma_k \leq 1$  (6) - decreasing head

- Operating range:  $\underline{H}_i \leq H_i \leq \bar{H}_i$  (7)

$q$  - flow  
 $H$  - head

# Network supply-demand control

minimize  
 $H, q, \beta, \gamma, s$

$$\sum_{k \in E_{pump}} c_{1k} q_k \beta_k^{m_{1k}} + \sum_{i \in N_{demand}} c_{4i} d_i s_i^{-m_{4i}} + \sum_{i \in E_{valve}} c_{2i} \gamma_i^{-m_{2i}} + \sum_{i \in N} c_{3i} H_i^{-m_{3i}}$$

subject to

eq. (1) – (7)

$$q_k \beta_k \leq \overline{Power}$$

$$\underline{s}_i \leq s_i \leq 1$$

Energy cost

Demand shedding penalty

Relaxation penalty

available resources

demand shedding

$s_i = 1$  zero demand shedding;  
 $s_i = 0$  full demand shedding;

- This problem formulation has a special structure conforming with geometric programming modeling constraints
- Current formulation is suitable for tree network topology

# Application

## Controls:

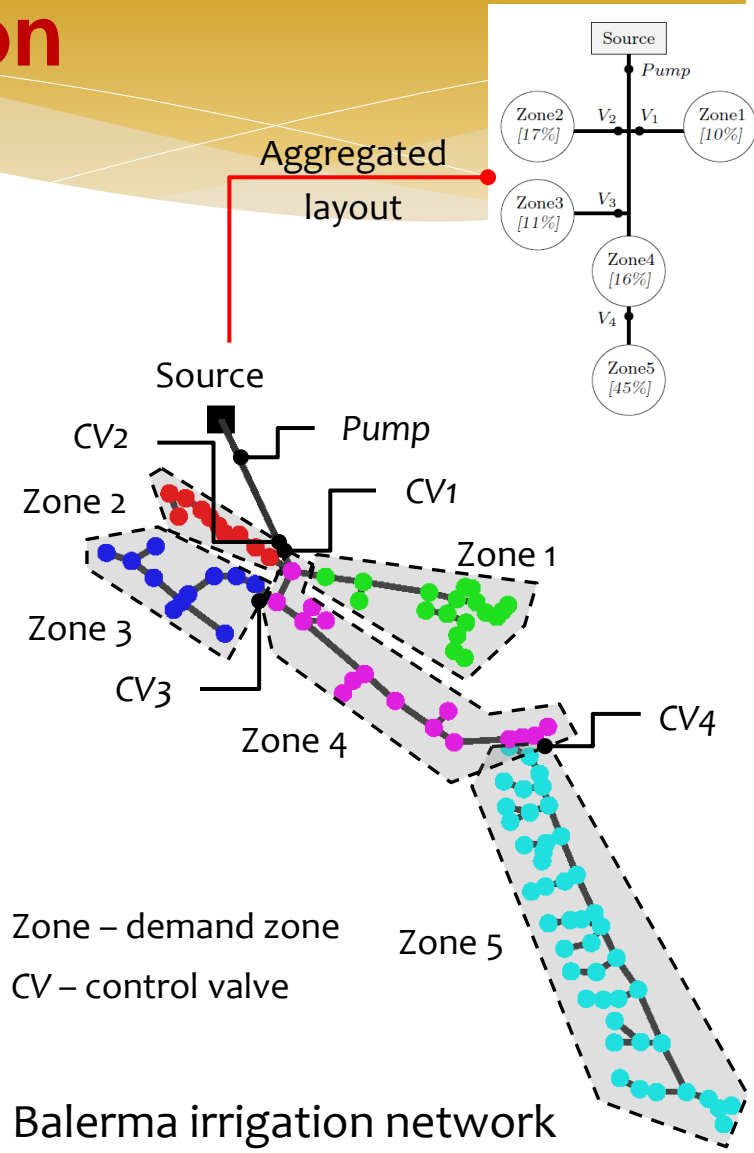
- Pumps – adding power to the system
- Control valves – decreasing pressure
- Demand shedding for each demand zone

## Costs:

- Energy cost for operating the pump
- Penalty cost for demand shedding
- Penalty cost for relaxing equality constraints

## Constraints:

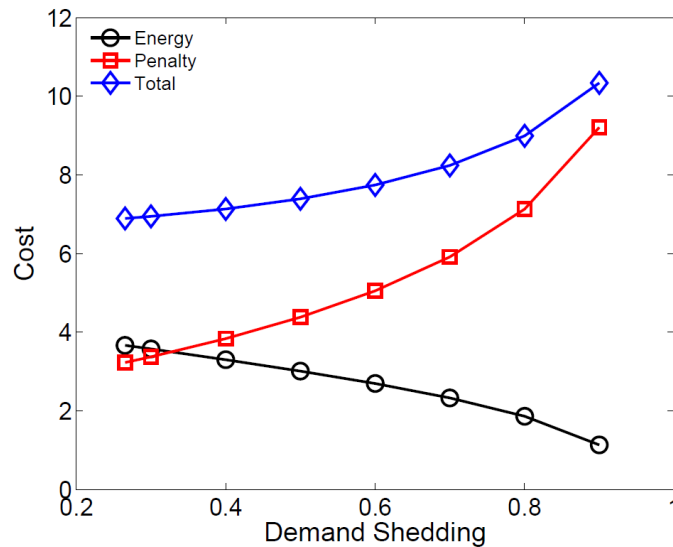
- Physical constraints
- Maximum available resources
- Maximum allowed demand shedding





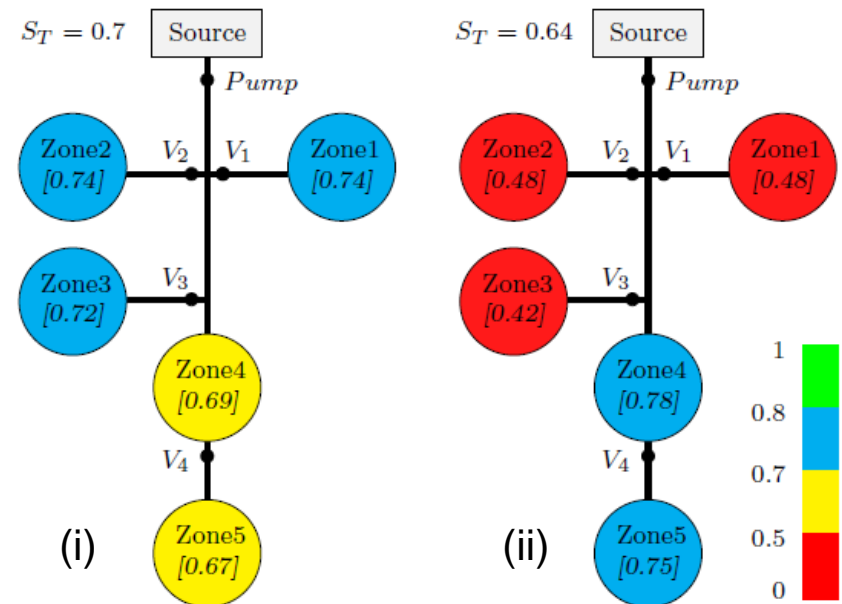
# Controlled demand shedding

## Energy cost and supply deficiency penalty



- Trade-off between cost of energy and water resources and penalty for supply shortage

## Individual demand shedding



- (i) Equal penalty – downstream consumers suffer more
- (ii) Mixed penalties – variable allocation

# Future work

## Sensor placement

- Better approximation of the physical disturbance model
- Robustness to sensor failures
- Heterogeneous sensors

## Network control

- Extension to looped topologies
- Supply-demand management for different operational regimes
- Demand response through water pricing schemes



**Thank you!**