# A Supervisory Control Approach to Dynamic Cyber-Security 

Demosthenis Teneketzis joint work with Mohammad Rasouli and Erik Miehling<br>Dept. of Electrical Engineering \& Computer Sciences, University of Michigan, MI, USA



VANDERBILT
UNIVERSITY

## Outline

- Introduction/Motivation
- Literature review
- Contribution
- Model
- Problem formulation
- Results
- Summary/conclusion


## Outline

Introduction／Motivation

## Model

The Defender＇s Problem（ $P_{D}$ ）

Defender Optimal Policy

## Conclusion

GFORCES

## Motivation

- Increasing importance of safety of many modern technological systems
- computer networks
- the internet
- mobile networks
- power grids
- implantable medical devices
- ...

- Strengthen resiliency of systems against attacks, intentional and unintentional misuses, and inadvertent failures.


## Motivation

- Key issues in cyber-security systems
- Progressive attacks
- Dynamic/adaptive defense
- Imperfect knowledge (for attacker and/or defender) of system status
- Non-strategic vs. strategic attacker (control vs. game theory)


## Literature Review

- Static models: perfect vs. imperfect information
- One-agent-model: resource allocation for infrastructure protection [Bier et al 2007, Bohme-Felegyhazi 2010, Chen-Jamil 2006, Bloem et al 2007, Bloem et al 2009, Chen-Jamil 2006, Mastroleon 2009 and many others]
- Based on game theory: [Bier et al 2007, Chen-Jamil 2006, Roy et al. 2010, Schwartz 2013 and many others].
- Dynamic models: perfect vs. imperfect information
- Based on control theory: [Khouzani et al. 2012, Ligatti et al. 2005, Ligatti 2009, Rowe et al. 2012, Schneider 2000 and many others]
- Based on game theory: [Khouzani 2012, Roy et al. 2010, Van Dijk et al. 2013, Yin et al. 2010 and many others].


## Literature Review

- Static models: perfect vs. imperfect information
- One-agent-model: resource allocation for infrastructure protection [Bier et al 2007, Bohme-Felegyhazi 2010, Chen-Jamil 2006, Bloem et al 2007, Bloem et al 2009, Chen-Jamil 2006, Mastroleon 2009 and many others]
- Based on game theory: [Bier et al 2007, Chen-Jamil 2006, Roy et al. 2010, Schwartz 2013 and many others].
- Dynamic models: perfect vs. imperfect information
- Based on control theory: [Khouzani et al. 2012, Ligatti et al. 2005, Ligatti 2009, Rowe et al. 2012, Schneider 2000 and many others]
- Based on game theory: [Khouzani 2012, Roy et al. 2010, Van Dijk et al. 2013, Yin et al. 2010 and many others].


## Contribution

A supervisory control approach for cyber-security from the point of view of the defender with

- progressive attacks,
- defender's imperfect knowledge of the state of the system,
- dynamic defense,
- conservative approach to security,
- quantification of the cost incurred at every possible state of the system and every possible defender action,


## Contribution

A supervisory control approach for cyber-security from the point of view of the defender with

- progressive attacks,
- defender's imperfect knowledge of the state of the system,
- dynamic defense,
- conservative approach to security,
- quantification of the cost incurred at every possible state of the system and every possible defender action,
- quantification of the performance of various defender policies,
- determination of the defender's optimal control policy (within a restricted set of policies) for a min-max performance criterion.


## Contribution

A supervisory control approach for cyber-security from the point of view of the defender with

- progressive attacks,
- defender's imperfect knowledge of the state of the system,
- dynamic defense,
- conservative approach to security,
- quantification of the cost incurred at every possible state of the system and every possible defender action,
- quantification of the performance of various defender policies,
- determination of the defender's ontimal control nolicy (within a restricted set of policies) for a min-max performance criterion.


## Contribution

A supervisory control approach for cyber-security from the point of view of the defender with

- progressive attacks,
- defender's imperfect knowledge of the state of the system,
- dynamic defense,
- conservative approach to security,
- quantification of the cost incurred at every possible state of the system and every possible defender action,
that achieves
- quantification of the performance of various defender policies,
- determination of the defender's optimal control policy (within a restricted set of policies) for a min-max performance criterion.


## Outline

## Introduction／Motivation

Model

The Defender＇s Problem（ $P_{D}$ ）

## Defender Optimal Policy

## Conclusion

## G FORCES

## Model: Network Structure

$$
\begin{aligned}
& \square s_{i}=\text { Normal }
\end{aligned}
$$

$$
\begin{aligned}
& s_{i}=\text { Remote compromised }
\end{aligned}
$$



Routing Layer

Computer Layer

- Possible states of each computer $i$ : Normal ( $N$ ), Compromised (R), Fully Compromised (W), Remote Compromised (F).


## Model: System State

- System of $K$ computers, $\mathcal{N}=\{1,2, \ldots, K\}$
- state of the system $Z=\left\{s_{1}, s_{2}, \ldots, s_{K}\right\}$
- $s_{i}$ state of computer $i$
- $s_{i} \in\{N, R, W, F\}$
- $N=$ Normal
- $R=$ compromised
- $W=$ Fully compromised
- $F=$ Remote compromised


## Model

- One decision-maker
- Defender $\Rightarrow$ controller/decision maker
- Attacker $\Rightarrow$ nature
- Non-probabilistic dynamics
- Imperfect observation for defender


## Model: Timing

- Interaction rules between controller and nature



## Model: Defender Costs

- Cost of state $Z \Rightarrow C(Z)$
- Cost of controllable event $d \Rightarrow \hat{C}(d), d \in \mathcal{D}$
- Time horizon $\Rightarrow$ finite or infinite


## Model: Defender and Nature Actions

## Defender's Actions $\quad \mathcal{D}=\left\{N^{d},\left\{E^{i}\right\}_{i \in \mathcal{N}},\left\{R^{i}\right\}_{i \in \mathcal{N}}\right\}$



Nature's Events $\mathcal{A}=\left\{N^{a},\left\{P_{n}^{i}\right\}_{i \in \mathcal{N}, n \in \mathcal{B}},\left\{H^{i j}\right\}_{i, j \in \mathcal{N}}\right\}$

## Model: Discrete Event Systems

- All events/transitions $\mathcal{E}=\mathcal{A} \cup \mathcal{D}$.
- Controllable (by the defender) events: $\mathcal{D}$
- Observable (by the defender) events: $\mathcal{D} \cup\left\{\left\{H^{i j}\right\}_{i, j \in \mathcal{N}}\right\}$
- Defender's observation of nature's events:

$$
\mathcal{A}^{\prime}=\left\{X,\left\{H^{i j}\right\}_{i, j \in \mathcal{N}}\right\} \text { where } X=\left\{N^{a},\left\{P_{n}^{i}\right\}_{i \in \mathcal{N}, n=B_{1}, B_{2}, B_{3}}\right\}
$$

- Events admissible/allowable at each state
- $\mathcal{D} \cup\left\{N^{a}\right\}$ are admissible from every state
- Event $\left\{H^{i j}\right\}_{i, j \in \mathcal{N}}$ is only admissible when $s_{i}=F$ and $s_{j}=\{N, R, W\}$
- Probe $P_{B_{1}}^{i}, P_{B_{2}}^{i}$, and $P_{B_{3}}^{i}$ only admissible from $s_{i}=N, s_{i}=R$, and $s_{i}=W$, respectively.


## Model: System Automaton

System state before nature's event


System state before defender's action

## Outline

## Introduction／Motivation

## Model

The Defender＇s Problem（ $P_{D}$ ）

## Defender Optimal Policy

## Conclusion

G FORCES

## Defender's Optimal Policy

- Optimization problem

$$
\min _{g \in \mathcal{G}} \max _{\left\{Z_{t}^{g} \in \mathcal{Z}, t \in \mathcal{T}\right\}}\left\{\sum_{t \in \mathcal{T}} \beta^{t}\left[C_{Z_{t}^{g}}+\hat{C}\left(d_{t}\right)\right]\right\}
$$

subject to model dynamics

- Information state at $t^{+} \Rightarrow$ All system trajectories up to $(t-1)^{++}$ consistent with the history of observations and actions
- Using this information state one can in principle write the dynamic program for $P_{D}$.
- Computationally intractable dynamic program
- Restricting attention to defense policies with specific structure


## The Defender's Problem $\left(P_{D}^{\prime}\right)$

- Defender's observer: the possible states that the network can be in at time $t$ from the defender's perspective (defender has imperfect information).
- Observer dynamics
- $S_{t}$ : observer's state at $t$
- $d_{t}$ : defender's action at $t^{+}$
- $a_{t}^{\prime}$ : nature's move at $t^{++}$

$$
S_{t+1}=f\left(S_{t}, d_{t}, a_{t}^{\prime}\right)
$$

- Problem $\left(P_{D}^{\prime}\right)$

$$
\begin{equation*}
\min _{g \in \mathcal{G}^{\prime}} \max _{Z_{t}^{g} \in S_{t}}\left\{\sum_{t \in \mathcal{T}} \beta^{t}\left[C_{Z_{t}^{g}}+\hat{C}\left(d_{t}\right)\right]\right\} \tag{D}
\end{equation*}
$$

subject to model dynamics

$$
\begin{aligned}
& d_{t}=g_{t}\left(S_{t}\right), t \in \mathcal{T} \\
& S_{t+1}=f\left(S_{t}, d_{t}, a_{t}^{\prime}\right), t \in \mathcal{T}
\end{aligned}
$$

$\mathcal{G}^{\prime}:=\left\{g \mid g:=\left\{g_{t}, t \in \mathcal{T}\right\}, g_{t}: \mathcal{S} \rightarrow \mathcal{D}, d_{t}=g_{t}\left(S_{t}\right) \quad\right.$ for all $\left.\quad t \in \mathcal{T}\right\}$.

## Solution to $\left(P_{D}^{\prime}\right)$

- Defender's dynamic program

$$
\begin{equation*}
V(S)=\min _{d \in \mathcal{D}} \max _{Z \in S}\left[C_{Z}+\hat{C}(d)+\max _{S^{\prime} \in \mathcal{Q}(S, d, Z)} \beta V\left(S^{\prime}\right)\right] . \tag{1}
\end{equation*}
$$

- $\mathcal{Q}(S, d, Z)$ is the set of observer states that can be reached by $S$ when the defender's action is $d$ and the true system state is $Z$
- Right-hand side of Eq. 1 is a contraction mapping.
- Use value iteration to solve $P_{D}^{\prime}$.


## Outline

```
Introduction／Motivation
```


## Model

```
The Defender＇s Problem（ \(P_{D}\) ）
```

Defender Optimal Policy

## Conclusion

GFORCES

## Numerical Solution to $P_{D}^{\prime}$

- Build observer automaton from the system automaton (details in appendix)
- Number of observer states grows exponentially with number of computers
- Two computer network: 87 states and 1207 transitions
- Example: $\{R F, W F, F F\},\{R N, W N, R R, W R, R W, W W, R F\}$.
- Three computer network: 1423 states and 65602 transitions
- Example: $\{R F W, W F W, F F N\},\{R N W, W N N, R R W\}$.


## Numerical Sensitivity Analysis for Two Computers



Figure: Optimal defender policy (Reimage, Sense, Null) with increasing cost of Reimage.

## Numerical Sensitivity Analysis for Two Computers



Results: Threshold property switching from Reimage to other defense actions

- Switching from Reimage to Sense or Null actions happens at different costs
- Duality of control (Reimage) and estimation (Sense)
- No Sense when there is no Reimage in the policy


## Outline

## Introduction/Motivation

## Model

The Defender's Problem ( $P_{D}$ )

## Defender Optimal Policy

Conclusion

## G FORCES

## Summary

- Supervisory control approach to dynamic cyber-security from defender's perspective with imperfect information, progressive attacks, and min-max performance criterion
- Dynamic programming with numerical results for determining defender's optimal min-max actions at each instant of time
- Threshold behavior with varying cost of actions/states


## Future Research

- Address exponentially growing number of states and transitions with the number of computers
- qualitative properties of optimal defender strategies to accommodate large networks
- hierarchical decomposition
- approximate dynamic programming methods
- Game theoretic formulation
- dynamic game with asymmetric information


## Thank you!!

GFORCES

## Appendix: observer automaton

Construction of observer automaton based on system automaton using UMDES-LIB software library available on https://www.eecs.umich.edu/umdes/toolboxes.html.


