Optimization of Large-Scale Interdependent Systems: from Supermarket Refrigerators to Electrical Grids

Insoon Yang

Department of Electrical Engineering and Computer Sciences
UC Berkeley

Trends in power and energy systems

1. Renewable sources: wind/solar generation

California: 33% by 2020

[wikipedia.org]

Trends in power and energy systems

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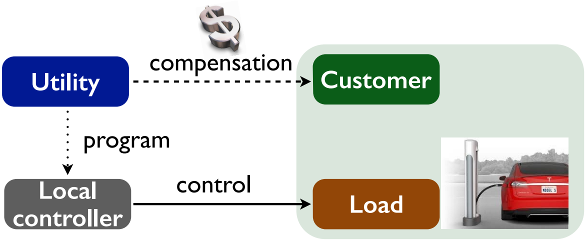
2. **Demand-side resources: air conditioners, EVs**

   Electric Vehicle-Grid Integration (VGI) [San Diego Gas & Electric]



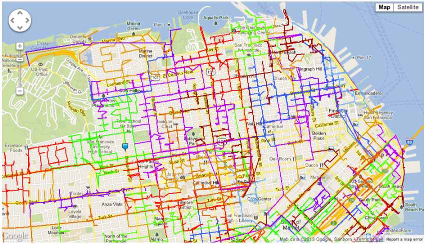
[teslamotors.com]

Trends in power and energy systems

1. **Renewable sources: wind/solar generation**
   California: 33% by 2020

2. **Demand-side resources: air conditioners, EVs**
   Electric Vehicle-Grid Integration (VGI) [San Diego Gas & Electric]

3. **Automation**
   Distribution system automation [PG&E, CIEE]



SF distribution networks [pge.com]

micro synchrophasor (sensor):
distribution grid visibility [pqubepmu.com]

Challenge: system interdependency

Generators

Power System

Combinatorial

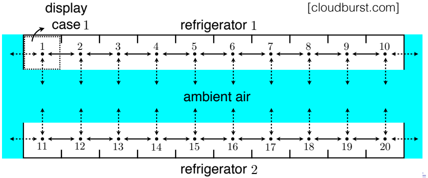
Demand

Large-scale interconnected loads

Supermarket refrigeration systems consume 7% of total commercial energy consumption in the US

DOE, 2014

[cloudburst.com]



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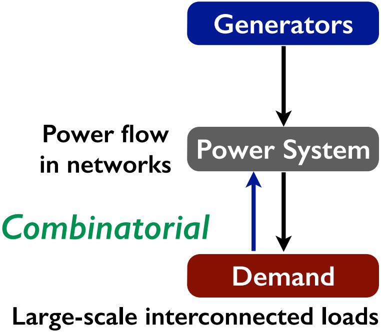

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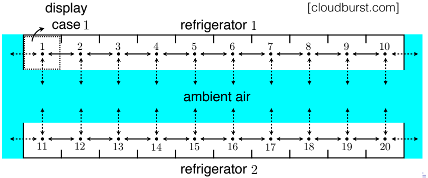


Challenge: system interdependency

Generators

Power flow in networks — **Power System**

Combinatorial

Demand

Large-scale interconnected loads

[cloudburst.com]

Scalable optimization tools for interdependent systems

Towards control mechanisms for sustainable power systems

Goal:

- **real-time optimization of interdependent systems (e.g., loads, power networks)**

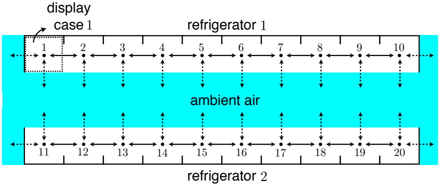
Challenges:

- **large-scale discrete control**
- **interdependent dynamics**
- **real-time requirements**

Engineering objectives:
Performance (optimality) + **Scalability (computation)**

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

ON/OFF control of supermarket refrigeration systems



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Good demand response potential: Pacific Gas & Electric - Safeway

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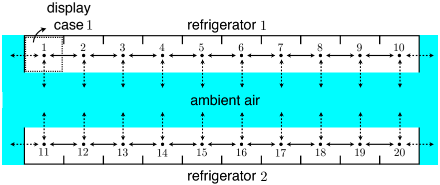

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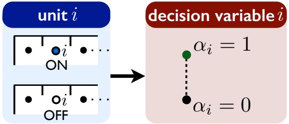

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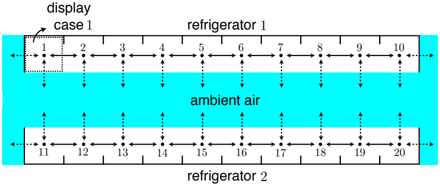

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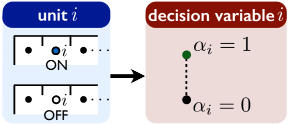

Temperature hybrid dynamics
(combinatorial dynamical system)

$$\dot{x}_i = \begin{cases} -A_{ii}(x_i - \theta_i) - \sum_{j=1}^{n} A_{ij}(x_i - x_j) - b_i & \text{if } \alpha_i = 1 \\ -A_{ii}(x_i - \theta_i) - \sum_{j=1}^{n} A_{ij}(x_i - x_j) & \text{if } \alpha_i = 0 \end{cases}$$

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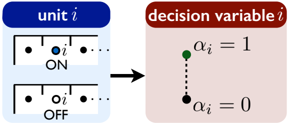
ON/OFF control of supermarket refrigeration systems

Demand Response Problem

One-step look ahead optimization at k (receding-horizon)



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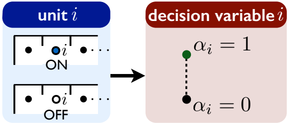

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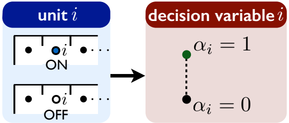
ON/OFF control of supermarket refrigeration systems

Demand Response Problem

One-step look ahead optimization at k (receding-horizon)

$$\max_{\alpha \in \{0,1\}^m} \int_{(k-1)\Delta t}^{k\Delta t} \text{Freshness}(x(t))dt$$

$$\text{subject to} \quad \text{lower bound} \le \underbrace{\text{\# of ON units}}_{\sum_{i=1}^{n} \alpha_i} \le \text{upper bound}$$



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Towards scalability: linear approximation

Optimization of CDS

$$\max_{\alpha \in \{0,1\}^m} \quad J(\alpha) := \int_0^T r(x^\alpha)dt + q(x^\alpha(T))$$

$$\text{subject to} \quad \mathbf{A}\alpha \leq \mathbf{b}$$

$$\dot{x}^\alpha(t) = f(x^\alpha(t), \alpha)$$

combinatorial dynamical systems (CDS)

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$\bar{\alpha}$: linearization pt

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$\bar{\alpha}$: linearization pt $\qquad J(\alpha) \approx J(\bar{\alpha}) + DJ(\bar{\alpha})^\top(\alpha - \bar{\alpha})$

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$$\max_{\alpha \in \{0,1\}^m} \quad DJ(\bar{\alpha})^\top \alpha$$

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Q) How to define $DJ(\bar{\alpha})$?

Towards scalability: linear approximation

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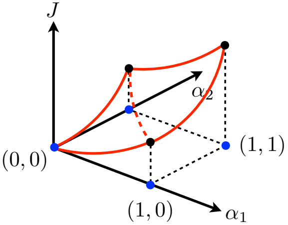

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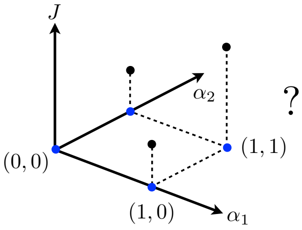
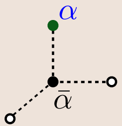


Q) How to define $DJ(\bar{\alpha})$?

Nonstandard derivative: variation in vector fields

$\{0,1\}^m$ discrete space



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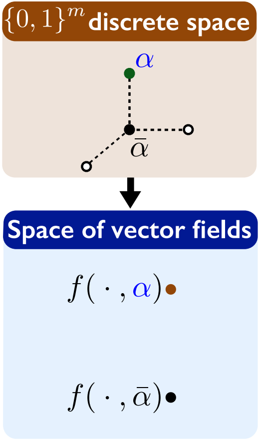

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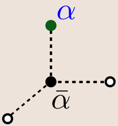


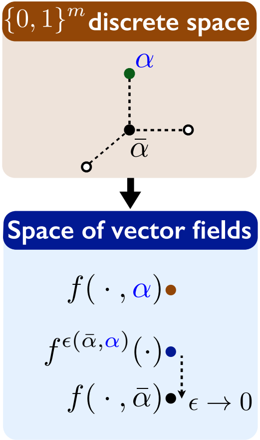
Space of vector fields

$f(\,\cdot\,, \alpha)$

$f^{\epsilon(\bar{\alpha}, \alpha)}(\cdot)$

$f(\,\cdot\,, \bar{\alpha})$

Nonstandard derivative: variation in vector fields



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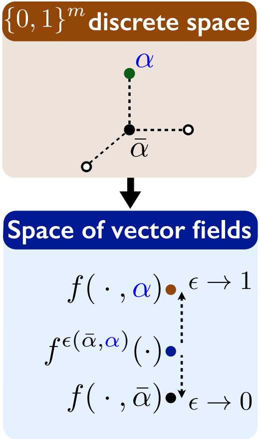

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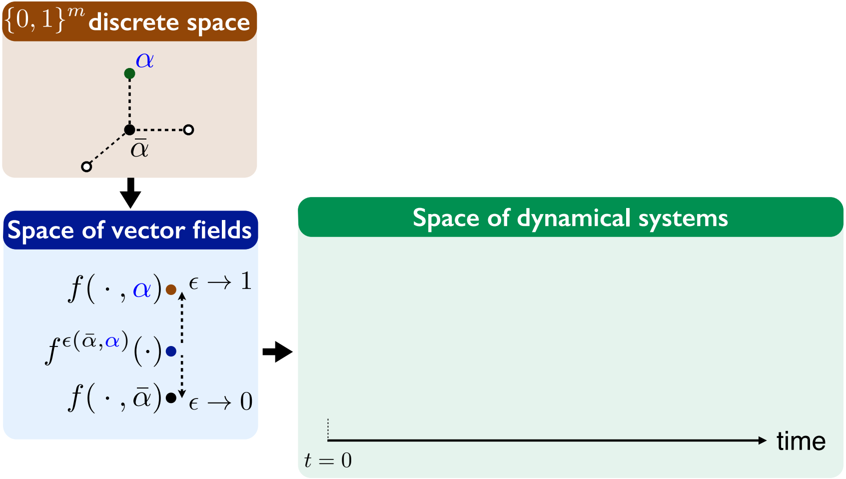

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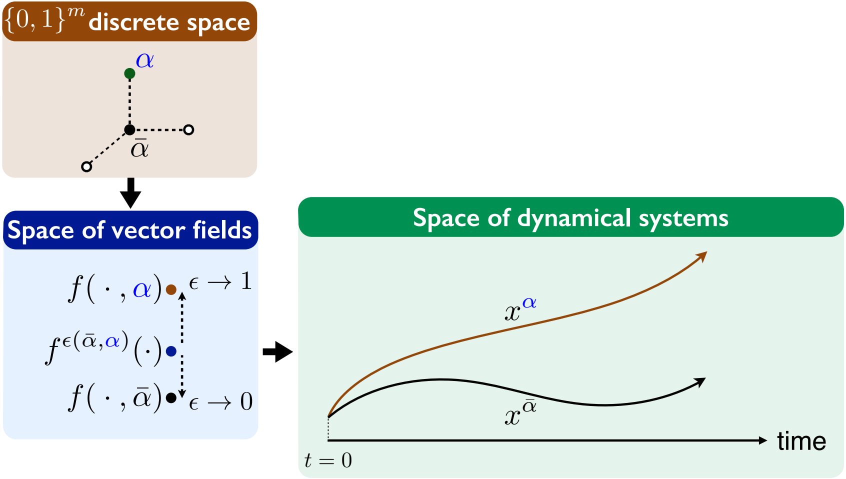

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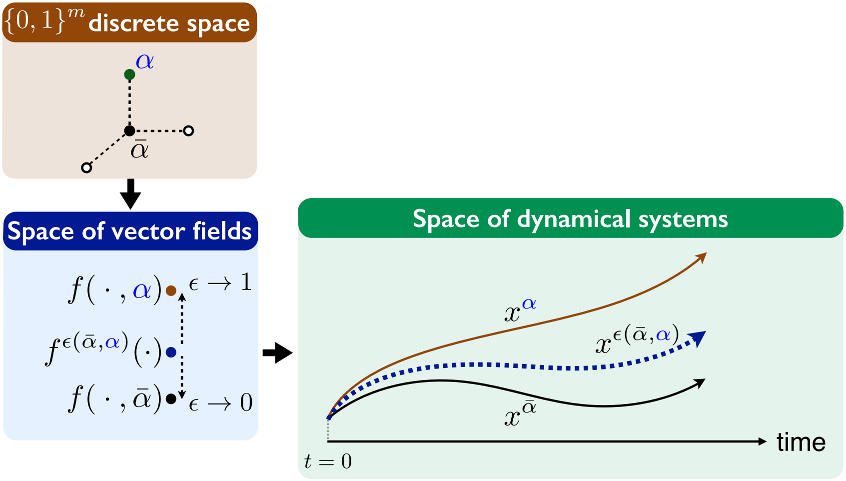

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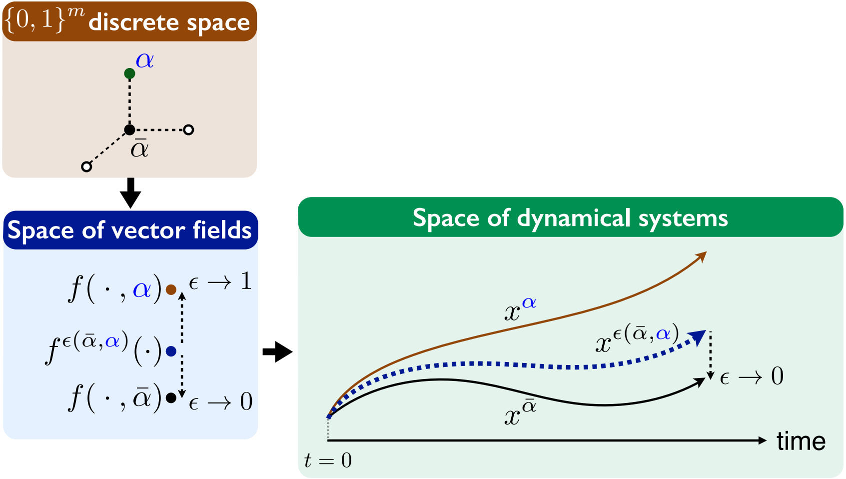

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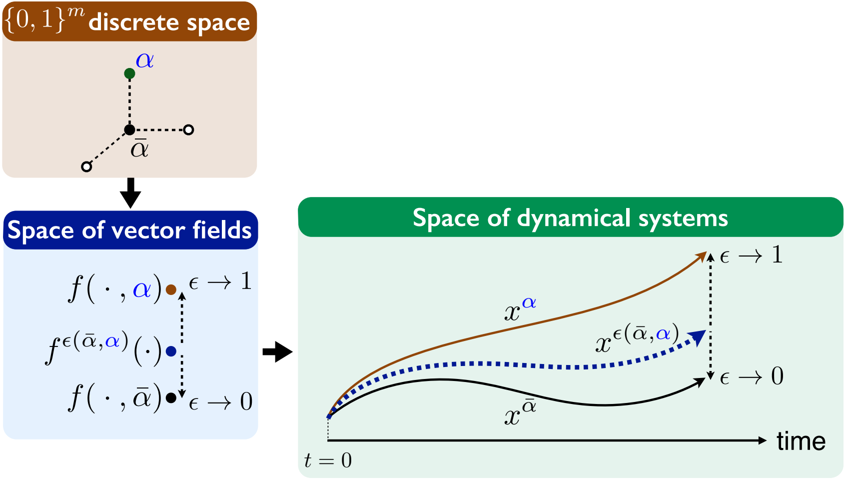

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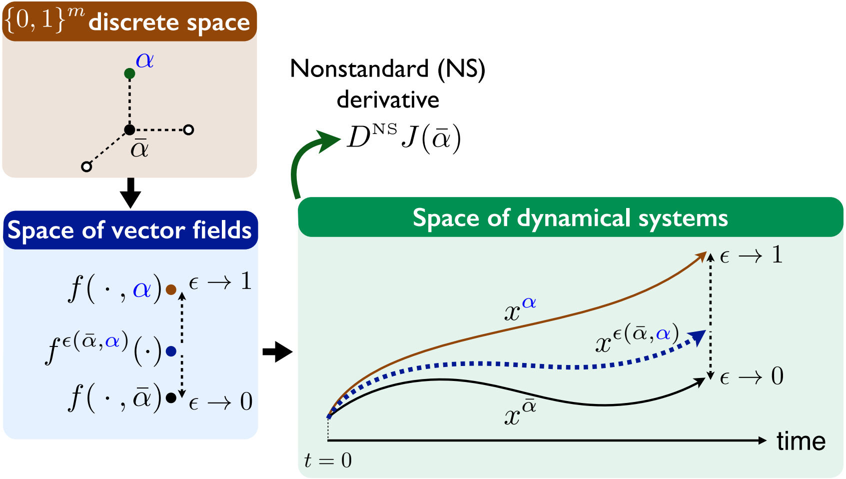

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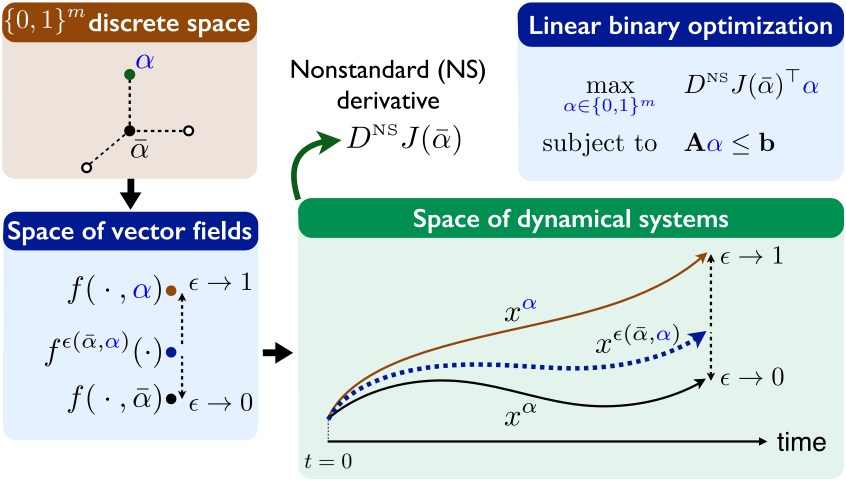

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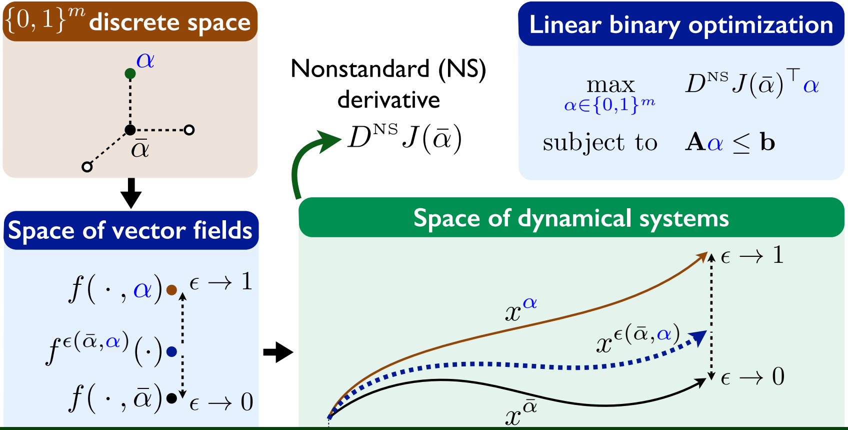


Contribution I.

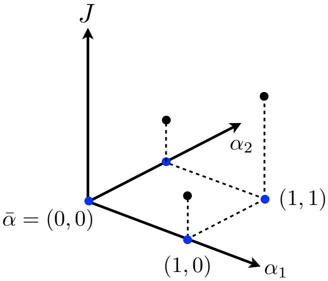
Scalability: linear approximation

Efficiency: no need to repeatedly solve ODEs

[Yang, Burden, Sastry, Tomlin, CDC, 2013]

[Yang, Burden, Rajagopal, Sastry, Tomlin, IEEE Trans. Automatic Control, conditionally accepted (arXiv:1409.7861)]

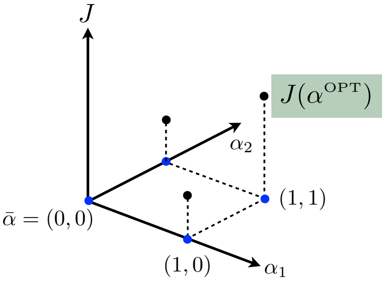
Suboptimality bound



α^{OPT}: optimal solution

α^* : approximate solution

Suboptimality bound



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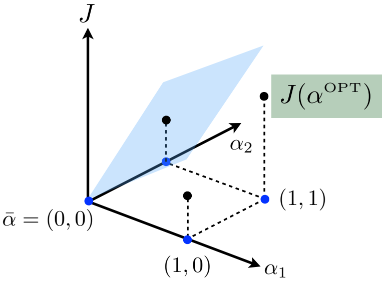

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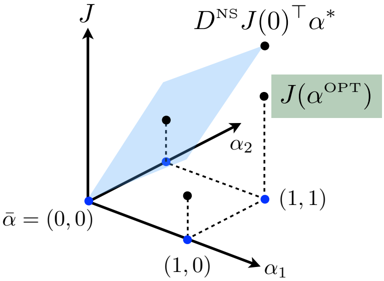

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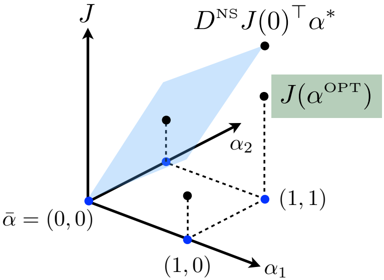

α^{OPT}	: optimal solution
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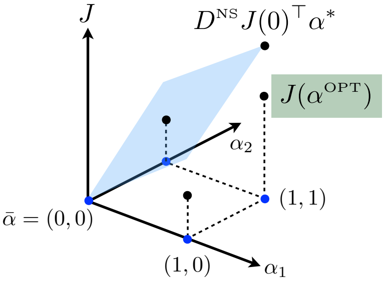
If $J(\alpha) - J(\bar{\alpha}) \leq D^{\mathrm{NS}} J(\bar{\alpha})^\top (\alpha - \bar{\alpha}) \quad \forall \alpha \in \{0,1\}^m$,
then the following suboptimality bound holds:

$$\rho J(\alpha^{\mathrm{OPT}}) \leq J(\alpha^*), \quad \rho := \frac{J(\alpha^*)}{D^{\mathrm{NS}} J(\bar{\alpha})^\top (\alpha^* - \bar{\alpha})}.$$

Suboptimality bound



$\hat{J}$: reformulated problem

$$D^{\mathrm{NS}}J \equiv D^{\mathrm{S}}\hat{J}$$

$$J|_{\{0,1\}^m} \equiv \hat{J}|_{\{0,1\}^m}$$

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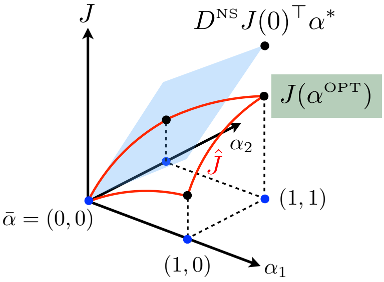

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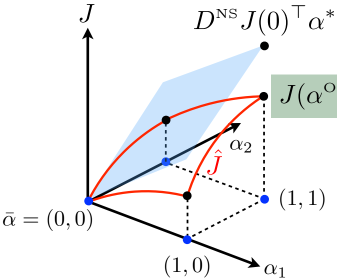

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[Yang, Burden, Rajagopal, Sastry, Tomlin, IEEE Trans. Automatic Control, conditionally accepted (arXiv:1409.7861)]

Theorem (Suboptimality bound)

If the reformulated problem is concave,
then the following suboptimality bound holds:

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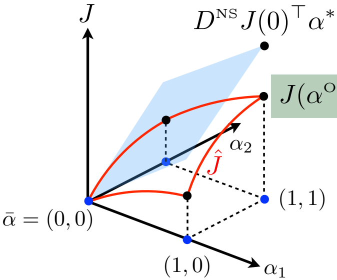
Suboptimality bound



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Contribution IV.
Performance guarantee: suboptimality bound

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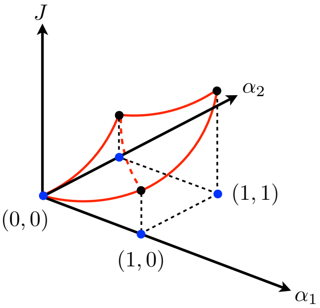
$$\rho J(\alpha^{\mathrm{OPT}}) \leq J(\alpha^*), \quad \rho := \frac{J(\alpha^*)}{D^{\mathrm{NS}} J(\bar{\alpha})^\top (\alpha^* - \bar{\alpha})}.$$

Example



$$x^\alpha = \alpha_1^2 + \alpha_2^2$$

$$J(\alpha) = x^\alpha - \frac{\alpha_1^2 + \alpha_2^2}{2}$$

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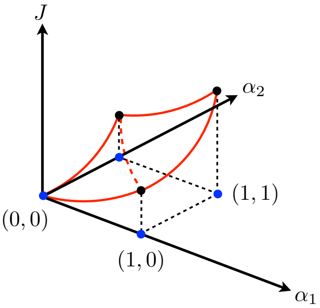

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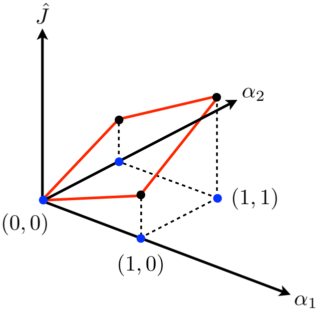
Reformulated



FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

ON/OFF control of supermarket refrigeration systems

Computation time (2.3GHz Intel Core i7, 16GB RAM, MATLAB):

$(m = 20)$ **Proposed:** 0.015s, **Greedy:** 0.57s, **Exhaustive search:** 3112s

$(m = 1000)$ **Proposed:** 0.81s

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90+% of oracle's performance

65+% suboptimality bound

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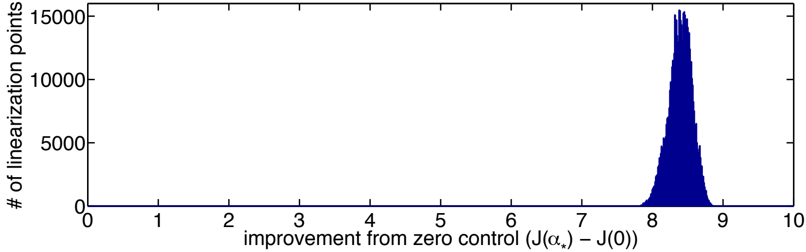
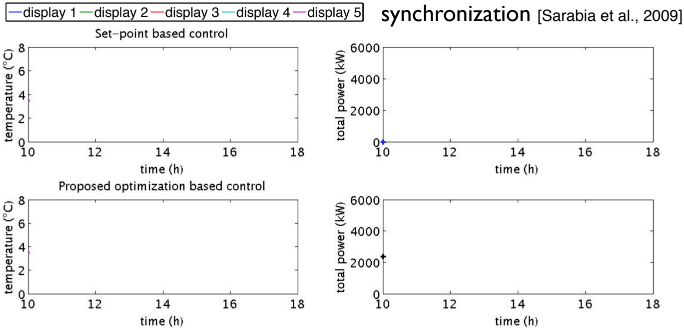
Robustness with respect to the linearization point



Energy efficiency and peak load reduction

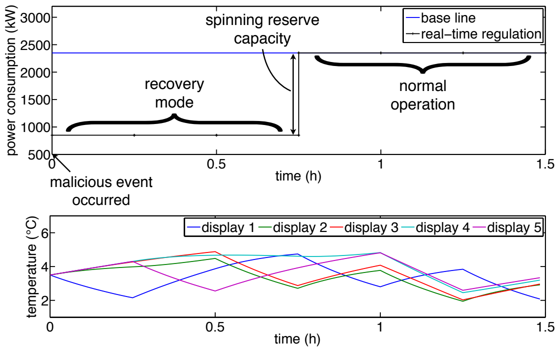
(per refrigerator)	energy	peak demand charge	reserve	arbitrage
Set-point based	143MWh/yr	$720/yr		
Proposed	137MWh/yr	$338/yr		

synchronization [Sarabia et al., 2009]



Spinning reserve services to the electrical grids

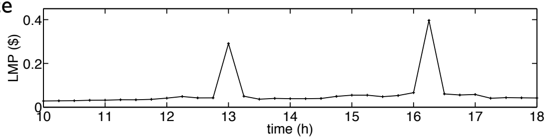
(per refrigerator)	energy	peak demand charge	reserve	arbitrage
Set-point based	143MWh/yr	$720/yr	N/A	
Proposed	137MWh/yr	$338/yr	15kW	

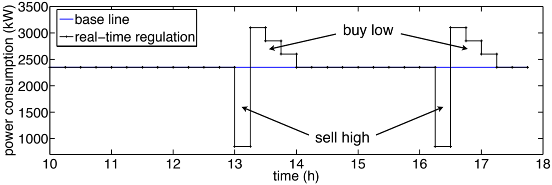


Energy arbitrage in wholesale electricity markets

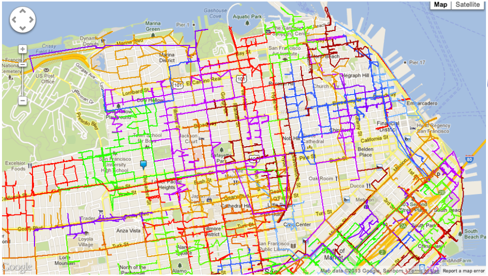
(per refrigerator)	energy	peak demand charge	reserve	arbitrage
Set-point based	143MWh/yr	$720/yr	N/A	N/A
Proposed	137MWh/yr	$338/yr	15kW	$3201/yr

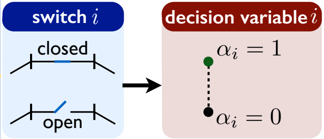
wholesale price



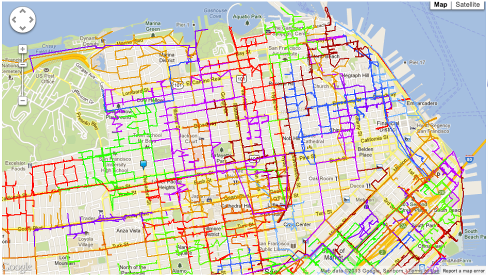


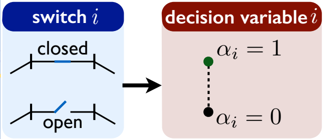
Power network topology optimization





Power network topology optimization





Power losses	33 nodes	94 nodes	118 nodes	880 nodes
Initial	203kW	532kW	1297kW	895kW
lower bound	123kW	463kW	820kW	463kW
proposed	161kW	471kW	898kW	484kW
saving	$37K/yr	$53K/yr	$350K/yr	$360K/yr

[Baran, Wu, 1989] [Chiou, Chung, Su, 2005] [Zhang, Fu, Zhang, 2007]
[REDS: repository of distribution systems, http://venus.ece.ndsu.nodak.edu/~kavasseri/reds.html]

Conclusion

- Goal:
  real-time optimization of interdependent systems
- Contribution:
  1. scalability & efficiency
  2. performance guarantee
- Computation & validation:
  1. online computation
  2. 1000-dimensional problems in 1 sec.
- Future work:
  1. experiments with supermarket refrigerators
  2. autonomous anti-cascading failure mechanisms for power grids
  3. limited sensing/communication

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USC Socio-Technical Systems Laboratory

– Towards sustainable CPS interfaced with human decision-makers

- **The Lab:**
  supermarket refrigerators, air conditioners ('living' laboratory)
- **Viterbi School of Engineering:**
  USC Dynamic Demand Response, USC Microgrids
- **Los Angeles Department of Water & Power**
- **Looking for highly motivated students!**