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Economics of resource allocation for CPS in the presence of strategic attackers

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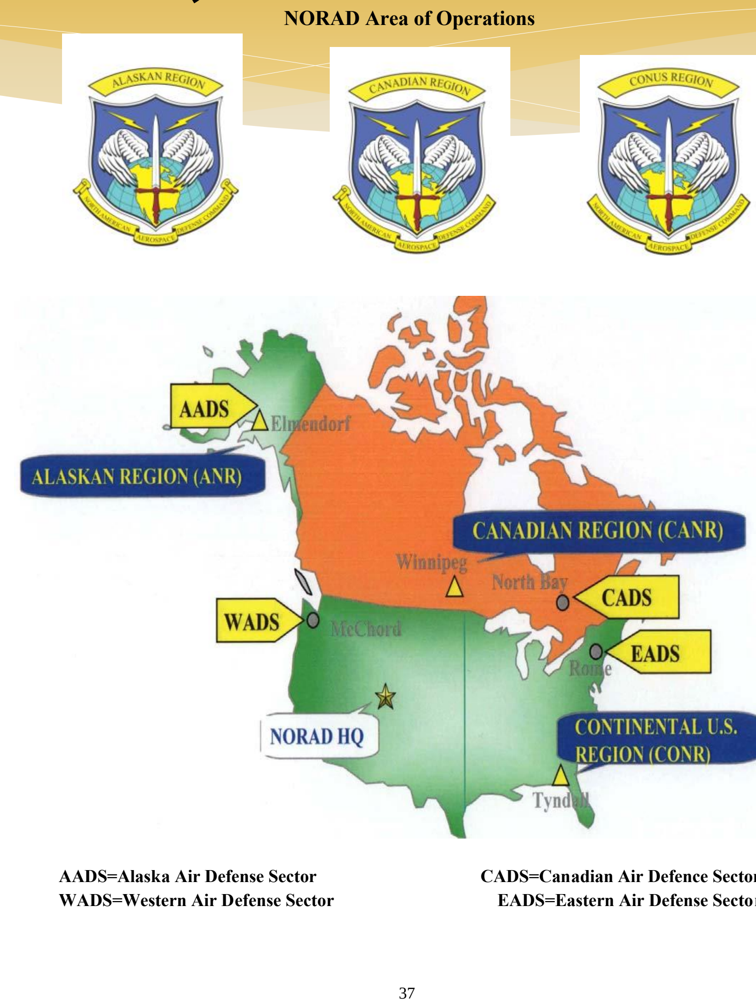
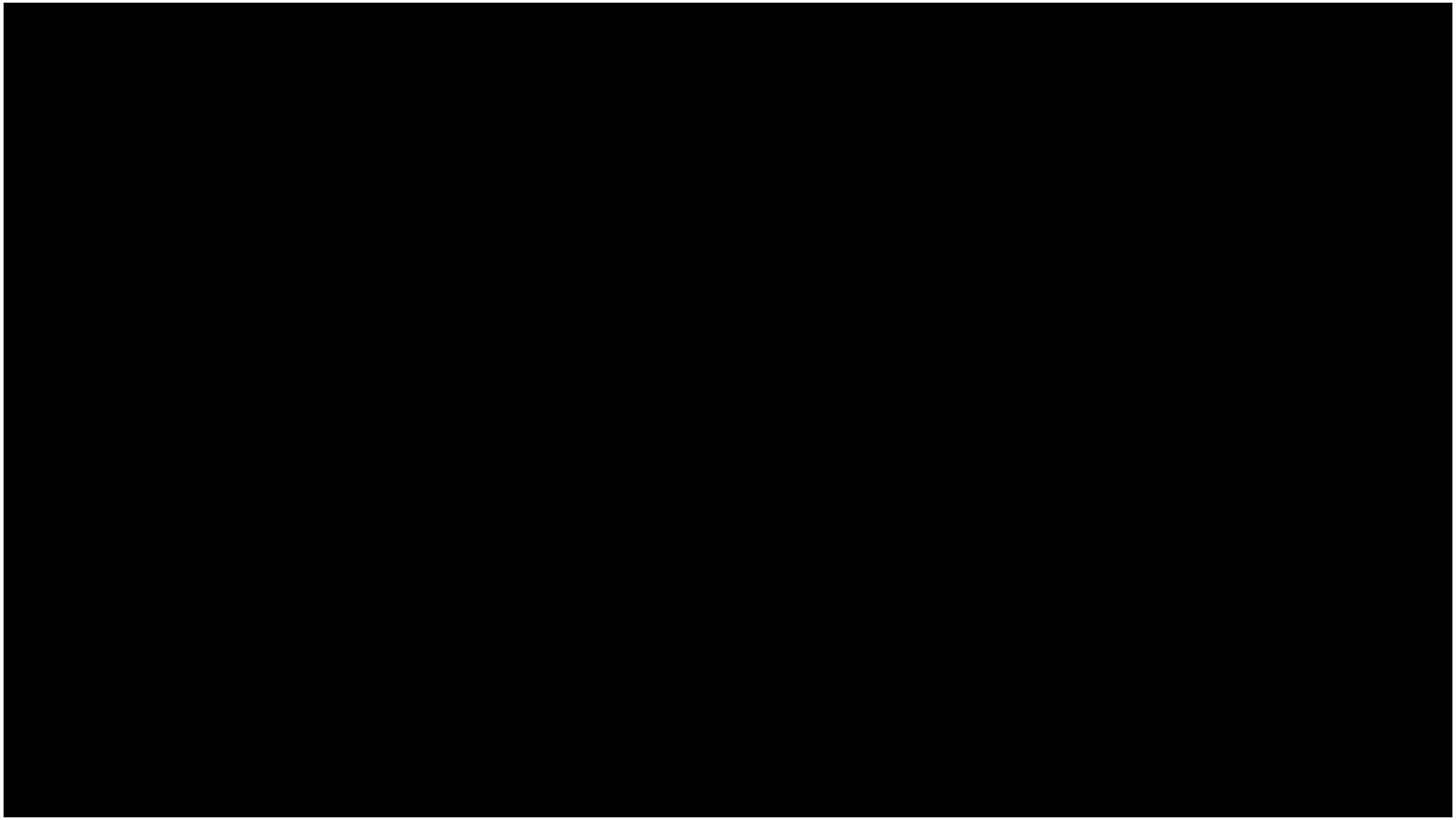
North American Aerospace Defense Command(NORAD)

* NORAD



September 11 airspace shutdown [a movie]

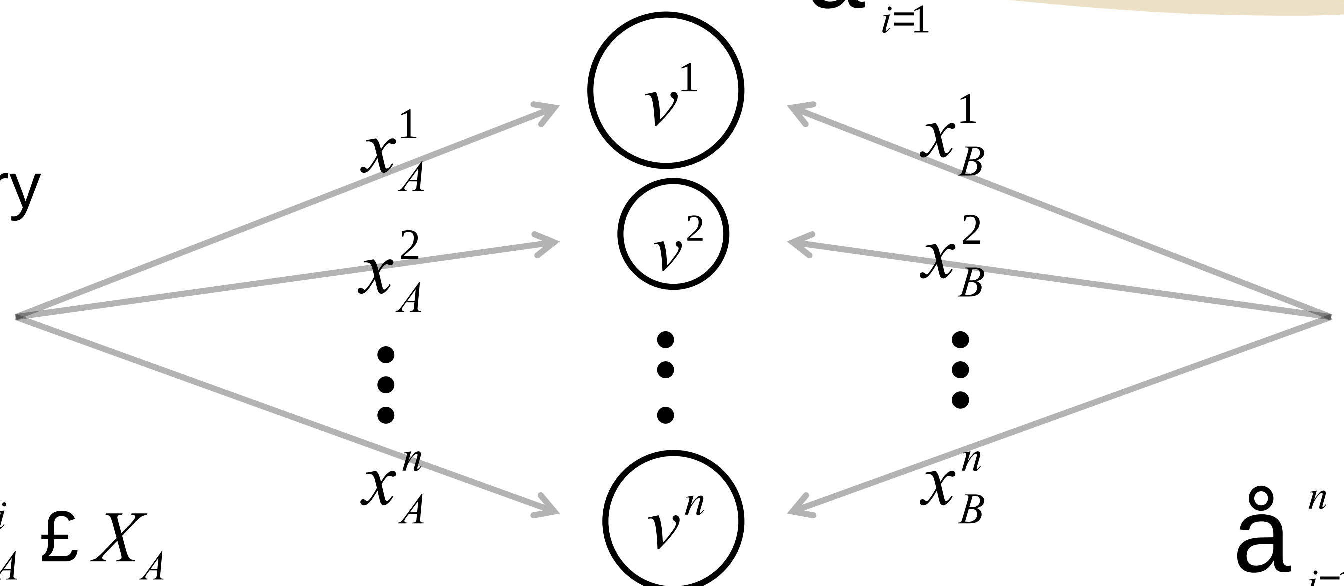


The Colonel Blotto game
Original formulation [Borel (1921)]

n battlefields $\quad \sum_{i=1}^{n} v^i = V$

Adversary

A



Blotto

B

$\sum_{i=1}^{n} x_A^i \le X_A$ $\qquad$ $\sum_{i=1}^{n} x_B^i \le X_B$

Resource constraint $(X_A \le X_B)$

- If $x_A^i \le x_B^i$ B wins battlefield *I*
- *(a tie is resolved in favor of a stronger player)*
- Payoffs: a sum of values of won battlefields

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North American Aerospace Defense Command (NORAD)

NORAD Area of Operations

* NORAD

Western Air Defense Sector	Canadian Air Defense Sector	Eastern Air Defense Sector

AADS=Alaska Air Defense Sector
WADS=Western Air Defense Sector

CADS=Canadian Air Defence Sector
EADS=Eastern Air Defense Sector

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Western Air Defense Sector	Canadian Air Defense Sector	Eastern Air Defense Sector

Example (intuition)

- 3 identical fields: $v^1 = v^2 = v^3 = 1 \Rightarrow V = 3$
- 2 identical players: $X_A = X_B = 1$
- → there exists a symmetric equilibrium w/ equal expected payoffs $\mathrm{P}_A = \mathrm{P}_B = 3/2$

- Pure strategy is not an equilibrium. Let $x_A^{1,2,3} = 1/3$
- Then, player B optimal response: $x_B^{1,2} = 1/2 \quad and \quad x_B^3 = 0$

$$\mathrm{P}_B = 2 \quad and \quad \mathrm{P}_A = 1$$

- Player A could improve by using a mixed strategy

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Western Air Defense Sector	Canadian Air Defense Sector	Eastern Air Defense Sector

Example continued (simplified, for intuition)

* 3 identical fields: $v^1 = v^2 = v^3 = 1 \;\; => \;\; V = 3$
* 2 identical players: $X_A = X_B = 1$
* Simplification: budget constraint holds "on average" (in expectation). Then, equilibrium strategies are:

$$x_A^i \sim Uniform\left(\left[0, \frac{2v^i}{V} X_B\right]\right) = Uniform\left(\left[0, \frac{2}{3}\right]\right)$$

$$x_B^i \sim Uniform\left(\left[0, \frac{2v^i}{V} X_B\right]\right) = Uniform\left(\left[0, \frac{2}{3}\right]\right)$$

* I.e., uniformly distributed on [0, 2/3]. in expectation, each field has 1/3. Payoffs: $P_A = P_B = 3/2$

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Western Air Defense Sector	Canadian Air Defense Sector	Eastern Air Defense Sector

Discussion of the example

* 3 identical fields: $v^1 = v^2 = v^3 = 1 \;\Rightarrow\; V = 3$
* 2 identical players: $X_A = X_B = 1$
* A mixed strategy: each field uniform dist. on [0, 2/3]:

$$\mathrm{P}_A = \mathrm{P}_B = 3/2$$

* With player A pure strategy of 1/3: $\mathrm{P}_B = 2 \;\; and \;\; \mathrm{P}_A = 1$
* ➔ Mixing improves player A payoff:

$$from \;\; \mathrm{P}_A = 1 \;\; to \;\; \mathrm{P}_A = 3/2$$

* 1. If Adversary is strategic, randomization is essential!
* 2. If no simplification (budget constraint holds exactly), equilibrium will be similar, but subtle to construct.

Western Air Defense Sector	Canadian Air Defense Sector	Eastern Air Defense Sector

Recommendations: how to allocate recourses

* How to allocate resources [fixed manpower]
* Inefficiency of fixed resource allocation
  * At each sector for each date
* Payoff can be improved with no resources added!
* HOW? By employing mixing: each sector will still employ the same resources, but ON AVERAGE (in our example, uniform dist. on [0, 2/3] instead of 1/3 on each date)
* ➔ Mixing improves Defender's payoff if attackers are strategic
* I. If Adversary is strategic, randomization is essential!
* II. If budget constraint is exact: similar, but subtle
* III. Our paper constructs equilibrium with any sector values

Features of Colonel Blotto-type games

* A general resource allocation game
* A simultaneous move game
* A constant-sum game (extends to a non-constant sum)
* Not a finite game
* Contest functions and payoffs could be discontinuous

* Other resource allocation games:
  * **FLIPIT game:** continuous time game, contest function – as in Blotto
  * **Contest of teams game:** with fixed number of players, M and N, and a lottery–like contest function [Gladiator game]

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FlipIT

- M. van Dijk, A. Juels, A. Oprea, and R. Rivest. *Flipit: The game of stealthy takeover*, *Journal of Cryptology*, 26(4):655--713, 2013.

- One field (resource) only
- The game of timing
- Each player chooses when to flip
- Time is continuous, finite length T
- Costs of flip for each player are common knowledge
- Playoff: the fraction of time the player "owns" the resource

Contests between two teams

- K. S. Kaminsky, E. M. Luks, and P. I. Nelson. Strategy, nontransitive dominance and the exponential distribution, *Australian Journal of Statistics*, 26(2):111--118, 1984.
- Y. Rinott, M. Scarsini, and Y. Yu, A colonel Blotto gladiator game, Math. Oper. Res. [MOR], 37(4):574--590, November 2012.

- The rules of the game:
- Each team has fixed number of players
- A manager of each team
  - has fixed resources to distribute between players
  - decides how to allocate resources to players
  - If player resources are a and b, the probability of winning is a/(a+b)
  - The last winning player wins the entire game
- Only one battlefield

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Environments with Blotto-type settings

* Useful in environments where:
  * **Strategic attacks are present**
  * **Players move simultaneously**
  * **Fixed resources must be allocated**
* Why renewed interest in Blotto?
  * Recent analytical and computational advances allow to solve complex resource allocation problems
  * Global connectivity allows rapid aggregation of information from heterogeneous public and private sources.

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Applications

* Information technology (IT) security: resource (human, processor) allocation across tasks.
* Emergency relief allocation of state / federal resources: equipment, water, food, medical supplies, air fleet allocation
* Air and sea (underwater) fleet: [patrolling and warfare]
* Anti-terror defenses [under a strict resource constraint]: Blotto allows to consider simultaneous games
* Air space patrolling / monitoring

Blotto game: a first step: Gross & Wagner (1950)

- For 2 battlefields (n=2): complete solution of heterogeneous game X_B, X_A *and* v_1, v_2
- if X_B ³ $2X_A$ pure strategy equilibrium
- If $X_B < 2X_A$ mixed strategy equilibrium (mixing has a final number of mass points in its support)

- For 3 battlefields (n=3): a solution for players with identical resources $X_B = X_A$

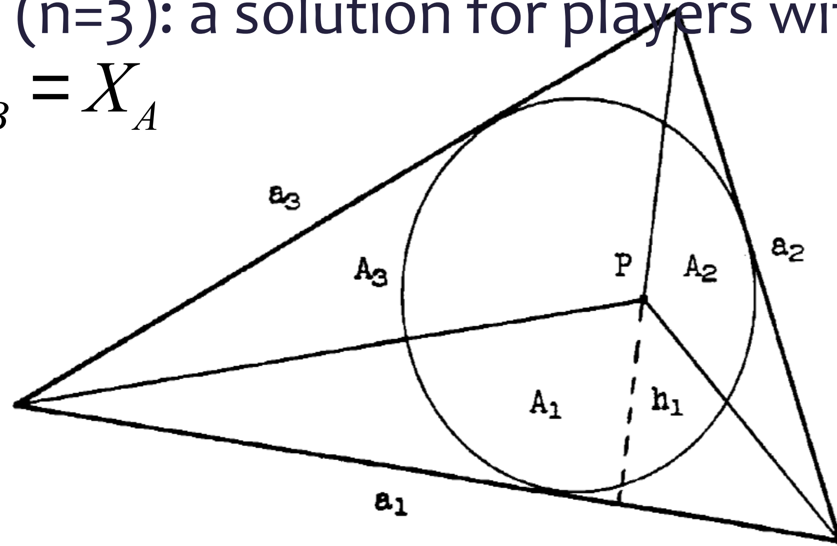


Illustration: courtesy of Gross & Wagner '50

Marginals

* Gross & Wagner (1950) solution for n=3

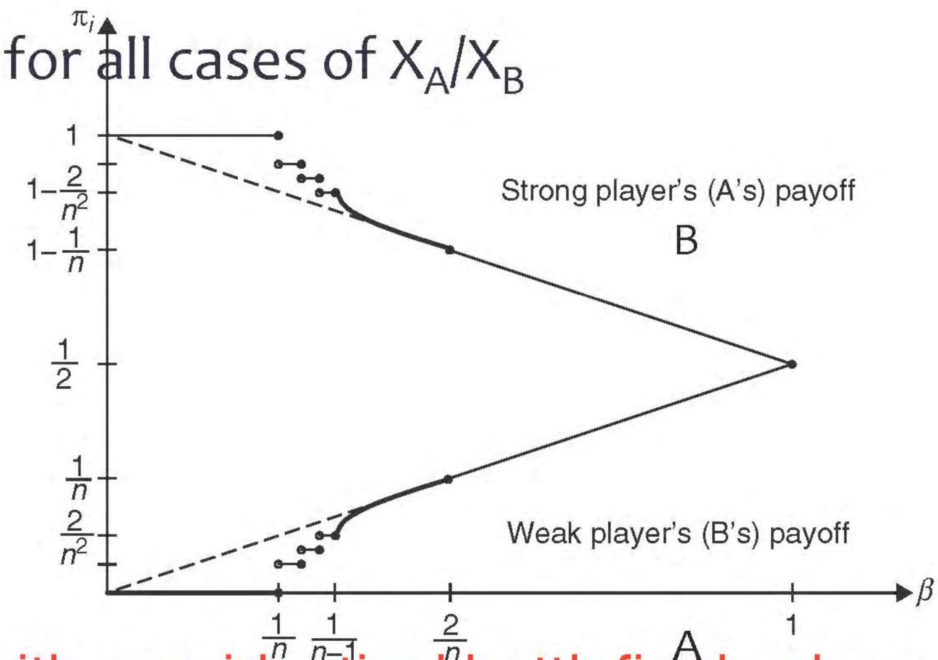
$$x_A^i \sim Uniform\left[0, \frac{2v^i}{V} X_A\right] \text{ same for } x_B^i$$

* Easy (relatively): to show that a distribution with these marginal is an equilibrium
* Difficult: to find a joint distribution with these marginals respecting budget constraints
* Extensions of Gross & Wagner (1950)
  * Laslier & Picard (2002)
  * Thomas (2013)

Step 2: asymmetric player resources; identical battlefields [Roberson (2006)]

* Roberson (2006) a solution for all cases of X_A/X_B

Picture from
Roberson 2011



* No solution for the game with non-identical battlefield values

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Equilibrium characterization [by Roberson (2006)]

* n homogeneous battlefields: $n \geq 3$ $\quad v^i = v \quad and \quad V = nv = 1$
* Case 1: $X_B \geq nX_A$ [Extreme resources disparity] $\Pi_A = 0; \quad \Pi_B = V$
* Case 2: [Intermediate resource disparity] $\Pi_A > 0; \quad \Pi_B < V$
* Case 3: $\frac{2}{n} < \frac{X_A}{X_B} \leq 1$ [Similar resource endowments]

$$\Pi_A = \frac{X_A}{2X_B}V; \quad \Pi_B = \left(1 - \frac{X_A}{2X_B}\right)V$$

* Case 1 – pure equilibrium, Cases 2 & 3 – mixed equilibrium

Roberson (2006)
Case 1: extreme disparity of player resources

- case 1 : $P_A = 0; \quad P_B = V$
- If $X_B \geq nX_A$ pure strategy equilibrium exists
- Multiple payoff equivalent equilibria (possibly mixed)
- For example: a stronger player puts X_B / n on each field
- Weaker player has a zero payoff; he is indifferent between playing the game and staying out of the game.

Roberson (2006)
Case 2: Resources in intermediate range

- *n* homogeneous battlefields: n ³ 3 $v^i = v$ *and* $V = nv = 1$
- Case 2: Resources are in the range between Cases 1 and 3

$$\frac{1}{n} < \frac{X_A}{X_B} \text{ £ } \frac{2}{n}$$

- "Guerilla warfare equilibrium": A weaker player allocates no resources some (one or more) fields

Roberson (2006)
Case 3: Players with similar resources

* *n* homogeneous battlefields: $n \geq 3$ $\quad v^i = v \quad and \quad V = nv = 1$

* Case 3: $\frac{2}{n} < \frac{X_A}{X_B} \leq 1$ In equilibrium,

$$\Pi_A = \frac{X_A}{2X_B} V; \quad \Pi_B = \left(1 - \frac{X_A}{2X_B}\right) V$$

  * Unique payoffs
  * Unique mixed strategies

(i) For player A:

$$F_A^j(x) = \left(1 - \frac{X_A}{X_B}\right) + \frac{x}{\frac{2v^j}{V} X_B} \left(\frac{X_A}{X_B}\right), \quad x \in \left[0, \frac{2v^j}{V} X_B\right]; \tag{2a}$$

(ii) For player B:

$$F_B^j(x) = \frac{x}{\frac{2v^j}{V} X_B}, \quad x \in \left[0, \frac{2v^j}{V} X_B\right]. \tag{2b}$$

  * Strictly positive amount of resources on all battlefields
  * Proof: by constructing a joint distribution with these marginals; uniqueness follows from all-pay auctions results

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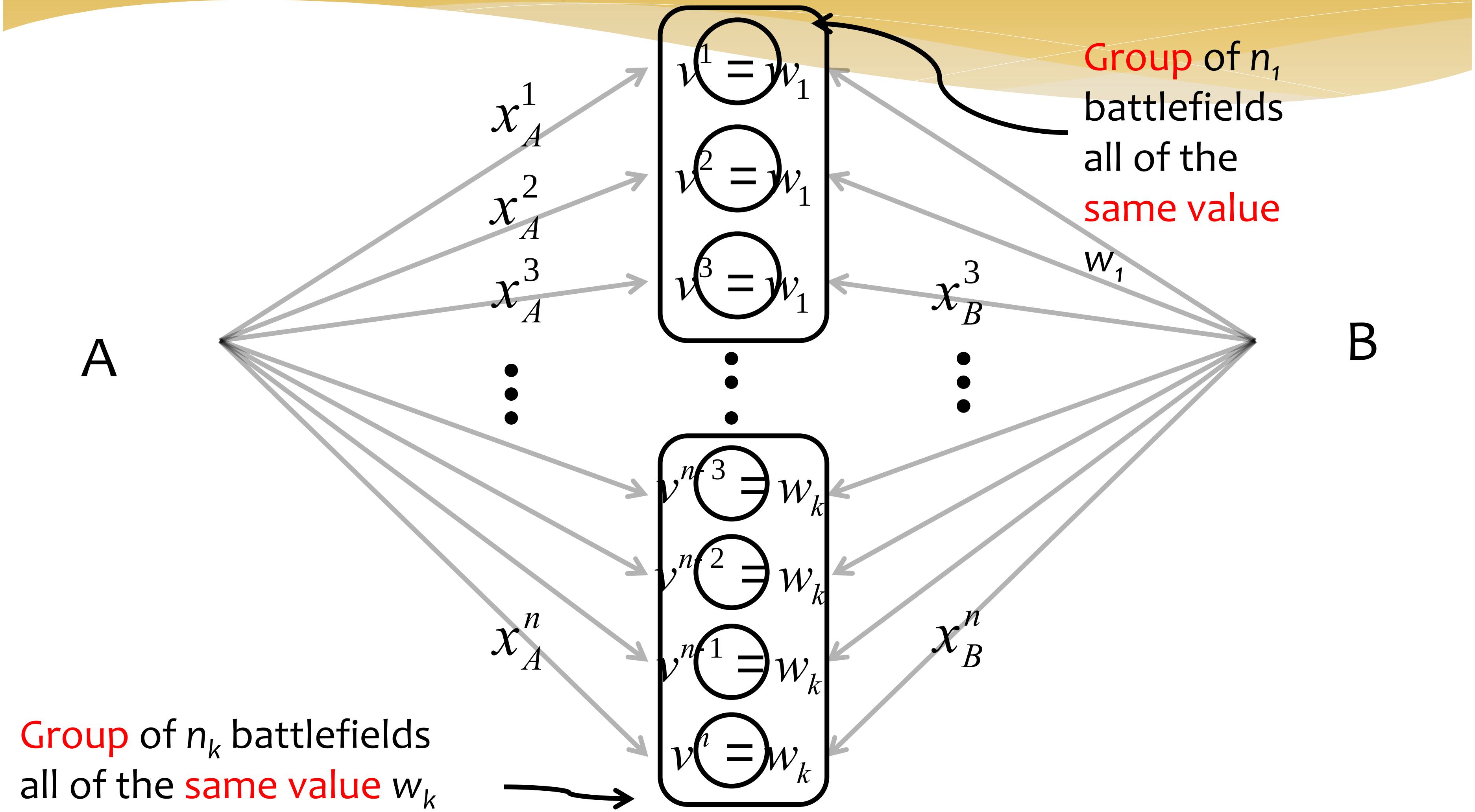
Our paper solves Blotto game with heterogeneous battlefields



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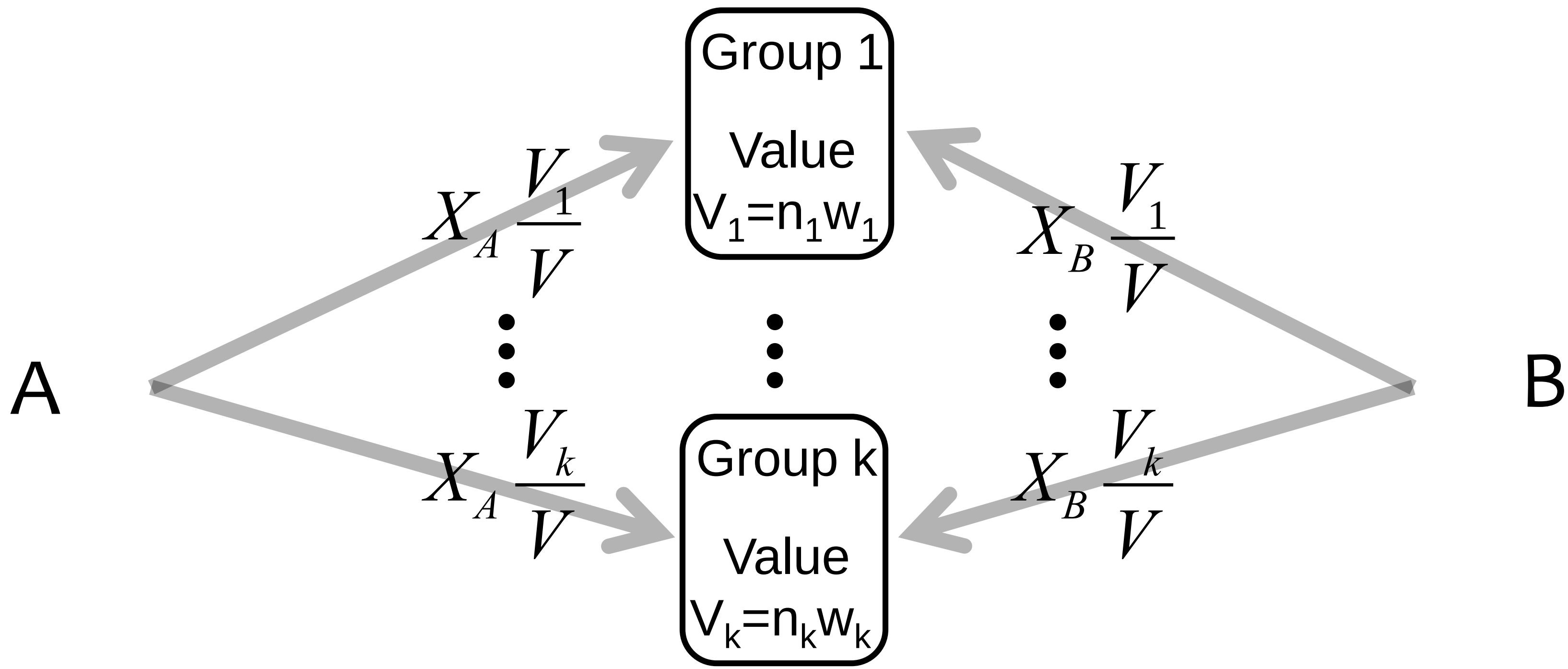
Our contribution: a solution of heterogeneous Blotto game

* Assume that for each group *j* $\frac{2}{n_j} < \frac{X_A}{X_B} \le 1$ then:
  * Equilibrium marginals: Similar to Robinson (2006)
  * Unique equilibrium payoffs $\text{A: } \frac{X_A}{2X_B}V; \quad \text{B: } \left(1 - \frac{X_A}{2X_B}\right)V$
  * There exists a valid joint distribution respecting budget constraints (proven by its construction)

Construction of joint distribution

- Step 1: allocate resources to each group of battlefields proportional to total value of the group



- Step 2: within each group, allocate as Roberson (2006)

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Remarks

* Remark: we require $\frac{2}{n_j} < \frac{X_A}{X_B} \le 1$
* We construct an equilibrium joint distribution
  * It gives correct marginals
  * It respects the budget constraint

* Require $n_j \ge 3$ for all *j*
* All groups (need to be in regime 3)
* Joint distribution is not unique

Recent Blotto Applications in Engineering

* Infrastructure protection (robustness of cyber-physical systems)
  * hybrid defensive allocation w/ partially strategic attackers [Shan & Zhuang (2013)]
  * interactions of teams / coalitions facing a common adversary [my presentation: FORCES June, 2014]
* Network defenses: effects network structure on security
  * Blotto on network w/ various topologies [Goyal & Vigler (2010)]
  * Blotto on network with propagation [Bachrach et.~al., (2012)]
* Fending terrorist attacks
  * Assisting resource allocation [Powell (2007, 2009)]
  * Blotto combined w/ Milind Tambe framework [Paruchuri et.~al., (2009), Jain et.~al.,(2010)]

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Conclusion

* Summary:
  * Blotto-type games are beautiful and useful!
  * We solve the game with asymmetric player endowments and heterogeneous battlefields, under minor restrictions
  * We provide an algorithm for allocating resources across battlefields
* An open problem:
  * An equilibrium for players with moderately asymmetric resources (guerilla warfare region)
* Future work
  * Players with unequal valuations of the battlefields
  * The limit of large number of battlefields

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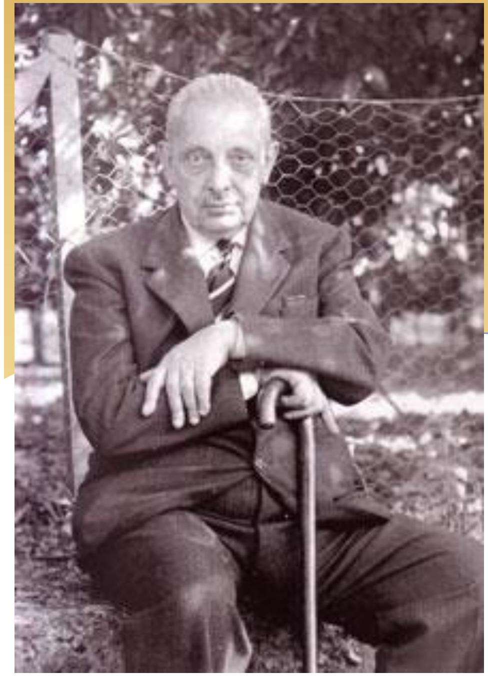
If we want things to stay as they are, things will have to change.



Il Gattopardo, [The leopard] 1958
Giuseppe Tomasi di Lampedusa

http://www.imdb.com/title/tt0057091/quotes

THANK YOU