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A Framework for Aperiodic Model Predictive Control

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Motivation- Why NMPC?

- It is intuitively attractive!

- It can handle nonlinearities and can cope with constraints on the controls and the states….

…<u>but</u> it is computationally demanding

- So, it would be of great interest if the control law would not be updated at each sampling instant, but only *when it is needed*.

(Nonlinear) MPC

$$x_{k+1} = f\left(x_k, u_k\right)$$

$$x_k \in X,\ u_k \in U$$

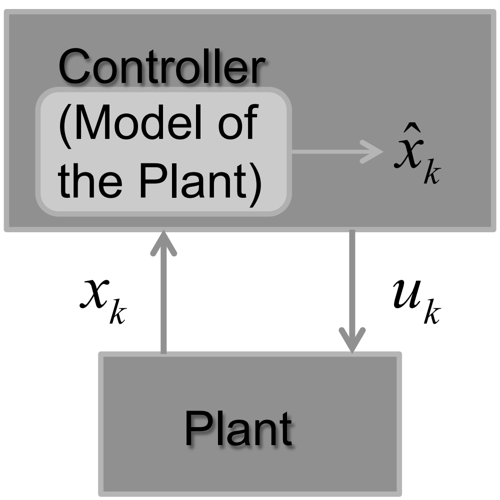
$$\min J_N\left(x_k, u_F\left(k\right)\right)$$

$$\downarrow$$

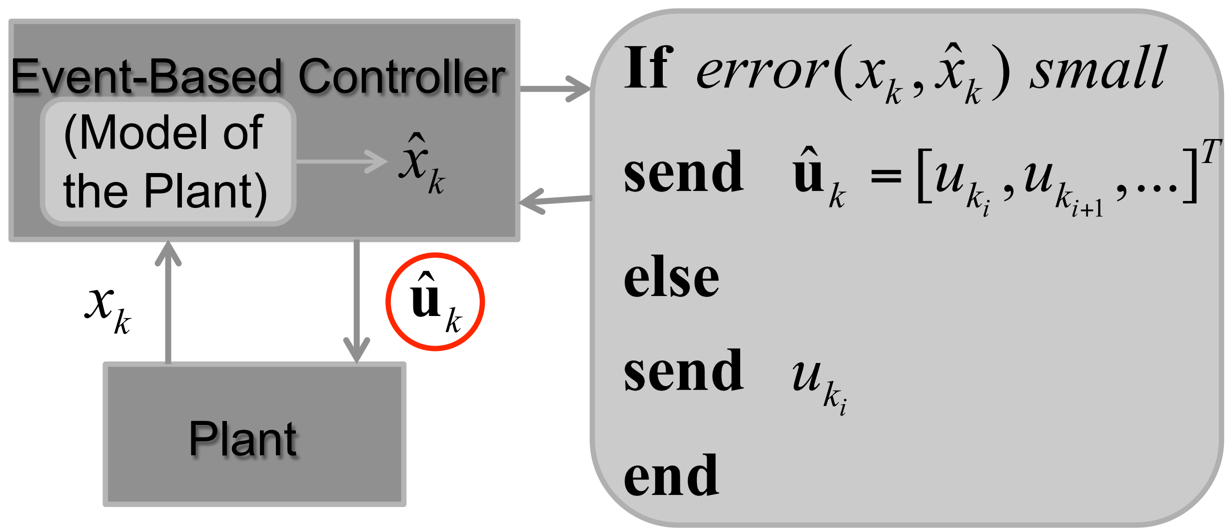
$$u_F\left(k\right)$$

Motivation (Centralized Case)

- In order to extend the inter-sample times of the MPC controllers, we adopt an event-driven framework.
- The key attributes of these approaches is that the decision for the time of execution of the control task is not made ad-hoc, but it is based an a certain condition.



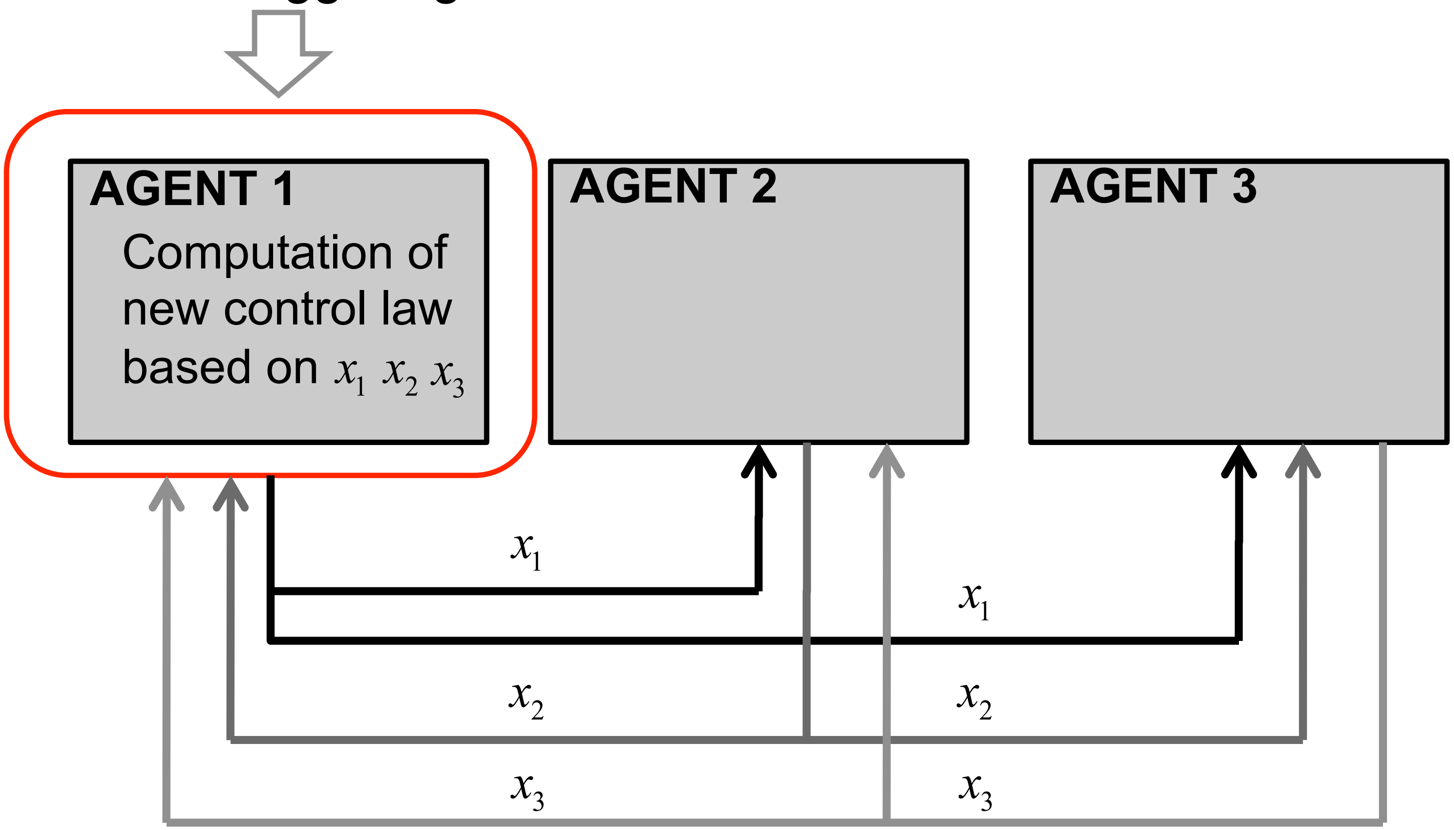
Classic Approach



Event-based Approach

Motivation (Distributed Case)

Violation of triggering condition



Where $x_i, \quad i = 1,2,3$ are the actual states of the agents.

Motivation

The update instants of the control law depend on the error of the actual and the predicted state trajectory of the neighboring agents

Compute trigger function (depends only on actual state x_i and predicted state $\hat{x}_i$ $\longrightarrow$ $e_i = \| x_i - \hat{x}_i \|$

Check trigger condition. If violated, send x_i and receive actual states from everyone else $x_{j \neq i}$

Compute control law, based on x_i and $x_{j \neq i}$

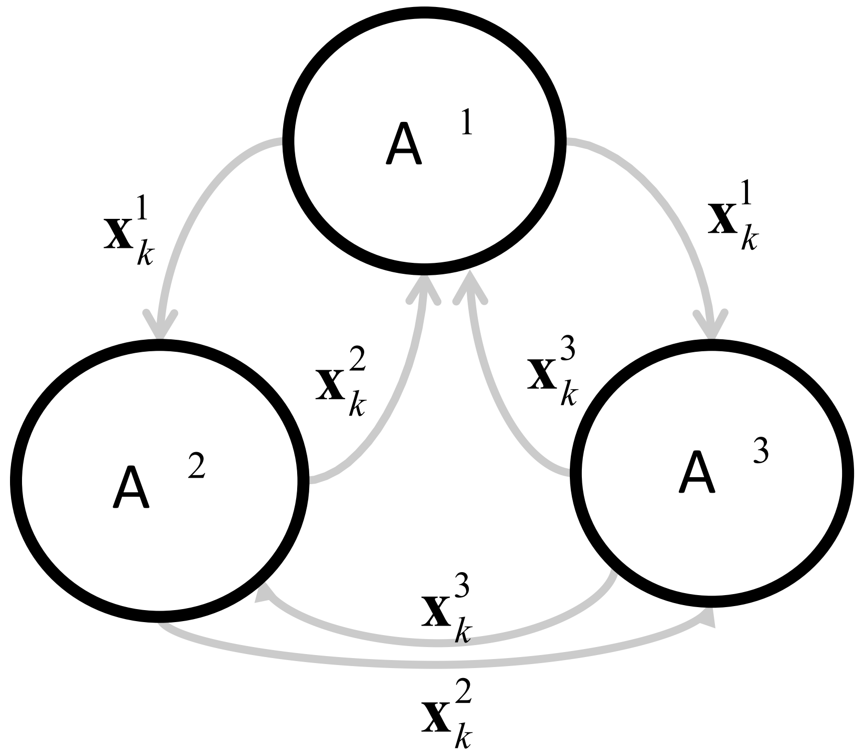
Motivation

Using the proposed event-triggered framework we aim at …

- achieving
  - an improvement on the requirements for computation resources
  - decrease of control related traffic, in the case of networks,
- while…
  - preserving Stability and Convergence of the system.

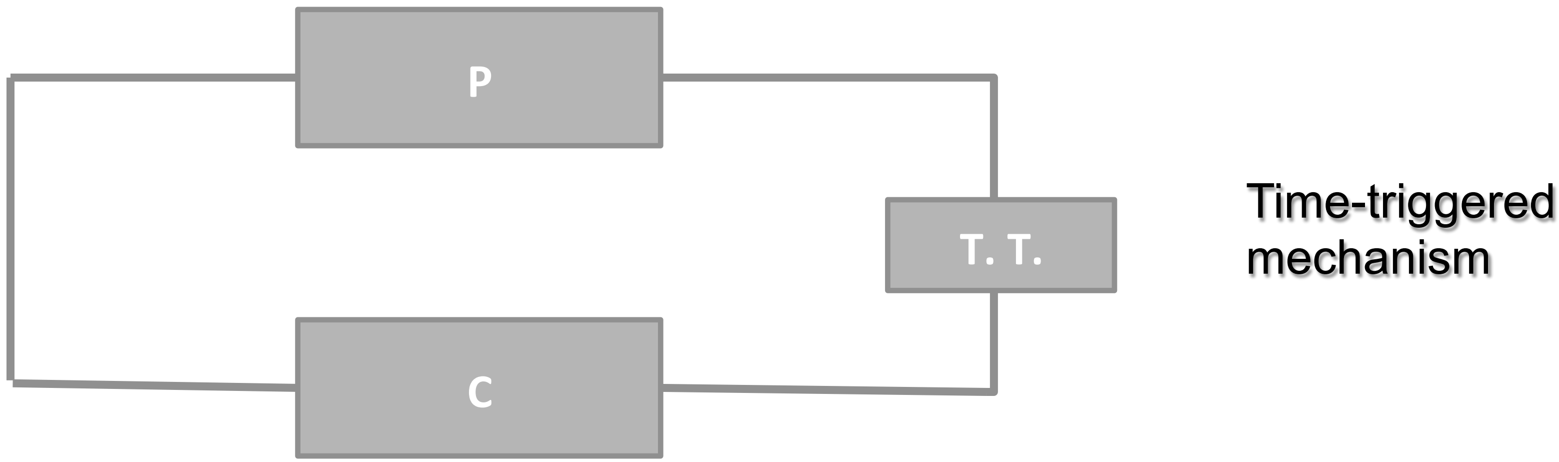
The Basic Idea

- A team of cooperating distributed agents, working in the same environment is considered.
- Each agent is controlled locally by NMPC laws. Each controller depends on information which is of nature, both
    - local and
    - coming from the neighboring agents.
- Sufficient conditions for triggering the NMPC law for each agent are given. They depend on the error between the actual and the expected state.



Preliminaries on Event-Triggered Control

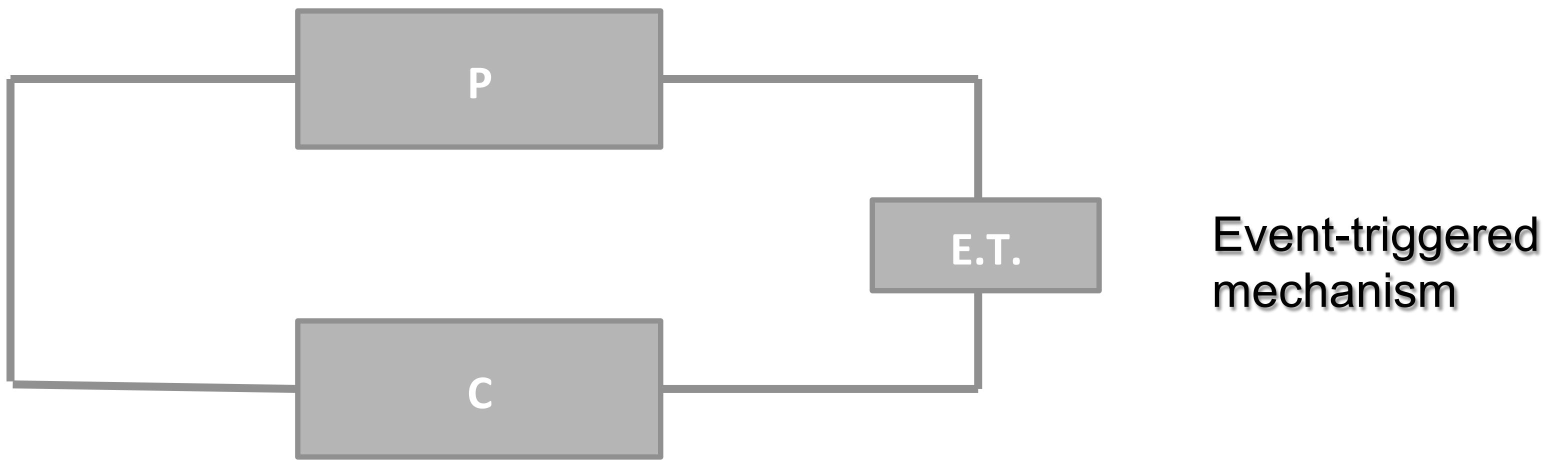
- **Time - triggered** : sampling at pre-specified instants does not take into account optimal resource utilization.



- Time-scheduled control might be conservative in terms of the number of control updates, since constant sampling period has to guarantee stability in the worst-case scenario.

Preliminaries on Event-Triggered Control

- **Event - triggered control** : better resource utilization.



- It takes into account state or output feedback in order to sample as “infrequently” as possible in an aperiodic fashion.

Event-Triggered Control[1]

Consider the system $x_{k+1} = f\left(x_k, u_k\right)$ and a feedback controller $u_k = p\left(x_k\right)$

that renders the closed-loop system $x_{k+1} = f\left(x_k, p\left(x_k + e_k\right)\right)$

Input-to-State Stable w.r.t. measurement errors $e \in \circ^{\,n}$.

Then, there exists a smooth ISS-Lyapunov function $V \in \circ$ that satisfies

$$V\left(f\left(x_k, p\left(x_k + e_k\right)\right)\right) - V\left(x_k\right) \le -a\left(\left\|x_k\right\|\right) + \gamma\left(\left\|e_k\right\|\right)$$

If we restrict the error to satisfy $\gamma\left(\left\|e_k\right\|\right) \le \sigma \cdot a\left(\left\|x_k\right\|\right)$ then

$$V\left(f\left(x_k, p\left(x_k + e_k\right)\right)\right) - V\left(x_k\right) \le \left(\sigma - 1\right) \cdot a\left(\left\|x_k\right\|\right)$$

Thus, guaranteeing that V decreases, provided that $0 < \sigma < 1$.

Triggering rule: $\boxed{\gamma\left(\left\|e_k\right\|\right) \le \sigma \cdot a\left(\left\|x_k\right\|\right)}$

γ, a are class $\mathcal{K}_\infty$ -functions.

1. P.Tabuada,2007, W.P.M.H. Heemels et.al,2007, X.Wang et.al.,2008, A. Eqtami et.al.,2010.

Preliminaries on NMPC

- NMPC – General Framework (Nominal Case)

$$x_{k+1} = f\left(x_k, u_k\right) \qquad \text{with} \qquad x_k \in X,\ u_k \in U$$

The optimal control problem (OCP):

$$J_N\left(x_k, u_F(k)\right)$$

$$= \min_{u_F(k)} \sum_{l=0}^{N-1} \left\{L\left(x\left(k+l \mid k\right), u\left(k+l \mid k\right)\right)\right\} + V\left(x\left(k+N \mid k\right)\right)$$

s.t.

$$x_{k+1} = f\left(x_k, u_k\right),\ x\left(k \mid k\right) = x_k,\ u_k \in U,\ x_k \in X,\ x\left(k+N\right) \in X_f$$

Preliminaries on NMPC

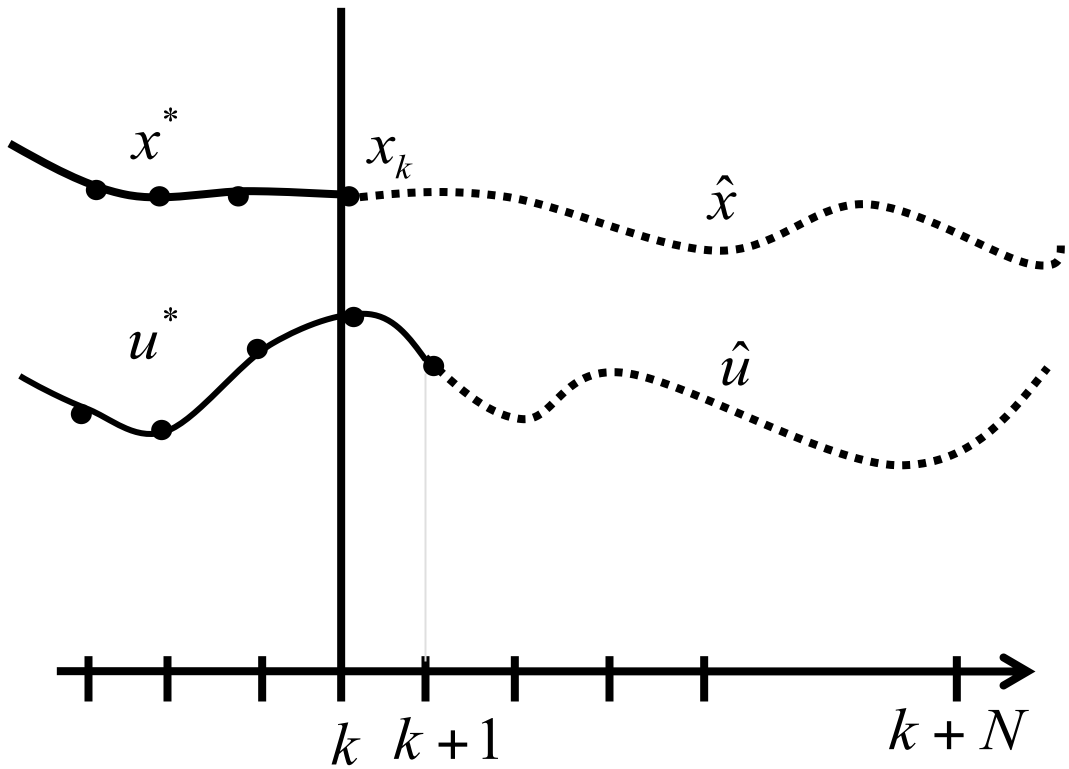
- MPC algorithm:

1. Measure the current state of the plant x_k .
2. Compute the open-loop optimal control sequence $u^*:[k, k+N]$ solution to the problem $J_N(x_k, N)$

   The control is applied to the plant for k until k+1:

   $$u(k) = u^*(k \mid k)$$

   (the remaining control is discarded).
4. The procedure is repeated from (1).



The MPC strategy.

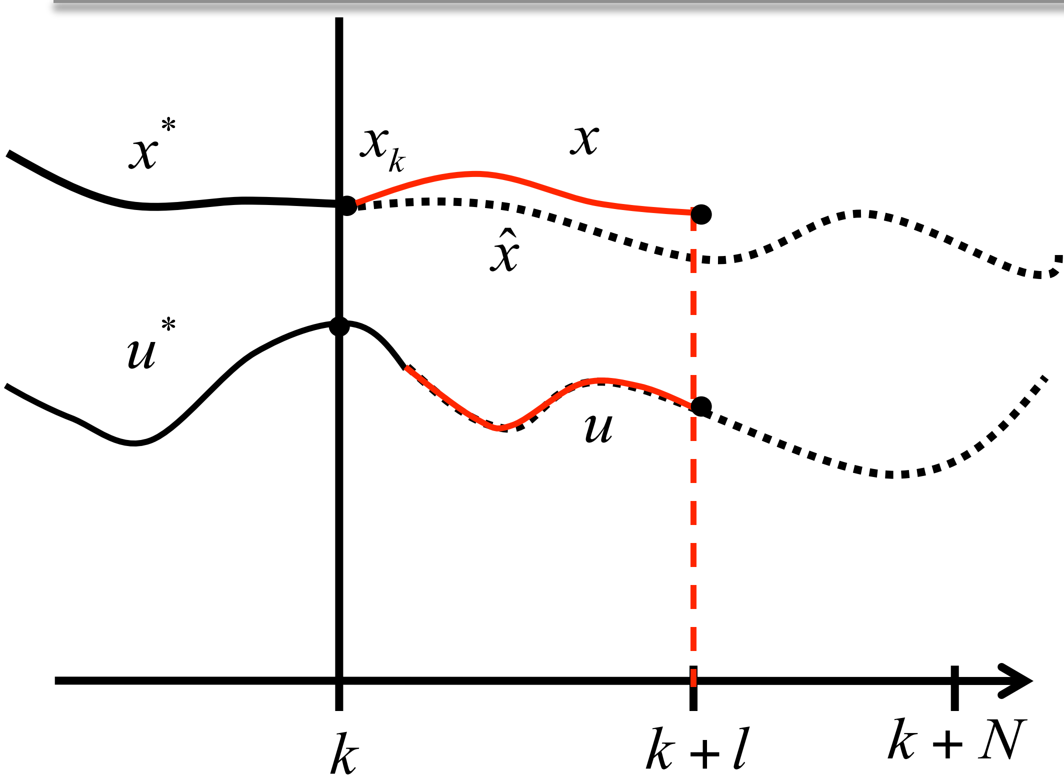
Event-Triggered MPC: The Main Idea

- In the aforementioned event-triggering techniques, the control value is held constant between the actuation updates.

However…

- Predictive controllers provide a control trajectory (sequence) at each time step.
- The control trajectory provided by MPC is applied to the plant in an open-loop fashion, until an event occurs…

Event-Triggered MPC: The Main Idea



The ETMPC strategy.

- **ETMPC algorithm:**

1. Measure the current state of the plant x_k .
2. Compute the open-loop optimal control $u^* : [k, k+N]$
3. Measure error: e_k
4. The control is applied to the plant $u(k+l) = u^*(k+l \mid k)$ (the remaining control is discarded)-until the error exceeds some limit.
5. The procedure is repeated from (1).

[2] P. Varutti et.al.,2009.

Event-triggered MPC

Relevant Developments on Event-based MPC:

- Event-triggered MPC for
  - Centralized Systems
    - Continuous (Eqtami et.al., CDC-ECC 2011)
    - Discrete-time (Eqtami et.al., IFAC World Conference, 2011)
  - Decentralized systems (Eqtami et.al., IFAC World Conference, 2011)
  - Linear Systems. (Eqtami et.al. ACC 2010)

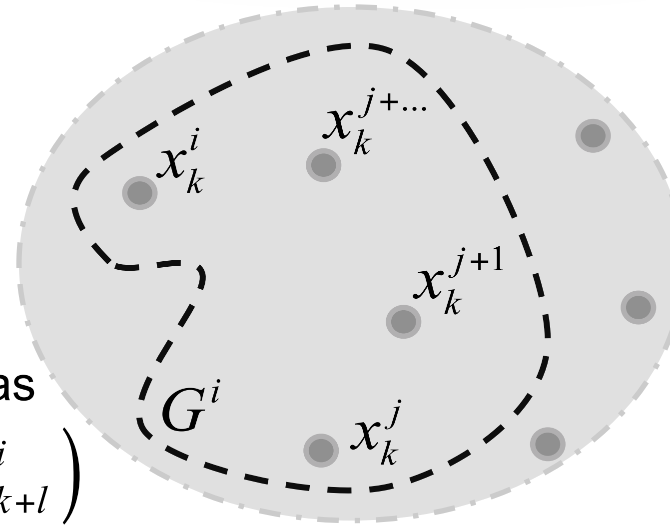
- Event-based MPC
  - ✓ NCS (P. Varutti et.al. ACC 2009)
  - ✓ wireless NCS (Y. Lino et.al. ACC 2009)
  - ✓ + Estimator between sensors (J. Sijs et.al. HSCC 2010)

- Self-triggered MPC for
  - Linear Systems (Barradas Berglind et.al. 4th IFAC NMPC Conference, 2012)
  - Network Scheduling (Henriksson et.al. 8th IFAC Symp.Adv.Cntrl.Chem.Proc., 2012.)

System & Problem Statement

- Local Subsystems: $i = 1, \ldots, M$
- Model of the subsystem $x_{k+1}^i = f^i\left(x_k^i, u_k^i\right)$

s.t. $x_k^i \in X^i,\ u_k^i \in U^i,\ f^i\left(0,0\right) = 0$



The **predicted state** at time $k + l + 1$ is denoted as

$$\hat{x}^i\left(k + l + 1\right) = f^i\left(\hat{x}^i\left(k + l \mid k\right), u_{k+l}^i\right)$$

Each agent exchanges state information with a set of neighboring agents. The state information received by an agent i, at time step k is denoted as

$$\boxed{w_k^i \ @ col\left(x_k^j, j \in G^i\right)}$$

with $w_k^i \in W^i \ @ \Pi_{j \in G^i} X^j$

Where G^i denotes the set of indexes, identifying the agents belonging to the neighborhood of agent i .

NMPC for subsystem i

- The Optimal Control Problem :

$$J_N^i\left(x_k^i, u_F^i(k), w_k^i\right)$$

$$= \min_{u_F^i(k)} \sum_{l=0}^{N^i-1} \{L^i\left(x^i(k+l\,|\,k), u^i(k+l\,|\,k)\right) + Q^i\left(x^i(k+l\,|\,k), w^i(k+l\,|\,k)\right)\}$$

$$+V^i\left(x^i\left(k+N^i\,|\,k\right)\right)$$

s.t.
$$u^i(k+l\,|\,k) \in U^i,$$
$$x^i(k+l\,|\,k) \in X^i,$$
$$x^i\left(k+N^i\,|\,k\right) \in X_f^i \quad \forall l = 1,\dots,N^i-1$$
$$w^i(k+l\,|\,k) \in W^i \rightarrow \boxed{w^i(k+l\,|\,k) = w^i(k\,|\,k), \forall l = 0,\dots,N^i-1}$$

Event-triggered MPC for Distributed Agents

To assure that the proposed system is ISS stable wrt the measurement errors of the neighboring agents under the NMPC strategy, we consider the difference:

$$\Delta J_t^i = J_N^{i*}(k+t) - J_N^{i*}(k-1), t \in [0, N^i - 1]$$

and prove that ΔJ_t^i has to be bounded to assure that the optimal cost $J_N^{i*}(\text{g})$ is an ISS Lyapunov function with respect to an error, i.e.

$$\Delta J_t^i \leq (N^i - t - 2) L_{qw}^i e_w^i (k+t \mid k-1) - \sum_{\rho=0}^{t} \left\{ r^i \left(\left\| x_{k-\rho+t-1} \right\| \right) \right\}$$

where the error e_w^i between the predicted and the actual trajectories of the **neighboring** agents was appropriately defined.

To maintain stability of the closed loop system

$$\Delta J_{t+1}^i \leq \Delta J_t^i$$

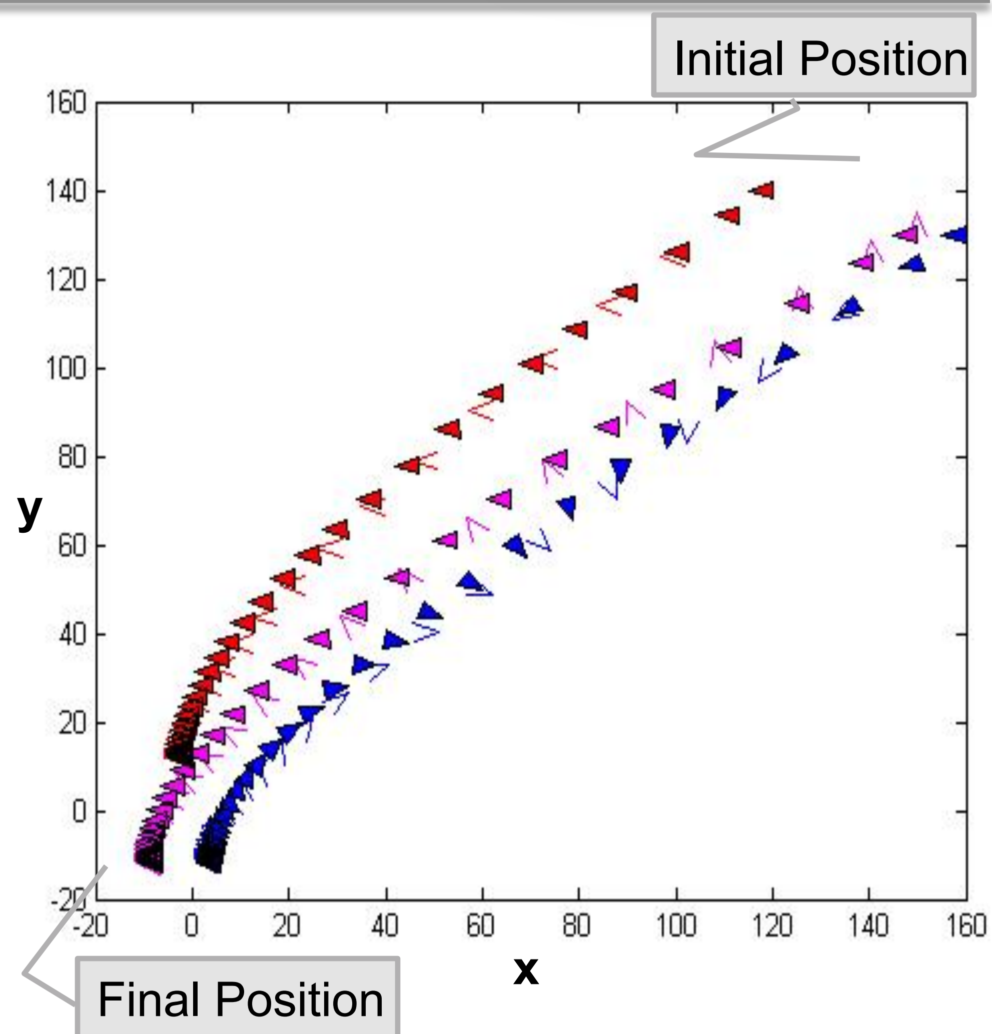
Those two essentially constitute the triggering conditions.
OCP is triggered whenever any of them are violated.

Example- 3 Agents

Set of 3 cooperating agents (holonomic case).

- Filled triangles represent agents under the event-based framework.
- Empty triangles represent the agents under classic time-driven MPC.

- Both frameworks have comparable results; they achieve the goal; all agents reach to the desired configuration, while keeping relative distance between them.



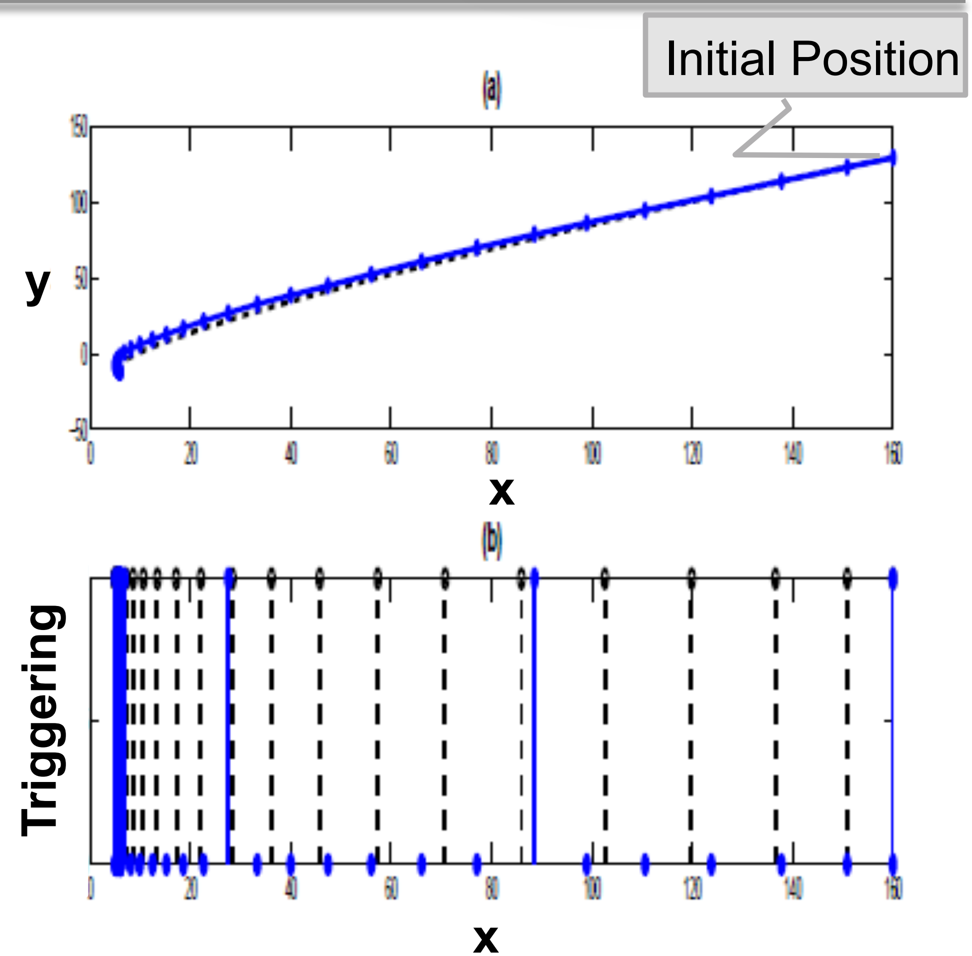
Example-Triggering Instants

- Same scenario.

Figure depicts only one agent and its triggering instants.

- Blue solid line represents the trajectory of the agent under the Event-based MPC. Black solid line represents the trajectory of the same agent under classic MPC.

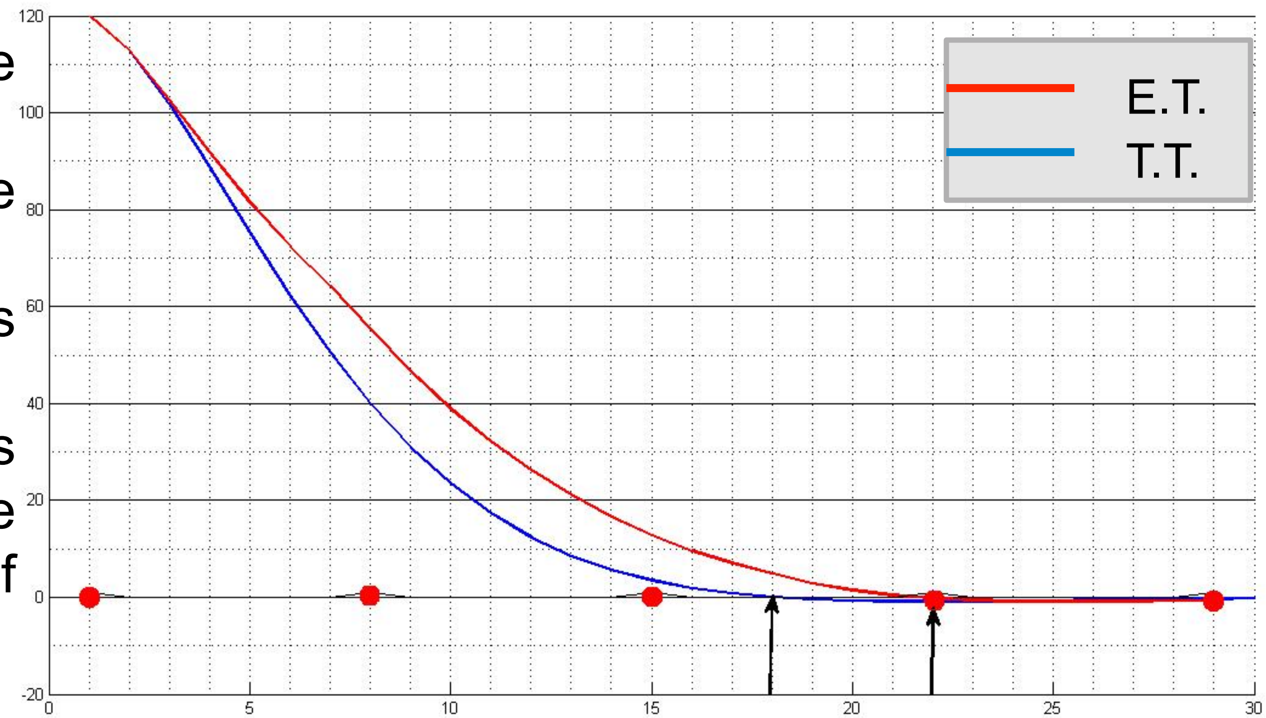
- Blue stems the triggering instants of ET-MPC. Black dash stems represent the classic time-based MPC.



Example- Comparison

- Blue line represents the time-triggered setup,
- Red line represents the event-based setup.
- The Time-triggered has faster convergence.
- The Event-based offers overall reduction to the need for computation of the control law.

State of an agent versus time-steps.



Conclusion & Future Work

- *An event-based approach was presented for Model Predictive Controllers: The Input-to-State Stability w.r.t. to a measurement error was utilized in order to reach to sufficient conditions for triggering the computation of the control law.*

[1] A. Eqtami et.al., "Aperiodic MPC via Perturbation Analysis", CDC 2012

- Between the controller updates, the last computed control trajectory is applied to the system, in conjunction with a correction term. This term consists of a perturbation solution of the nominal system which depends on the error[1]. This results to faster convergence for the event-based set-up!

$$u(k+l) = u^*(k+l \mid k)$$: Even-based execution of the control law

$$u(k+l) = u^*(k+l \mid k) + \underbrace{\delta u(k+l)}_{K^*(k+l)dx(k+l)}$$: Enhanced even-based execution of control

$$K^*(k+l)dx(k+l) \longrightarrow \|dx(k+l)\| @ e(k+l \mid k)$$

- Self-triggered MPC: Both the control input & the time of the next control update are evaluated to avoid continuous supervision based on continuously taking state measurements.

Relevant Publications

- Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "**Event-triggered Control for Discrete-Time Systems**", 2010 American Control Conference, Baltimore, MD, USA, pp. 4719-4724, July 2010.
- Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "**Event-Triggered Strategies for Decentralized Model Predictive Controllers**", 18th IFAC World Congress, Milano, Italy, August 2011.
- Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "**Novel Event-Triggered Strategies for Model Predictive Controllers**", 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, FL, USA, pp. 3392-3397, December 2011.
- Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "**Event-Based Model Predictive Control for the Cooperation of Distributed Agents**", 2012 American Control Conference, Fairmont Queen Elizabeth, Montréal, Canada, pp. 6473-6478, June 2012.
- Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "**Aperiodic Model Predictive Control Via Perturbation Analysis**", 51st IEEE Conference on Decision and Control, Maui, Hawaii, December 2012, to appear.