CPS: Breakthrough: Distributed computing under uncertainty: a new paradigm for cooperative cyber-physical systems

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Abstract. The main objective of this research is to develop the Science of CPS by proposing and developing new (dynamical) models of computation systems, integrated with the physical dynamics of cooperative multi-agent systems over communication networks. We intend to develop novel dynamical systems that solve distributed optimization and other computational problems and are resilient to noise and uncertainty. Toward this end, we have developed a new approach to distributed solution of systems of linear equations over unreliable networks. We have proposed a distributed computing system for the Optimal Power Flow problems and analyzed its convergence. Finally, we are developing an optimal and convex control synthesis method for systems over packet drop networks.

Distributed Solution of Linear Equations with Packet Drops and Noise

Main idea: Wang and Elia IFAC 2014

Solve linear equations via constrained optimization

General approach

- A need not to be symmetric
- Solution need not to be unique
- Solution assumed to exist

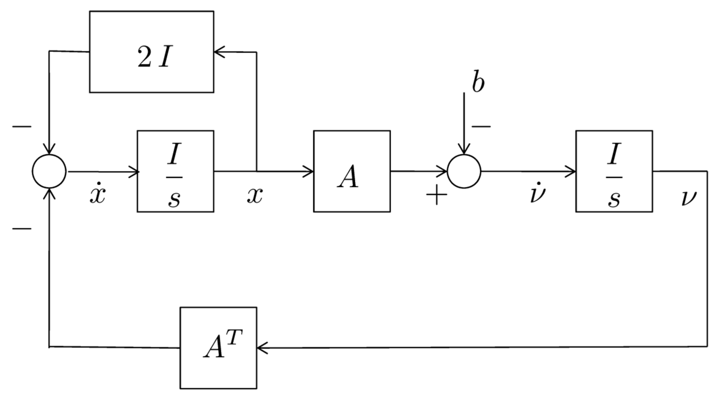
$$p^* = \min_{x\in\mathbb{R}^n} \|x\|_2^2 \quad Ax = b.$$

Lagrangian: $F(x,\nu) = x'x + \nu'(Ax-b).$

CT Optimization system

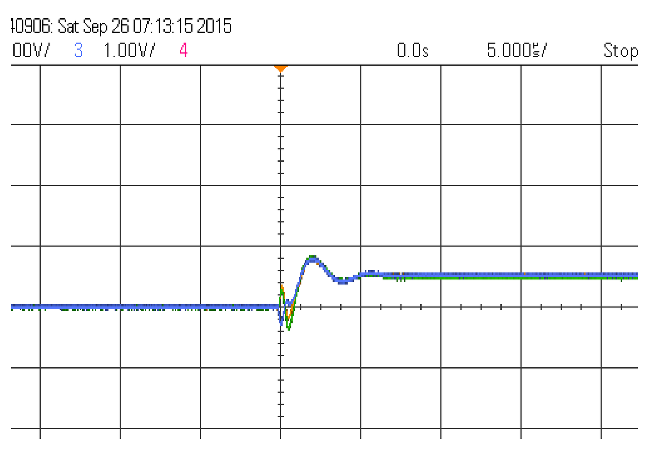
$$\dot{x} = -2x - A'\nu$$
$$\dot{\nu} = Ax - b$$

LTI Dynamics



- Exponential convergence

Analog circuit implementation



DT Optimization system

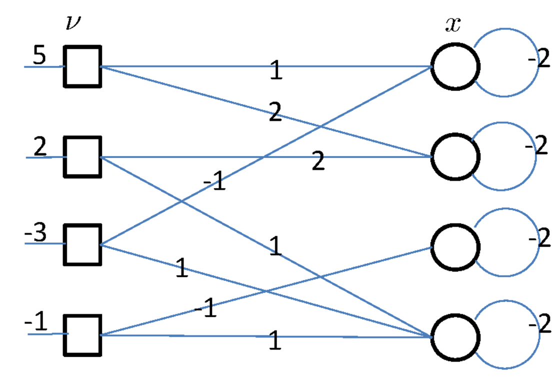
$$x(k+1) = (1-2\gamma)x(k) - \gamma A'\nu(k)$$
$$\nu(k+1) = \nu(k) - \gamma(Ax(k) - b)$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 2 \\ -3 \\ -1 \end{bmatrix}$$

- Natural Distributed Implementation

Bipartite graph

$\dot{\nu}_i = -b_i + v_i$, $v_i = \sum_{j\in\mathcal{N}_i} a_{ij}x_j$



$\dot{x}_i = -2x_i - u_i$, $u_j = \sum_{i\in\mathcal{N}_j} a_{ij}\nu_i$

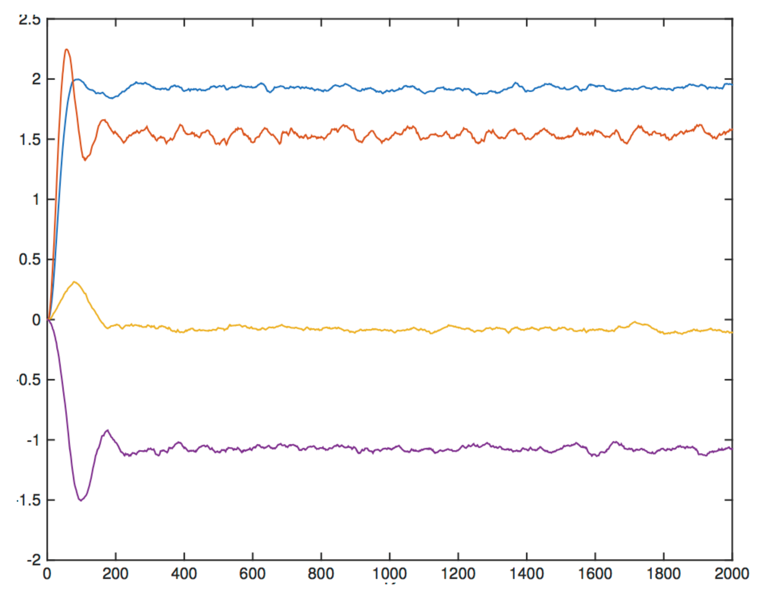
Resilience to randomly switching links

Under mild assumptions and γ small enough

- System is MS stable under Bernoulli drops
- System converges to solution a.s. (without additive noise)
- Almost-asynchronous convergence

Wang and Elia ACC 2016 (submitted)

Resilience to packet drops and additive noise



Distributed Optimal Power Flows

Formulation of optimal power flows

$$\underset{U\in\mathbb{R}^{2n}}{\text{minimize}} \sum_{l\in\mathcal{G}} f_l\left(U^T\mathbf{Y}_l U + P_l^d\right)$$

subject to

$$P_k^{\min} - P_k^d \le U^T\mathbf{Y}_k U \le P_k^{\max} - P_k^d$$
$$Q_k^{\min} - Q_k^d \le U^T\bar{\mathbf{Y}}_k U \le Q_k^{\max} - Q_k^d$$
$$(V_k^{\min})^2 \le U^T\mathbf{M}_k U \le (V_k^{\max})^2$$
$$(U^T\mathbf{Y}_{lm}U)^2 + (U^T\bar{\mathbf{Y}}_{lm}U)^2 \le (S_{lm}^{\max})^2$$

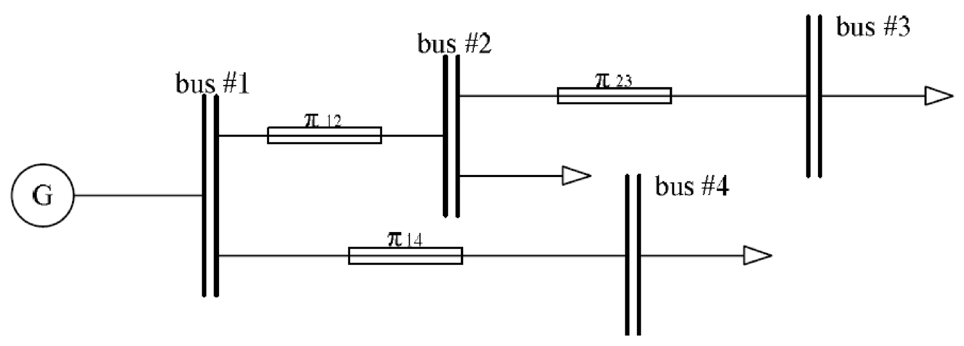
- Quadratic cost for power generation
- Non-convex quadratic constraints
- Can be solved by SDP dual relaxations

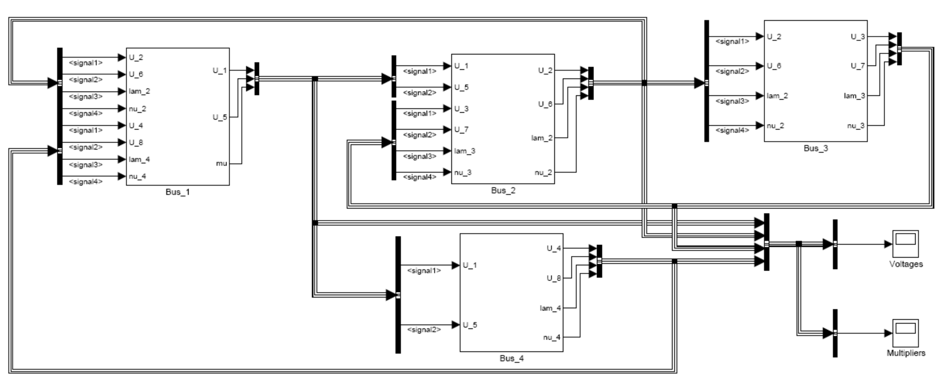
Define the primal-dual gradient dynamics

$\dot{U} = -2\mathbf{B}(U,\lambda,\gamma,\mu,\nu)U$ (the primal part)

- Any equilibrium point is a KKT point
- Saddle point (primal-dual optima) ?

Distributed computing topology = power network topology



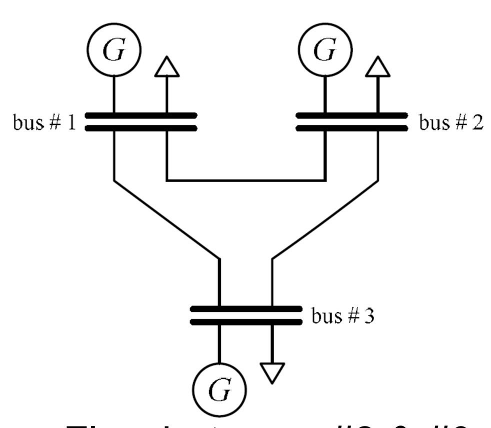


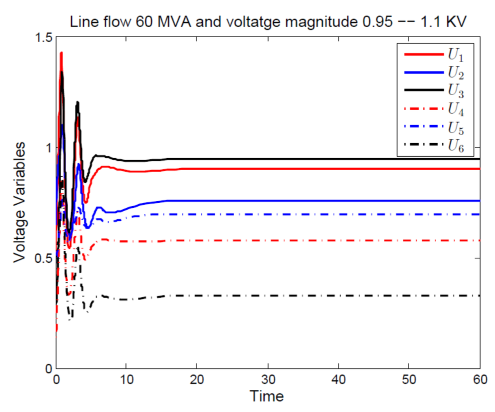
- Distributedness due to the sparsity of the network
- Each bus involves as a computing agent
- Implementation mimics the network topology

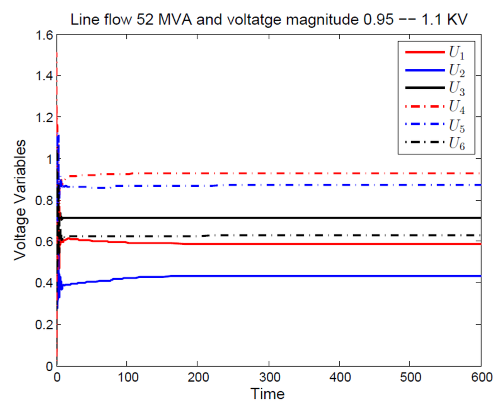
Saddle point characterizations

SDP zero Duality gap $\Leftrightarrow$ The optimal $\mathbf{B}^* \succeq 0$ $\Rightarrow$ Saddle points

Counterexample







- Flow between #2 & #3
- 60-MVA: SDP no gap
- 52-MVA: SDP has gap
- By sum-of-squares (SOS) we prove that saddle point still exists

Convergence

Under mild conditions

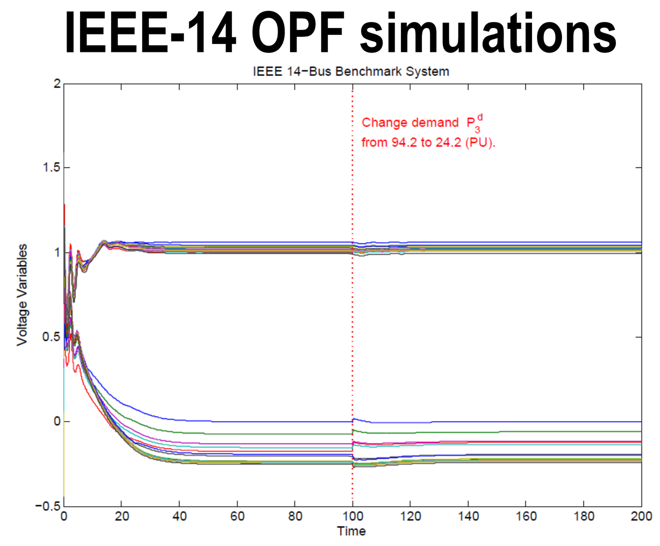
If system converges to a point with $\mathbf{B}^* \geq 0$

Then solution is optimal

If an equilibrium point results in $\mathbf{B}^* \geq 0$,

Then it is optimal and locally asymptotically stable;

IEEE-14 OPF simulations

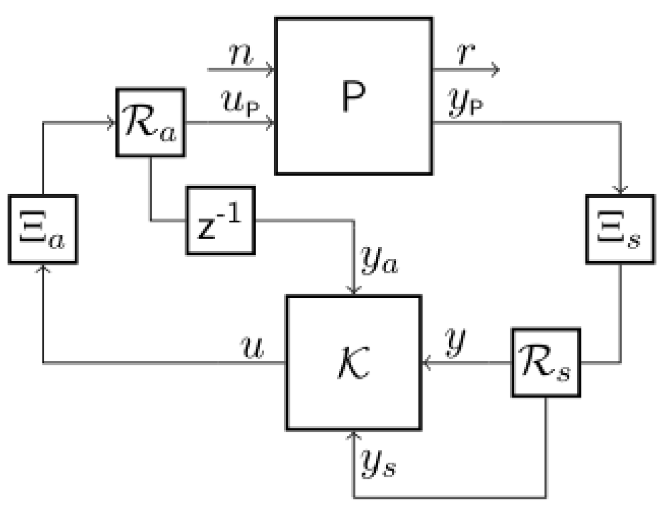


- Large domain of attraction
- Quick responses to demand changes

Ma and Elia CDC 2014, ECC 2015

Optimal Convex Controller Synthesis for Mean-square Performance over Packet Drops Networks

Networked Control Setup



$$\mathrm{P}: \begin{bmatrix} x_{\mathrm{P}}^+ \\ r \\ y_{\mathrm{P}} \end{bmatrix} = \left[\begin{array}{c|cc} A & B_n & B_u \\ \hline C_r & D_{rn} & D_r \\ C_y & D_n & 0 \end{array}\right] \begin{bmatrix} x_{\mathrm{P}} \\ n \\ u_{\mathrm{P}} \end{bmatrix}$$

- MIMO Fading Channels : Ξ_a Ξ_s
- Receivers with Acknowledgment : $\mathcal{R}_a$ $\mathcal{R}_s$
- LTI-switching Controller : $\mathcal{K}$
- Exogenous Noise : $n(k)$
- Performance Output : $r(k)$

Rich and Elia ACC 2015, ACC 2016 (submitted)

Convex Optimal MS Performance Synthesis via Separation Principle

Mean-square (MS) performance :

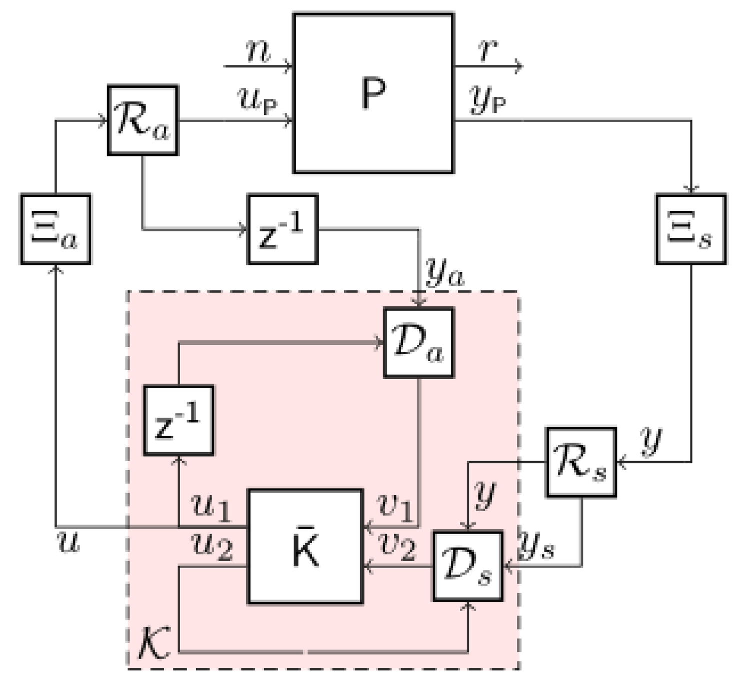
$$\nu = \lim_{k\to\infty} \mathbf{Tr}\left(\mathbf{E}\left[r(k)r^T(k)\right]\right)$$

- An analogue of H_2 performance for stochastic systems

Optimal MS performance synthesis :

- Control design accomplished via two convex semidefinite programming problems
- The overall performance cost a sum of costs from the two design sub-problems, analogous to classical H_2 performance synthesis

LTI-switching Control Implementation



$$v_1(k) = \Xi_1(k-1)u(k-1)$$
$$v_2(k) = \Xi_2(k)u_2(k) + y(k)$$

$$\bar{\mathrm{K}}: \begin{bmatrix} x_{\mathrm{K}}^+ \\ u_1 \\ u_2 \end{bmatrix} = \left[\begin{array}{c|cc} A & AB_u & -L \\ \hline F & FB_u & F_0 \\ -C_y & -C_yB_u & 0 \end{array}\right] \begin{bmatrix} x_{\mathrm{K}} \\ v_1 \\ v_2 \end{bmatrix}$$

Quasi-Convex Analysis of Limitations