Compositionality for Cyber-Physical Systems

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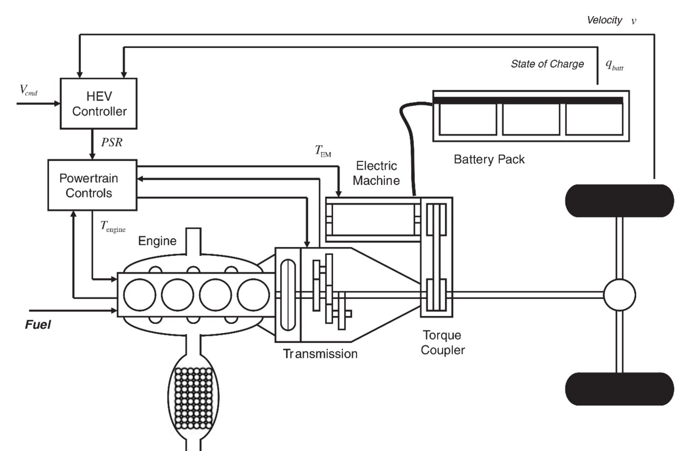
CPS Program Information

- CPS Breakthrough: ***Compositional Modeling of Cyber-Physical Systems*** *(NSF Grant: CNS-1446665)*
- PIs: Rance Cleaveland and Steve Marcus

Cyber-Physical Systems are Compositional

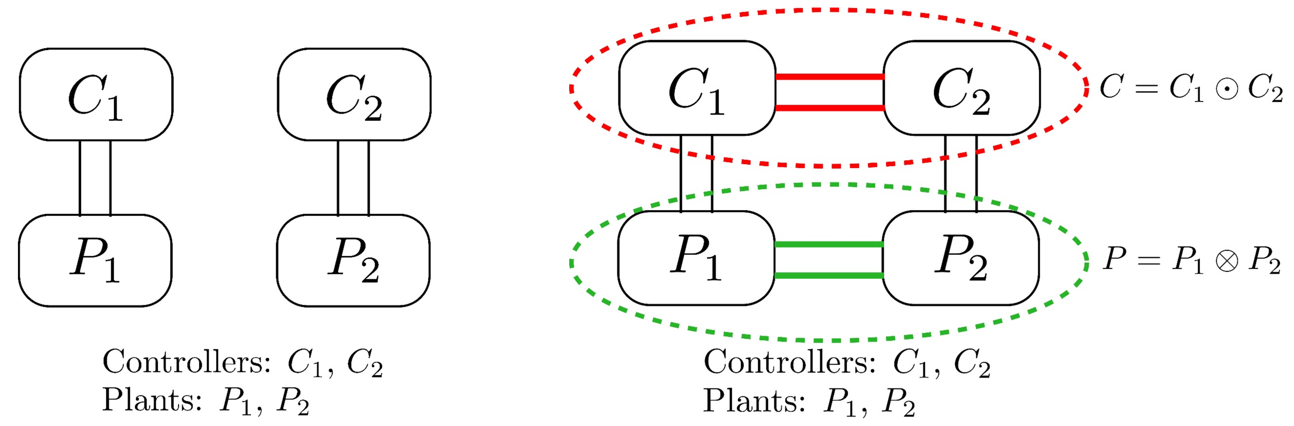
- For example, hybrid powertrains (see e.g. [3]):



(Diagram taken from [6])

Compositional Reasoning for CPSs

We need to reason about a complicated system based on models/behaviors of components:



- Can the composed system be analyzed in a rigorous way?

Algebraic Composition of Transition Systems

Famously, Milner [2] devised synchronization trees for labeled transition systems (subsequently known as Process Algebra):

Definition:

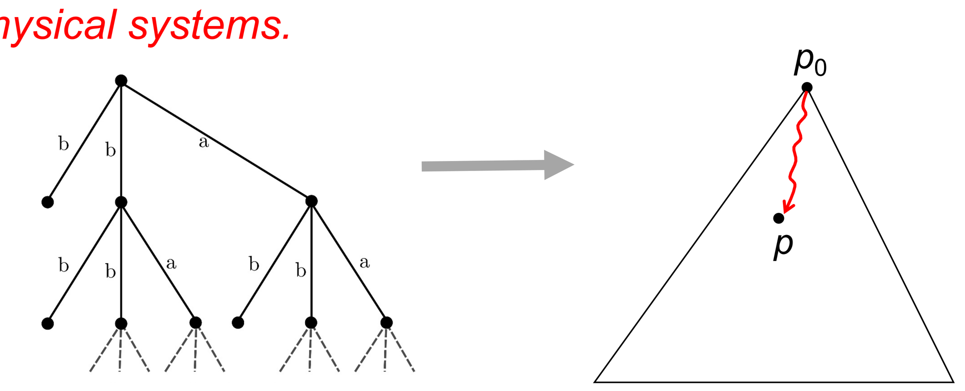
A **Synchronization Tree (ST)** over a set of labels L is a tuple $(V, E, \mathcal{L})$ where:

- (V, E) is an undirected, connected, acyclic graph with a specially identified root node r *and*
- $\mathcal{L}$ is a function $\mathcal{L}: E \to L \cup \{\varepsilon\}$

- Bisimulation is a natural (observational) notion of equivalence between trees.
- Composition: algebraic operations on synchronization trees. E.g. SOS rules:

$$\frac{P \xrightarrow{a} P' \quad a \notin S}{P|S|Q \xrightarrow{a} P'|S|Q} \qquad \frac{Q \xrightarrow{a} Q' \quad a \notin S}{P|S|Q \xrightarrow{a} P|S|Q'} \qquad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{a} Q' \quad a \in S}{P|S|Q \xrightarrow{a} P'|S|Q'}$$

- *Idea: generalize synchronization trees to enable algebraic treatment of cyber-physical systems.*



Generalized Synchronization Trees (GSTs)

Definition:

A **tree** is a partially ordered set $(P, \leq)$ with the following two properties:

1) There is a $p_0 \in P$ s.t. $p_0 \leq p$ for all $p \in P$. p_0 is the root of the tree.
2) For each $p \in P$, the set $\{p' \in P \mid p' \leq p\}$ is linearly ordered by $\leq$.

Definition:

A **Generalized Synchronization Tree (GST)** [1] over a set of labels L is a tree $(P, \leq)$ along with a labeling function $\mathcal{L}: P \backslash \{p_0\} \to L$.

Different Notions of Bisimulation for GSTs

Let $G_P = (P, p_0, \leq_P, \mathcal{L}_P)$ and $G_Q = (Q, q_0, \leq_Q, \mathcal{L}_Q)$ be two GSTs. Furthermore, let $(p, p'] \stackrel{\text{def}}{=} \{r \in P | p \leq r \leq p'\}$.

Definition:

G_P **weakly simulates** G_Q if there is a relation $R \subseteq P \times Q$ s.t. $(p_0, q_0) \in R$ and

- For any $(p, q) \in R$ and $q' \geq q$, there is a $p' \geq p$ such that $(p', q') \in R$, and there is an order-preserving bijection $\lambda: (p, p'] \to (q, q']$.

A new, semantically different kind of simulation for GSTs [1]:

Definition:

G_P **strongly simulates** G_Q if there is a relation $R \subseteq P \times Q$ s.t. $(p_0, q_0) \in R$ and

- For any $(p, q) \in R$ and $q' \geq q$, there is a $p' \geq p$ s.t. $(p', q') \in R$, and there is an order-preserving bijection $\lambda: (p, p'] \to (q, q']$ s.t. $\forall r \in (p, p'].\,(r, \lambda(r)) \in R$.

Bisimulation and Hennessy-Milner Logic

Definition:

Hennessy-Milner Logic (HML) is a set of formulas defined inductively by the rule:

$$\boldsymbol{\varphi} := \perp \mid \boldsymbol{\varphi_1} \to \boldsymbol{\varphi_2} \mid \Box\, \boldsymbol{\varphi}.$$

HML has a special connection to bisimulation between STs:

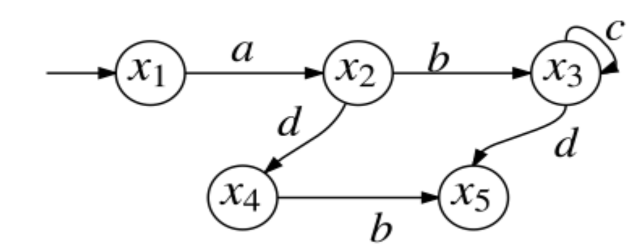
- If two STs are bisimilar, then they satisfy the same HML formulas;
- If two *image-finite* STs satisfy the all of the same HML formulas, then they are bisimilar.

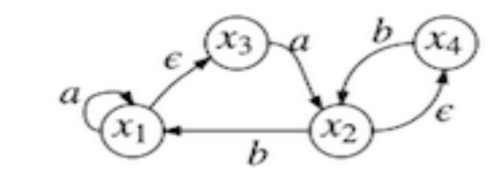
Similar relationships are currently being investigated for weak and strong bisimulation.

Why Should a CPS be Secure?

- A well-designed system must safeguard information critical to nominal operation.
- Cyber physical systems (CPSs) integrate communication, control, and computation with physical processes.
- ⇒ remote cyber attacks can cause physical damage to the system [4].
- ***Opacity*** [5]: Can a passive adversarial observer infer a *"secret"* of the system by observing the system behavior?
- *Current state of the art*: Opacity for Discrete Event Systems (DESs).
- **Present Work**: formulate notion of opacity in linear time invariant systems.
- **Future**: extend to nonlinear and hybrid systems.

Opacity for Discrete Event Systems





$\Sigma_o = \{a, b, c\}$
LBO: $L_s = \{abd\}, L_{\{ns\}} = \{abcc^*d, adb\}$
Not LBO: $L_s = \{abcd\}, L_{ns} = \{adb\}$

$\Sigma_o = \{a, b\}$
ISO: $X_s = \{x_3\}, X_{ns} = X \backslash X_s$
Not ISO: $X_s = \{x_1\}, X_{ns} = X \backslash X_s$

Language Based Opacity (LBO) ≡ Initial State Opacity (ISO) [6]

Opacity for Linear Systems

- States in a DES are discrete!
- A new framework for opacity in continuous state CPSs [7]:

$$x(t+1) = Ax(t) + Bu(t)$$
$$x(0) = x_0 \in X_0$$
$$y(t) = Cx(t)$$

- $\mathcal{K} \subset \mathbb{Z}_+$: times at which adversary observes system.
- $X_s, X_{ns} \subset X_0$: sets of initial secret, nonsecret states.
- $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n}$.

Definition:

Given $X_s, X_{ns} \subset X_0$ and $k \in \mathcal{K}$, X_s is **strongly k-initial state opaque (k-ISO)** with respect to X_{ns} if for every $x_s(0) \in X_s$ and admissible controls $u_s(0), \ldots, u_s(k)$, there exists $x_{ns}(0) \in X_{ns}$ and admissible controls $u_{ns}(0), \ldots, u_{ns}(k)$, such that $y_s(k) = y_{ns}(k)$.
X_s is **strongly $\mathcal{K}$-ISO** w.r.t. X_{ns} if X_s is strongly k-ISO w.r.t. X_{ns} for all $k \in \mathcal{K}$.

- Adversary must determine $x(0)$ from only snapshots of output.
  - ➢Might not want to reveal its presence.
  - ➢Might not have resources to make continuous observations.

Theorem:

1. Verifying k-ISO is equivalent to checking membership of the output at time k in a set of states reachable at time k, starting from X_s and X_{ns}.
2. k-ISO (under mild additional conditions) is equivalent to output controllability.

- **The Road Ahead:** Opacity in the presence of multiple adversaries [8]:
  - ➢ presence or absence of centralized coordinator.
  - ➢ presence or absence of collusion among adversaries.

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