Convex optimization for traffic estimation

Towards autonomous mobility-on-demand

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Objectives

- Estimate route-level traffic flow (demand) in urban setting
- Develop a convex optimization approach to large-scale under-determined state estimation problem
- Exploit pervasiveness of mobile network data, in particular when fused with existing loop detector data.
- *Motivation*: Accurate traffic demand estimates for urban transportation networks will enable a more effective use of traffic infrastructure (throughput, energy, reliability, robustness)

Modeling of transportation network ($\mathcal{M}$)

Given:

- $\mathcal{O}$: Origins and destinations
- $\mathcal{R}$: Routes of interest

Sensor inputs:

- b: Link flow from loop detectors (vehicle count)
- f: Origin-destination (OD) flow estimated from mobile network data

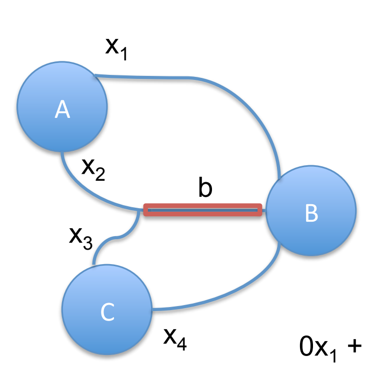
Traffic demand estimation

Problem: Given transport model $\mathcal{M}$, estimate route-level demand (flow split) x

- Convex formulation for the static problem:

$$\begin{aligned} \min_x \quad & \tfrac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} \quad & Ux = \mathbf{1} \\ & x \geq 0 \end{aligned}$$

- Small example (noiseless case)



$f_{AB} = 500$ vehicles/hr, $f_{CB} = 1000$ vehicles/hr — Flow between origin and destination (OD)

$b = 300$ vehicles/hr — Flow on link

$x_1+x_2 = 1$, $x_3+x_4 = 1$, $x_1,x_2,x_3,x_4 \geq 0$ — Distribution of flow among routes

$0x_1 + f_{AB}x_2 + f_{CB}x_3 + 0x_4 = b$ — Flow constraints

Matrix form: $\underbrace{\begin{bmatrix} 0 & f_{AB} & f_{CB} & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} = \mathbf{b}$ $\quad \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}_{\mathbf{U}} \mathbf{x} = \mathbf{1}$ $\quad \mathbf{x} \geq 0$

- Equality constraint elimination and a simplifying particular solution x_0:

$$\begin{aligned} \min_z \quad & \tfrac{1}{2}\|A(x_0 + Nz) - b\|_2^2 \\ \text{s.t.} \quad & 0 \leq z_1^{(k)} \leq \cdots \leq z_{n_k-1}^{(k)} \leq 1, \forall k \quad (\Omega) \end{aligned}$$

where $N = \text{Null(U)}, x_0 = \begin{pmatrix} \mathbf{e}_{n_1}^T & \ldots & \mathbf{e}_{n_p}^T \end{pmatrix}^T$,
$\mathbf{e}_{n_k}^T = \begin{pmatrix} 0 & \ldots & 0 & 1 \end{pmatrix} \in \mathbb{R}^{n_k}$, and $z = \begin{pmatrix} z^{(1)} & \ldots & z^{(p)} \end{pmatrix}^T$

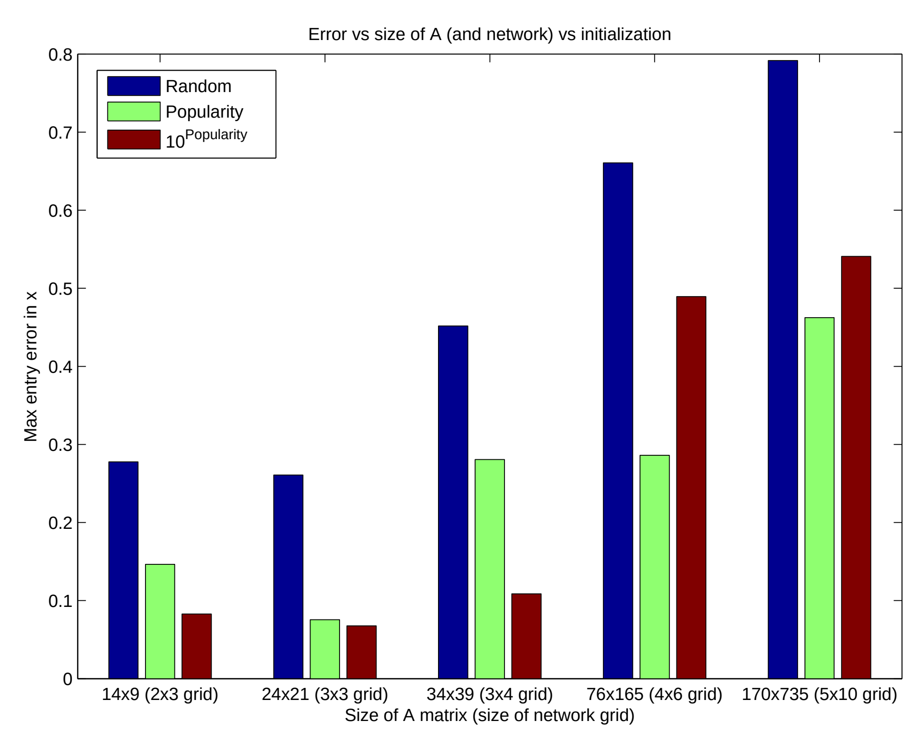
Algorithm: Spectral projected gradient (SPG) method

- Non-monotonic fast first-order descent method with super linear behavior on large scale problems
- Gradient: $\nabla f(z) = N^T A^T (A(x_0 + Nz) - b)$
- Separable constraints: projection onto the feasible set in the form of a block ordinal least squares problem

$$\begin{aligned} & P_\Omega(z) = (z^*(1), \ldots, z^*(p)) \\ & \text{where} \quad z^*(k) = \arg\min_u \|z^{(k)} - u^{(k)}\|_2^2 \\ & \text{s.t.} \quad 0 \leq u_1^{(k)} \leq \cdots \leq u_{n_k-1}^{(k)} \leq 1, \forall k \end{aligned}$$

- Isotonic regression with complete order (IRC), solved exactly in $O(n)$

Numerical experiments (preliminary)



- Synthetic data generated from grid network, considering 3 routes between all OD pairs and sensors (b) on all links
- Initialized SPG 1) randomly, 2) based on popularity, and 3) $10^{\text{popularity}}$, where popularity is a heuristic ranking of the routes
- Error metric is the worst performing route split
- High sensitivity to initialization point

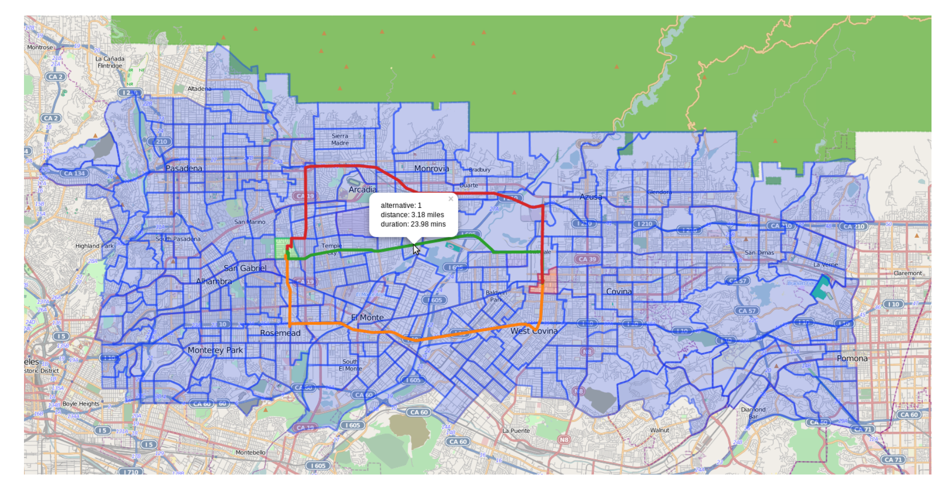
Extensions

- Iterative algorithm for higher accuracy of static flow estimation using block-coordinate descent algorithm

$$\begin{aligned} \min_{f,x} \quad & \|f - f_g\|_2^2 \\ \text{s.t.} \quad & A(f)x = b \\ & Ux = \mathbf{1}, x \geq 0 \end{aligned}$$

where f is the OD flow estimate, and $f_g(x, b)$ is the radiation model solution (heuristic)

- Dynamic problem: flows not stationary
- Noise modeling
- Deployment on I-210 corridor in California, US



I-210 corridor with 321 traffic analysis zones (TAZ) and an example of route assignment for a given OD pair

Project context

Demand prediction

- *Problem*: Given demand estimates (A, x) data and model $\mathcal{M}$, estimate the time-varying distribution of demand (per OD) P_f over a short time horizon
- Study parametric and non-parametric estimation techniques for prediction of traffic demand, using previously unavailable route-level flow estimates

Control of urban transportation

- *Problem*: Given demand forecasts P_f, demand estimates (A, x), and augmented model $\mathcal{M}'$, actuate to optimize for metrics
- Augmented transport model $\mathcal{M}'$ with actuators (autonomous vehicles, stop lights) and performance metrics (throughput, energy consumption, average user wait time, etc.)
- Pilot deployment on California I-210 corridor in the form of an advanced decision support system to minimize congestion by coordinating heterogeneous capacity vehicles
- Study emergent behaviors in multi-agent systems as a means to control traffic, parameterized by level of control, e.g. flocking, shepherding, load balancing, virtual stop lights

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