Differential Privacy in Cyber-Physical Systems



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- 5 Billion people to be connected by 2015 (Source: NSN)
- 7 trillion wireless devices serving 7 billion people in 2017 (Source: WWRF)
 - 1000 wireless devices per person?

[Courtesy: Niko Kiukkonen, Nokia]



TerraSwarm Challenges





Privacy Concerns - Cyber-physical applications



[image: Inst. Mech. Eng.]



Privacy Concerns - SmartGrid





Privacy Concerns – Mobile Millenium







• What is privacy, formally?

• Is there a tradeoff between privacy and utility?

• Privacy-aware estimation and control

• Systems and control tools for privacy

Privacy is not anonymity





- Privacy breaches generally due to existence of side information
 - Mass. GIC medical db w/ voter registration db (Sweeney, 1997)
 - Netflix prize w/ IMDB (Narayana & Shmatikov, 2008)
 - Individual online transactions w/ changes in public recommendation systems (Calandrino et al., 2011)
 - Anonymity in location based services
- Can't know what the adversary knows, or might know in the future.

Differential Privacy

Cynthia Dwork. "Differential privacy." Automata, languages and programming. Springer Berlin Heidelberg, 2006. 1-12.



Differential Privacy

Cynthia Dwork

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Abstract. In 1977 Dalenius articulated a desideratum for statistical databases: nothing about an individual should be learnable from the database that cannot be learned without access to the database. We give a general impossibility result showing that a formalization of Dalenius' goal along the lines of semantic security cannot be achieved. Contrary to intuition, a variant of the result threatens the privacy even of someone not in the database. This state of affairs suggests a new measure, differential privacy, which, intuitively, captures the increased risk to one's privacy incurred by participating in a database. The techniques developed in a sequence of papers [8, 13, 3], culminating in those described in [12], can achieve any desired level of privacy under this measure. In many cases, extremely accurate information about the database can be provided while simultaneously ensuring very high levels of privacy.

1 Introduction

A statistic is a quantity computed from a sample. If a database is a representative sample of an underlying population, the goal of a privacy-preserving statistical database is to enable the user to learn properties of the population as a whole, while protecting the privacy of the individuals in the sample. The work discussed herein was originally motivated by exactly this problem: how to reveal useful information about the underlying population, as represented by the database, while preserving the privacy of individuals. Fortuitously, the techniques developed in [8, 13, 3] and particularly in [12] are so powerful as to broaden the scope of private data analysis beyond this orignal "representatitive" motivation, permitting privacy-preserving analysis of an object that is itself of intrinsic interest. For instance, the database may describe a concrete interconnection network – not a sample subnetwork – and we wish to reveal certain properties of the network without releasing information about individual edges or nodes. We therefore treat the more general problem of *privacy-preserving analusis of data*.

A rigorous treatment of privacy requires definitions: What constitutes a failure to preserve privacy? What is the power of the adversary whose goal it is to compromise privacy? What auxiliary information is available to the adversary (newspapers, medical studies, labor statistics) even without access to the database in question? Of course, utility also requires formal treatment, as releasing no information or only random noise clearly does not compromise privacy; we

M. Bugliesi et al. (Eds.): ICALP 2006, Part II, LNCS 4052, pp. 1–12, 2006. © Springer-Verlag Berlin Heidelberg 2006

Differential Privacy, informally



• Set-up:



- Key Idea:
 - A differentially private mechanism randomly perturbs the answers to a query so that the output distribution over answers does not vary much if any given individual participates or not
 - Hard to infer if the data of any individual was used or not to answer the query

Differential Privacy, informally





Differential Privacy, formally



• Set-up:



• Formally

- Adj(d,d') a symmetric binary relation on the set D of databases
- Adjacent databases differ by the data of a single individual
- A mechanism $M: D \times \Omega \to (R, \mathcal{M})$ is (ϵ, δ)-DP if

for all sets $\forall S \in \mathcal{M}$ and all databases d, d' s.t. Adj(d,d'), we have

 $\mathbb{P}(M(d) \in S) \le e^{\epsilon} \mathbb{P}(M(d') \in S) + \delta$



$\mathbb{P}(M(d) \in S) \le e^{\epsilon} \mathbb{P}(M(d') \in S) + \delta$

- Constant ε is typically small (i.e. ~ 0.1) multiplicative error
- Constant δ is very small (i.e. ~0.01) additive error
- If δ =0 then we have (ϵ ,0)-DP or simply ϵ -DP
- Privacy definition depends on adjacency relation

Two Basic Differentially Private Mechanisms



- Database of CPS PI salaries d = [d₁,...d_n]
- Adjacency: $\operatorname{Adj}(d, d')$ iff for some i, $|d_i d'_i| \le \rho_i$ $d_j = d'_i, j \ne i$

 $-R = max \{\rho_i\}$

• Analyst Query:

$$q(d) = \frac{1}{n} \sum_{i=1}^{n} d_i$$

- The mechanism $M(d) = q(d) + Lap(R/\epsilon)$ is ϵ -DP
- The mechanism M(d) = q(d) + N(0,($\kappa(\delta,\epsilon)$ R)²) is (ϵ,δ)-DP

$$\kappa(\delta,\epsilon) \le 2\sqrt{2\ln(2/\delta)}/\epsilon$$
 $\left(\text{Lap(b) pdf: } \frac{1}{2b}e^{-|x|/b}\right)$



If mechanism M(d) is (ε, δ) -differentially private and f is an arbitrary function, then f (M(d)) is also (ε, δ) -differentially private

- f as the adversaries : Models arbitrary auxiliary or side information information the adversary may have. Privacy guarantee holds no matter what adversary does.
- f as our algorithm: If we access the database in a differentially private way, we don't have to worry about how our algorithm post-processes the result.





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Differentially Private Filtering

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Abstract-Emerging systems such as smart grids or intelligent transportation systems often require end-user applications to continuously send information to external data aggregators performing monitoring or control tasks. This can result in an undesirable loss of privacy for the users in exchange of the benefits provided by the application. Motivated by this trend, this paper introduces privacy concerns in a system theoretic context, and addresses the problem of releasing filtered signals that respect the privacy of the user data streams. Our approach relies on a formal notion of privacy from the database literature, called differential privacy, which provides strong privacy guarantees against adversaries with arbitrary side information. Methods are developed to approximate a given filter by a differentially private version, so that the distortion introduced by the privacy nechanism is minimized. Two specific scenarios are considered First, the notion of differential privacy is extended to dynamic systems with many participants contributing independent input signals. Kalman filtering is also discussed in this context, when a released output signal must preserve differential privacy for the measured signals or state trajectories of the individual participants. Second, differentially private mechanisms are described to approximate stable filters when participants contribute to a single event stream, extending previous work on differential privacy under continual observation

Index Terms-Privacy, Filtering, Kalman Filtering, Estimation

I. INTRODUCTION

RAPIDLY growing number of applications requires A users to release private data streams to third-party applications for signal processing and decision-making purposes. Examples include smart grids, population health monitoring, online recommendation systems, traffic monitoring, fuel consumption optimization, and cloud computing for industrial control systems. For privacy, confidentiality or security reasons, the participants benefiting from the services provided by these systems generally do not want to release more information than strictly necessary. In a smart grid for example, a customer could receive better rates in exchange of continuously sending to the utility company her instantaneous power consumption, thereby helping to improve the demand forecast mechanism. In doing so however, she is also informing the utility or a potential eavesdropper about the type of appliances she owns as well as her daily activities [1]. Similarly, individual private signals can be recovered from published outputs aggregated from many users, and anonymizing a dataset is not enough to guarantee privacy, due to the existence of public side information. This is demonstrated in [2], [3] for example,

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where private ratings and transactions from individuals on commercial websites are successfully inferred with the help of information from public recommendation systems. Emerging traffic monitoring systems using position measurements from smartphones [4] is another application area where individual position traces can be re-identified by correlating them with public information such as a person's location of residence or work [4], [5]. Hence, the development of rigorous privacy preserving mechanisms is crucial to address the justified concerns of potential users and thus encourage an increasing level of participation, which can in turn greatly improve the efficiency of these large-scale systems.

Precisely defining what constitutes a breach of privacy is a delicate task. A particularly successful recent definition of privacy used in the database literature is that of *differential privacy* [6], which is motivated by the fact that any useful information provided by a dataset about a group of people can compromise the privacy of specific individuals due to the existence of side information. Differentially private mechanisms randomize their responses to dataset analysis requests and guarantee that whether or not an individual chooses to contribute her data only marginally changes the distribution over the published outputs. As a result, even an adversary cross-correlating these outputs with other sources of information cannot infer much more about specific individuals after publication than before [7].

Most work related to privacy is concerned with the analysis of static databases [6], [8]-[10], whereas cyber-physical systems clearly emphasize the need for mechanisms working with dynamic, time-varying data streams. Recently, the problem of releasing differentially private statistics when the input data takes the form of a binary stream describing event occurrences aggregated from many participants has been considered in [11]-[13]. This work forms the basis for the scenario studied in Section VI, and is discussed in more details in Section VI-B. However, most of this paper is devoted to a different situation where participants individually provide real-valued signals. A differentially private version of the iterative averaging algorithm for consensus is considered in [14]. In this case, the input data to protect consists of the initial values of the participants and is thus a single vector, but the update mechanism subject to privacy attacks is dynamic. Information-theoretic approaches have also been proposed to guarantee some level of privacy when releasing time series [15], [16]. However, the resulting privacy guarantees only hold if the statistics of the participants' data streams obey the assumptions made (typically stationarity, dependence and distributional assumptions), and require the explicit statistical modeling of all available side information. This task is very difficult in general as new, as-yet-unknown side information can become available after releasing the results. In contrast, differential privacy is a worst-case notion

Private Filtering : Event Streams Counters



- Processing binary input signal (0-1 events) DP linear filter approximation
 - Event-level privacy: each user contributes a unique event: two inputs differing by a single event must be hard to distinguish
 - Ex: Counter [Dwork et al., 2010], [Chan et al., 2011] (unstable stable filter)
 - Ex: Certain stable filters with slowly decreasing impulse response [Bolot et al., 2011]
 - Complicated algorithms (non-recursive, not finite memory), hard to generalize, poor performance of the approach for the approximation of stable filters





• Approximate filter $y = \sum_{i=1}^{n} G_i u_i$ by a differentially private version



Adjacency relation

 $\operatorname{Adj}^{b}(u, u')$ iff for some $i, ||u_i - u'_i||_2 \leq B$, and $u_j = u'_j$ for all $j \neq i$.

Differentially Private Filtering



• Approximate filter $y = \sum_{i=1}^{n} G_i u_i$ by a differentially private version

- Adjacency relation Adj^b(u, u') iff for some $i, ||u_i - u'_i||_2 \le B$, and $u_j = u'_j$ for all $j \ne i$.

Theorem : For (ε, δ) -differential privacy, can add white Gaussian noise proportional to the maximum incremental gain with respect to the input channels

- Incremental gain: $||(Gu)_{0:T} (Gu')_{0:T}||_2 \le \gamma ||u_{0:T} u'_{0:T}||_2, \forall u, u', \forall T$
- Generalizes mechanism of [Dwork et al., 2006] to dynamic setting with continuous streams of real-time data
- System and control theoretic tools can be used to design differentially private mechanisms for continuous data streams

Differentially Private Filtering



Approximate filter $y = \sum_{i=1}^{n} G_i u_i$ by a differentially private version



Adj^b
$$(u, u')$$
 iff for some $i, ||u_i - u'_i||_{r_i} \le b_i$,
and $u_j = u'_j$ for all $j \ne i$.

$$B \ge \max_{1 \le i \le n} \{\gamma_{r_i,1}(\mathcal{G}_i) \, b_i\}$$

Theorem: The mechanism M(u)=G(u)+w where w is white noise with

 $w_t \sim (Lap(B/\epsilon))^m$

is ε -differentially private.

Differentially Private Filtering



Approximate filter $y = \sum_{i=1}^{n} G_i u_i$ by a differentially private version



Adj^b
$$(u, u')$$
 iff for some $i, ||u_i - u'_i||_{r_i} \le b_i$,
and $u_j = u'_j$ for all $j \ne i$.

 $\sigma \ge \kappa(\delta, \epsilon) \max_{1 \le i \le n} \{ \gamma_{r_i, 2}(\mathcal{G}_i) \, b_i \}.$

Theorem: The mechanism M(u)=G(u)+w where w is white noise with

$$w_t \simeq N(0, \sigma^2 I_m)$$

is (ε, δ) -differentially private.

Differential Privacy Architectures



• Two basic architectures for (ε, δ) -differential privacy



Input perturbation mechanism Output perturbation mechanism

Event Streams Counters





$$||G_i||_2^2 = \frac{1}{l}, ||G_i||_{\infty} = 1$$

 \rightarrow output perturbation better than input perturbation iff n>l

Event Streams Counters





A Traffic Monitoring Example



• Traffic velocity estimation using individual location traces from smartphones (ex: Mobile Millenium, Bayen et al.)

$$\begin{aligned} x_{i,t+1} &= \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} x_{i,t} + \sigma_{i1} \begin{bmatrix} T_s^2/2 & 0 \\ T_s & 0 \end{bmatrix} w_{i,t}, \\ y_{i,t} &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_{i,t} + \sigma_{i2} \begin{bmatrix} 0 & 1 \end{bmatrix} w_{i,t} \end{aligned}$$

Estimate
$$\frac{1}{n} \sum_{i=1}^{n} x_{i,2,t}$$





• For Kalman Filtering, we have additional public information about the dynamics generating the user signals

- Estimation objective: $z_t = \sum_{i=1}^n L_i x_{i,t}$ $\min_{\hat{z}} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[\|z_t \hat{z}_t\|_2^2 \right]$
- Adjacency relation:

 $\operatorname{Adj}^{\rho}(x, x')$ iff for some $i, ||x_i - x'_i||_2 \le \rho_i$, and $x_j = x'_j$ for all $j \ne i$.

- Cannot distinguish between two sufficiently close state trajectories of a user



$$Adj_{\mathcal{S}}^{\rho}(x, x') \text{ iff for some } i, \|S_i x_i - S_i x'_i\|_2 \le \rho_i, \\ (I - S_i) x_i = (I - S_i) x'_i, \text{ and } x_j = x'_j \text{ for all } j \ne i.$$

Theorem 5. Let $\epsilon, \delta > 0$. A mechanism releasing $(\sum_{i=1}^{n} L_i \mathcal{K}_i y_i) + \gamma \kappa(\delta, \epsilon) \nu$, where ν is a standard white Gaussian noise independent of $\{w_i\}_{1 \leq i \leq n}, \{x_{i,0}\}_{1 \leq i \leq n}$, and $\gamma = \max_{1 \leq i \leq n} \{\gamma_i \rho_i\}$, with γ_i the \mathcal{H}_{∞} norm of $L_i \mathcal{K}_i C_i S_i$, is (ϵ, δ) -differentially private for the adjacency relation (11).



 Can add the previous input and output perturbation schemes to the standard Kalman filter

Input pert.





• For the input perturbation scheme, can take into account the additional privacy-preserving noise in the redesign of the KF





• For the output perturbation scheme, can redesign the filter to trade-off the estimation error and the H_{∞} norm of the filter – Overall MSE is

$$\left(\sum_{i=1}^{n} \|TF(w_i \to e_i)\|_2^2\right) + \kappa(\delta, \epsilon)^2 \max_{1 \le i \le n} \{\rho_i^2 \|TF(x_i \to \hat{z}_i)\|_{\infty}^2\}$$

- Multi-objective H_2/H_∞ optimization problem
 - Lyapunov shaping using Linear Matrix Inequalities
 - Distinguish between stable and unstable dynamics

A Traffic Monitoring Example

 Traffic velocity estimation using individual location traces from smartphones (ex: Mobile Millenium, Bayen et al.)

$$x_{i,t+1} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} x_{i,t} + \sigma_{i1} \begin{bmatrix} T_s^2/2 & 0 \\ T_s & 0 \end{bmatrix} w_{i,t},$$
$$y_{i,t} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_{i,t} + \sigma_{i2} \begin{bmatrix} 0 & 1 \end{bmatrix} w_{i,t}$$
Estimate $\frac{1}{n} \sum_{i=1}^n x_{i,2,t}$



$$\rho = 100m$$

$$\epsilon = \ln 3$$

$$\delta = 0.05,$$

$$T_s = 1s$$

$$\sigma_{i1} = \sigma_{i2} = 1$$

$$n = 200$$



Traffic Monitoring – input versus output architectures









Traffic Monitoring – convergence/privacy tradeoff









- Many cyber-physical applications raise privacy concerns that need to be addressed to encourage user participation
- Need privacy-preserving mechanisms for various types of dynamic systems and data
- Characterizing privacy-utility tradeoff requires a quantitative definition of privacy
- System and control theoretic tools (optimal estimators, system gains) can be used to design differentially private mechanisms



A science of CPS privacy



