# Distributed Algorithms for Wide-Area Control of Power System Oscillations

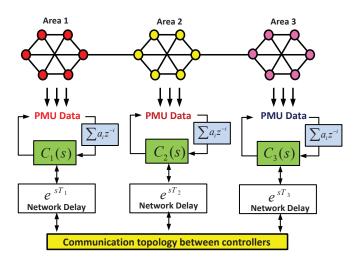
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Abstract—This paper presents a distributed optimization algorithm to address the highly pertinent problem of wide-area damping control of large-scale electric power systems using synchronized phasor measurements. Our approach consists of a three-step strategy. First, Synchrophasors from selected nodes in a power network are used to identify offline dynamic models of the dominant areas of the network. Thereafter, a linear controller is designed for this reduced-order model to shape the intermachine oscillation dynamics. Finally, algorithms are developed to invert this design to realistic local controllers in each area by optimizing the controller parameters until their interarea response matches the closed-loop inter-machine response achieved in the second step. A model reference control design following this three-step strategy was recently proposed in [1] using a centralized controller. Our results in this paper extend that design by posing the problem purely from a perspective of distributed optimization, and shows how excitation control parameters can be updated in a distributed way for inter-area damping. This idea will be particularly important as within the next few years the number of PMUs in the US transmission network scales to the thousands necessitating a distributed processing architecture for wide-area monitoring and control. We illustrate our results with a prototype power system models representing a transfer path in the US west coast grid.

*Index Terms*—Distributed control, identification, parameter tuning, optimization, transmission delay, Synchrophasors.

## I. INTRODUCTION

Following the Northeast blackout of 2003, Wide-Area Measurement System (WAMS) technology using Phasor Measurement Units (PMUs) has largely matured for the North American grid. However, as the number of PMUs scales up into the thousands in the next few years under the US Department of Energy's smart grid demonstration initiative. Independent System Operators (ISO) and utility companies are struggling to understand how the resulting gigantic volumes of real-time data can be efficiently harvested, processed, and utilized to solve wide-area monitoring and control problems for any realistic power system interconnection. It is rather intuitive that the current state-of-the-art centralized communication and information processing architecture of WAMS will no longer be sustainable under such a data explosion, and a completely distributed cyber-physical architecture will need to be developed. In the Eastern Interconnection (EI) of the US grid, for example, about 60 PMUs are currently streaming data via the Internet to a super phasor data concentrator (SPDC) which is handling about 100,000 data points per second.



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Fig. 1. Proposed mechanisms of distributed monitoring and control

This architecture will no doubt become untenable as the EI scales up to 300-400 PMUs by 2015. The North American Synchrophasor Initiative (NASPI) is currently addressing this architectural problem by developing new communication and computing protocols for WAMS through NASPInet and Phasor Gateway to facilitate PMU data communication between multiple utilities and control centers. However, almost no attention has yet been paid to perhaps the most critical consequence of this envisioned distributed architecture - namely distributed algorithms. Partly due to the priorities set forth by PMU installations, the NASPI community has not yet delved into investigating how centralized algorithms for wide-area control can be translated into a distributed computing framework once the decentralized WAMS architecture is realized in the next three to four years. Development of such algorithms will obviously be imperative not only for increasing reliability by eliminating single-point failures, but also for minimizing network transit. Transmitting data across a wide-area communication network (WAN) is expensive, the links can be relatively slow, and the bandwidth per dollar will indeed grow slower than other computing resources leading to distributed PMU data processing followed by transmission of full or partially processed outputs as a natural choice.

Motivated by this challenge, in this paper we address the problem of distributed wide-area damping control using Synchrophasor feedback. The proposed architecture is shown in Figure 1. We assume the system to be composed of multiple areas with a given set of PMU locations. Our strategy is to first derive an offline reference model for the closed-loop system

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using model reduction, categorize the available PMUs into area-level disjoint sets, and finally run a distributed optimization problem for tuning the controller parameters of each arealevel power system stabilizer (or group of stabilizers) (PSS) to match the cumulative output of the actual model with that of the reference model. The main idea behind our design is a so-called, novel control inversion framework which allows PMU-based linear power system stabilizers (PSS) designs, developed for reduced-order power systems, to be inverted to PSS controllers in higher-order systems via suitable optimization methods. A model reference control design following this strategy was presented for two-area power systems in our recent work, but the implementation was still centralized. This paper extends that design by posing the control problem purely from a distributed optimization perspective. The approach consists of three precise steps, namely:

1. *Model Reduction/Dynamic Equivalencing* - where PMU data are used offline to identify equivalent models of the oscillation clusters of the entire power system based on the differences in their coupling strengths. Our objective is to design the PSS control for damping the oscillations between these areas in a distributed fashion, for which we will simply assume that the area models are available to us by prior identification methods.

2. Aggregate Control - where output-feedback based linear PSS are designed to achieve a desired closed-loop transient response between every pair of clusters in the reduced-order system, and

3. *Control Inversion*- where the aggregate control design is distributed and tuned back to actual realistic controllers at the generator terminals until the inter-area responses of the full-order power system matches the respective inter-machine responses of the reduced-order system.

The most generic way to formulate the control inversion problem in the final step is to use functional optimization. This means that assuming standard second-order swing dynamics with first-order excitation for the  $j^{th}$  hypothetically aggregated machine in the reduced-order power system model, one may first design output-feedback excitation controllers:

$$u_{j} = f(y_{1}(t), y_{2}(t), ..., y_{m}(t), k_{1}, k_{2}, ..., k_{m})$$
(1)

where  $y_i(t)$  is a chosen set of variables (eg. voltage magnitude/phase angle, frequency, etc.) measured over time  $t \ge 0$ by a PMU installed at the  $i^{th}$  bus in the reduced network, and  $f(\cdot)$  is a smooth, nonlinear damping function producing a desired inter-machine transient response. Next,  $u_j$  needs to be distributed to each local machine belonging to the  $j^{th}$ area. A plausible approach for this would be to construct nonlinear functions  $\rho(\cdot)$  mapping each of the feedback gains  $(k_1, k_2, ..., k_m)$  to each such machine. Stacking these functions  $\rho(\cdot)$  and the gains  $k_j$  into vectors  $\mathcal{R}$  and  $\mathcal{K}$ , respectively, the problem that we must, therefore, solve is:

$$\min_{\mathcal{R}(\mathcal{K})} \sum_{i=1}^{n^*} \int_o^T ||x_{ij}(t, \mathcal{R}(\mathcal{K})) - \bar{x}_{ij}(t, \mathcal{K})||_2 \quad \text{st. } \mathcal{K} \in \mathcal{K}^*,$$
(2)

for all  $j \in \mathcal{N}_i$ , over time  $t \in [0, T]$ , where:  $n^*$  is the total number of areas,  $\mathcal{N}_i$  is the index set for the neighboring

### II. MAIN RESULT

We next describe a distributed optimization approach for designing PSS controllers interacting across areas for damping the interarea oscillation modes  $\lambda_s$ . The first step of the design is based on the reduce-order model identified using PMU data, as discussed in the previous section, followed by a control inversion strategy, recently developed in [1]. These may be summarized as follows:

1. A linear control design is performed for this reducedorder model to guarantee a desired dynamic performance for all the inter-machine power flows (which are equivalent to the inter-area flows in the full-order model). These damped power flow signals are used as references for the wide-area design in the next step.

2. A distributed optimization problem is solved for tuning actual PSS parameters in the full-order system until the *interarea* response of this system replicates the closed-loop *inter-machine* reference obtained in the previous step.

Mathematically, the second step can be posed as follows. Let the state-variable model for the  $j^{th}$  chosen generator with a tunable PSS, for j = 1, ...m, be given as

$$\dot{\delta}_j = \omega_j$$
 (3)

$$M_j \dot{\omega}_j = P_{mj} - D_j \omega_j - P_{ej} \tag{4}$$

$$\tau_{j}\dot{E}_{j} = -\frac{x_{dj}}{x'_{dj}}E_{j} + \frac{x_{dj} - x'_{dj}}{x'_{dj}}\cos(\delta_{j} - \theta_{j}) + E_{Fj}$$
(5)

where (5) represents the excitation system dynamics of the generator, and  $E_{Fj}$  is the excitation control feedback for the PSS. Let  $\mathcal{M}$  be the set of bus indices where a PMU is installed, and  $\mathcal{M}^j$  be the subset of  $\mathcal{M}$  that are available for output-feedback to the  $j^{th}$  PSS. The measurements in the set  $\mathcal{M}^j$  are denoted as  $y_{\mathcal{M}^j}(t)$ . Let the set of boundary buses separating the areas be  $\mathcal{E}_b$ , and the communication graph between the different controllers be  $\mathcal{G}$ . The distributed control problem then reduces to designing the function  $\psi(\cdot)$  for

$$E_{Fj}(t) = \psi_j(y_{\mathcal{M}^k}(t), x_{k \in \mathcal{N}_j}(t - \tau_{jk}), t), \ j = 1, .., m$$
 (6)

where  $\mathcal{N}_j$  is the neighbor set of  $j^{th}$  controller following from  $\mathcal{G}$ , and  $\tau_{jk}$  is the communication delay in the channel connecting the  $j^{th}$  and  $k^{th}$  controllers, such that all closedloop state responses are bounded over time, and the 'slow' oscillation component of the relative phase angle difference  $x_{pq}(t) \triangleq (x_p(t)-x_q(t))$  between every pair of boundary nodes  $(p,q) \in \mathcal{E}_b$  satisfy a desired response, which is precisely the corresponding inter-machine reference signal designed in Step 2. However, we must remember that in the actual system the signal  $x_{pq}(t)$  will contain the contribution of both local and inter-area modes. Hence, for an accurate tracking we must filter this signal through a band-pass filter (BPF), whose passband is designed to cover the typical inter-area frequency spectrum (0.1-1 Hz). We denote this BPF as G(s), and design it using standard Butterworth filters. In Figure 1 this filter is indicated in discrete-time as  $G(z) = \sum_i a_i z^{-i}$ . The filter coefficients can be designed, for example, using convex optimization. Furthermore, we consider our PSS designs to be linear, which means that essentially we need to design an output feedback controller of the form

$$C_{j}(s) = \frac{\rho_{j0} + \rho_{j1}s + \rho_{j2}s^{2} + \dots + \rho_{jj_{a}}s^{j_{a}}}{\vartheta_{j0} + \vartheta_{j1}s + \vartheta_{j2}s^{2} + \dots + \vartheta_{jj_{b}}s^{j_{b}}}$$
(7)

where  $j_a$  and  $j_b$  are fixed integers (practically, both should be less than or equal to 3 since high-order controllers increase processing delay). Denote the controller parameter set as

$$\mathcal{R}_j = \{\rho_{j0}, \dots, \rho_{jj_a}, \vartheta_{j0}, \dots, \vartheta_{jj_b}\}.$$
(8)

The distributed control design problem then simply reduces to a distributed parametric optimization problem for finding the optimal  $\mathcal{R}_j$ , j = 1, ..., m, that guarantees:

$$x(t) \in l_2, \quad \min ||G(s)[x_{pq}](t) - x_{pq}^d(t)||_2, \quad \forall (p,q) \in \mathcal{E}_b \quad (9)$$

over  $t \in [0, t_f]$ , where,  $x_{pq}^d(t)$  is the desired power flow response following from the pre-designed model in Step 2.

## III. EXAMPLES

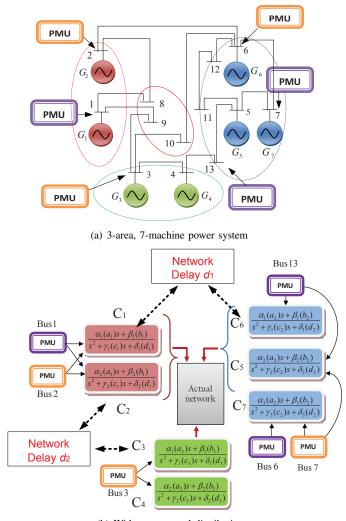
In this section we illustrate the results by considering a 3area model of Pacific AC intertie. The structure of this system is shown in Figure 2(a), and is based on the WA-CA north south power oscillation characteristics for which detailed PMU data analysis has been done. The system is first reduced to an equivalent 3-machine system characterized by 3 aggregate inertias. All the machines are classical generator models with identical parameters except for the machine inertias in each area, as given in the Appendix. A relatively low value of  $H_3$  makes the system act almost like a two-area system with a dominant slow mode of approximately 0.5 Hz. Hence, a second order Butterworth band-pass filter is designed for filtering the modes from the PMU measurements of the form:

$$G(s) = \frac{s(\omega_u - \omega_l)}{s^2 + (\omega_u - \omega_l)s + \omega_u\omega_l}$$
(10)

with  $\omega_u = 0.9$  Hz and  $\omega_l = 0.2$  Hz. A simple lead controller for Area 1 is designed to generate desired dynamic responses for the inter-machine power flows. For the actual system in Figure 2(b)  $C_7(s)$  is kept fixed, and  $C_1(s)$  through  $C_6(s)$  are designed using 2 zeros and 3 poles each, i.e.

$$C_i(s) = \frac{a_{i1}s + a_{i2}}{s^2 + b_{i2}s + b_{i3}}, \quad i = 1, .., 6.$$
(11)

The controller parameters are provided in the Appendix. An all-to-all communication between the 6 controllers is assumed. The communication between  $C_1$ - $C_6$  and  $C_2 - C_3$  are considered to be most prone to communication delays. It was observed that the closed-loop matching deteriorates as the delay increases, and after a certain threshold the matching becomes unacceptable. It can be verified that the integrated error between the distributed and centralized solution over  $t \in [0, 5]$  is less than a set threshold of  $\gamma = 10^{-4}$  for  $d_1 = d_2 = 0$ . The simulations are repeated for 450 MW power transfer between Area 1 and 2 with similar observations.



(b) Wide-area control distribution

Fig. 2. Distributed parametric optimization

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