

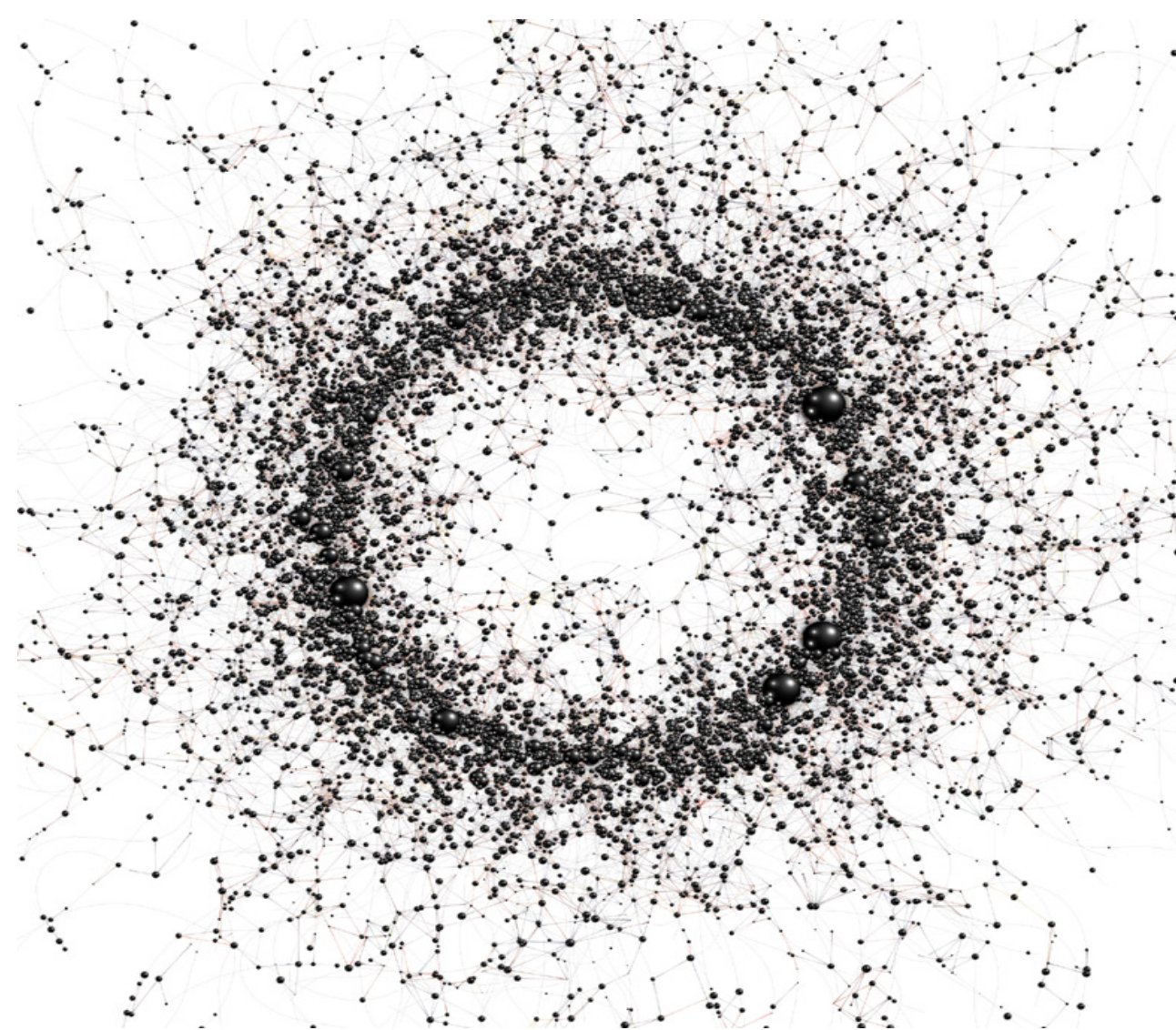
# Efficient Information Spread Control in Cyber-Physical Systems

Ali Khanafer and Tamer Başar

Coordinated Science Laboratory, ECE, University of Illinois at Urbana-Champaign

## Networks in CPS

- Reliable networks are vital for **information exchange** among system components
- Future generation networks will comprise **millions** of users and connections
- Efficient information propagation affects many networked systems
  - Directing **traffic**
  - Quarantining** patches in networks
  - Regulating **spam** and rumor spread

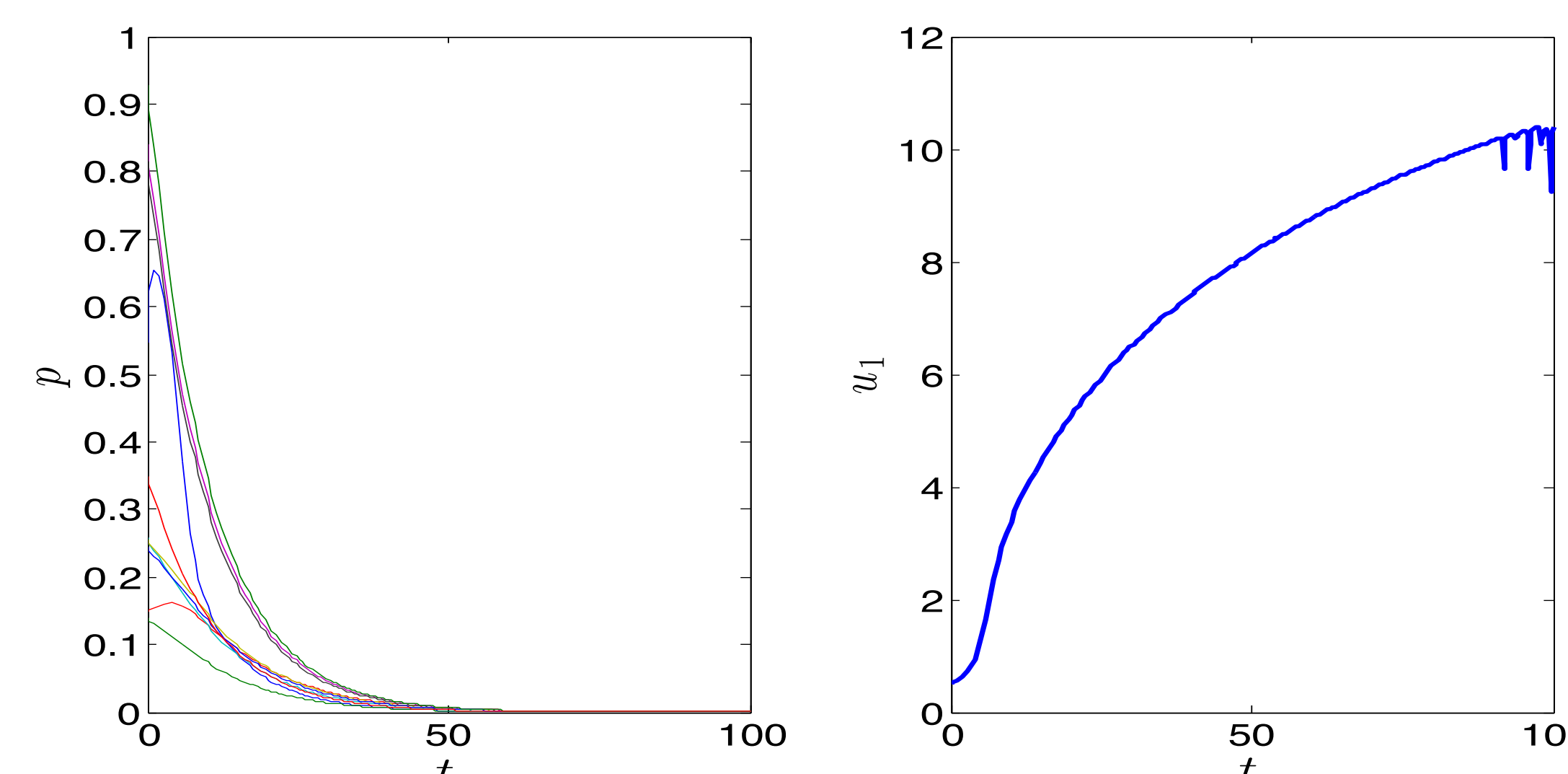


(www.complexification.net)

## Low Cost Network Curing

### Theorem

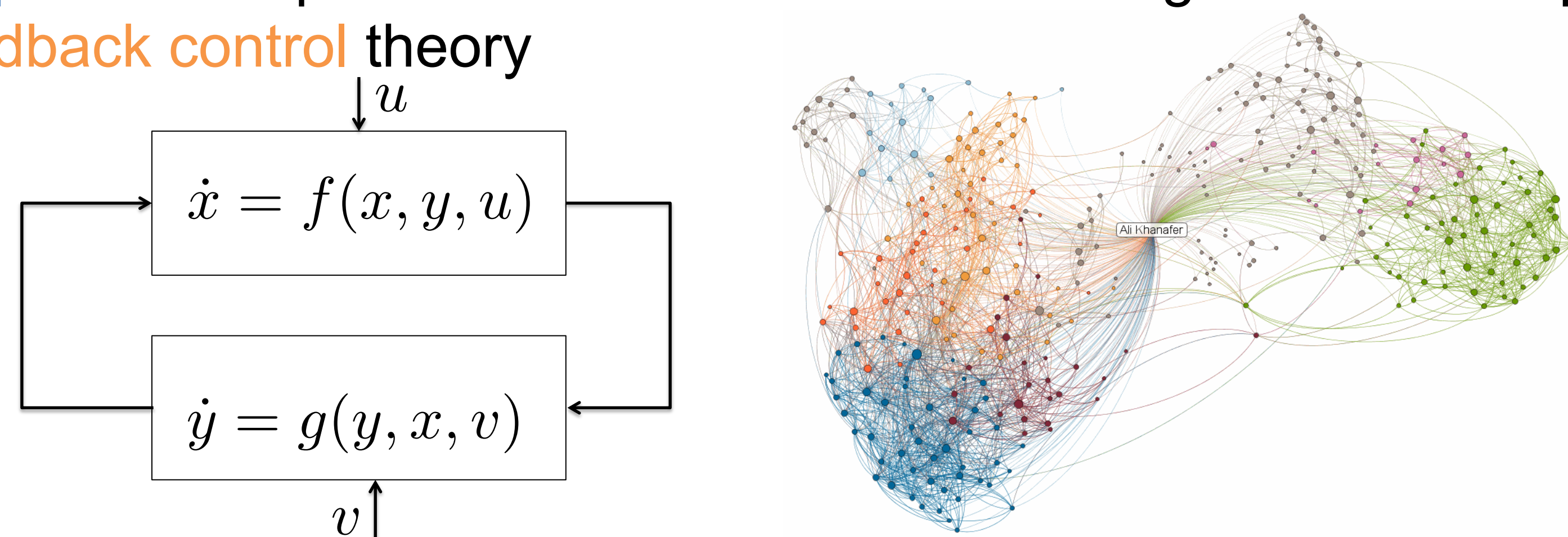
Stabilization achieved by placing controls so that **no path exists** between two uncontrolled nodes



- Implication** on required number of control nodes:  
Star: 1; Path:  $n/2$ ; Binary Tree:  $\frac{1}{3}(2^\ell - 1), \ell > 2$  (even)

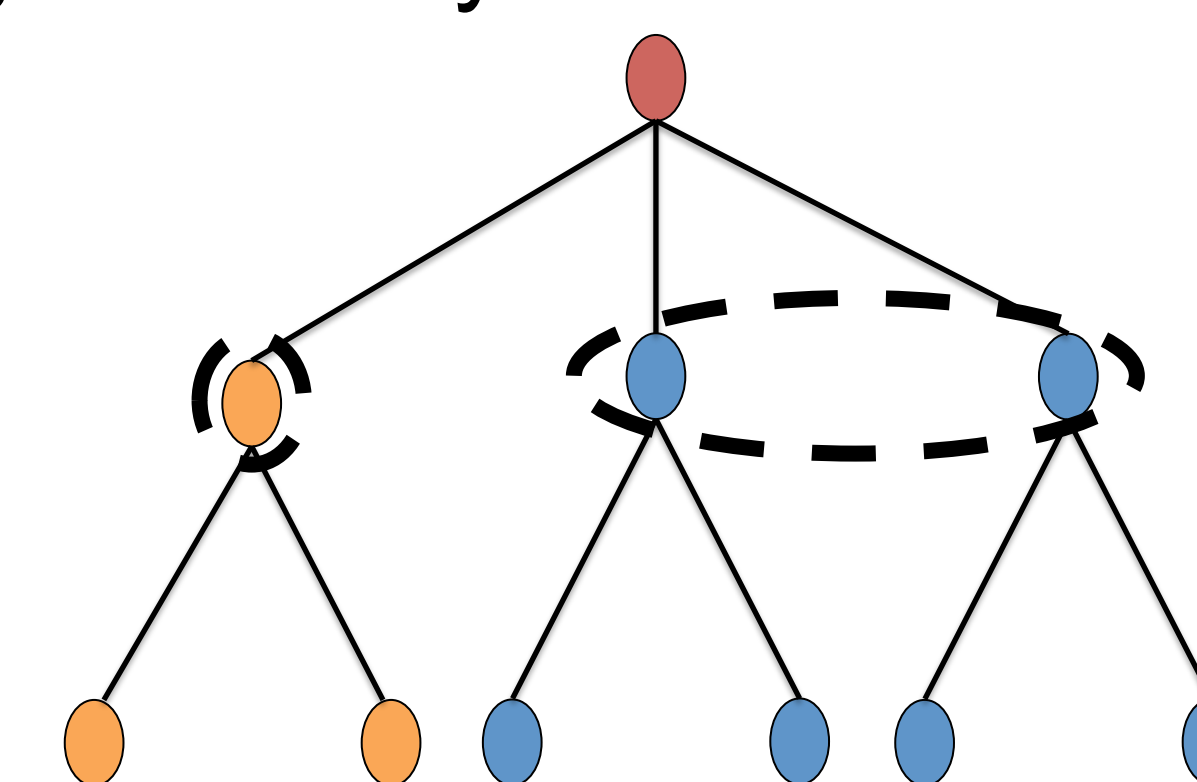
## Goal I: Controllability via Limited Control

- Information spread control schemes must be **scalable**
- Common theme: control **every** node – infeasible and expensive
- Two fundamental questions
  - Q1:** What is the minimum number of controllers required?
  - Q2:** Which nodes should be controlled?
- Approach:** Exploit advances in **classification** algorithms to employ **feedback control theory**



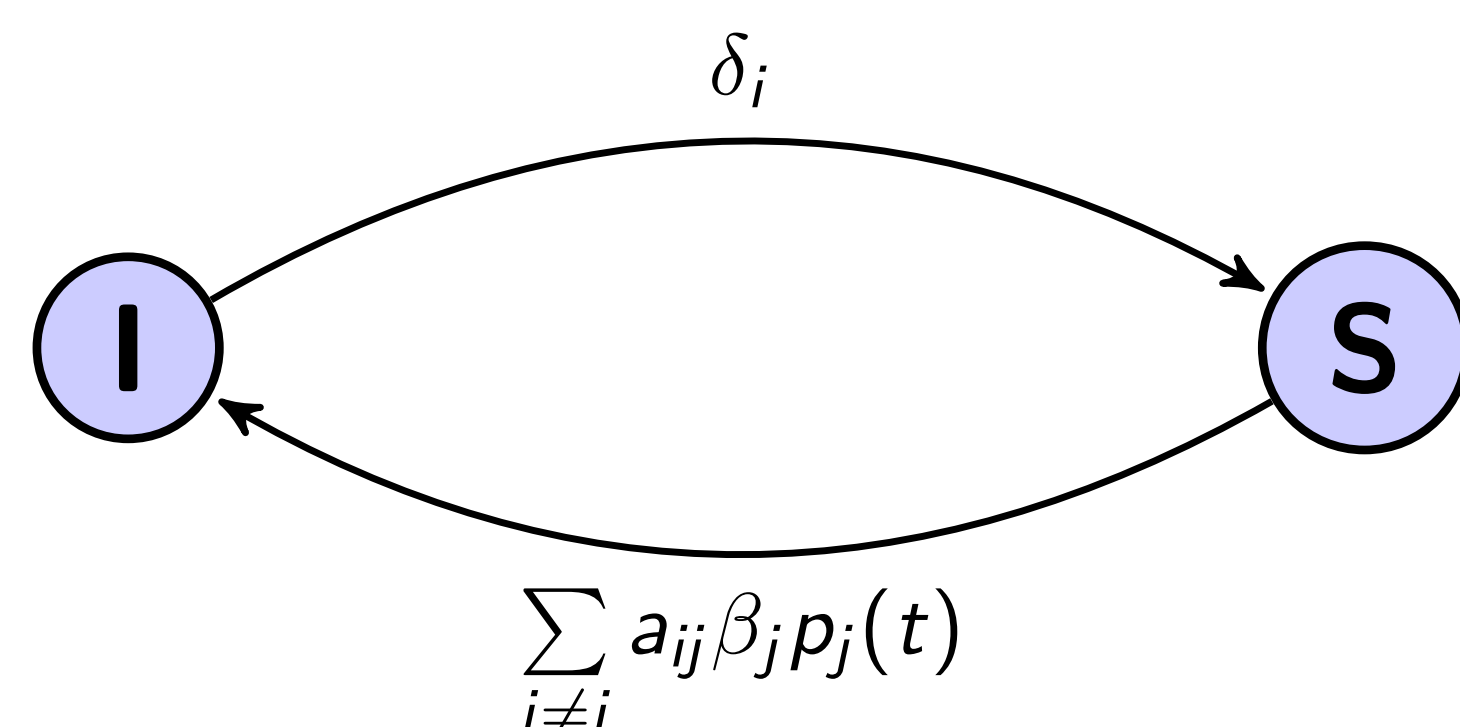
## Goal II: Robust Distributed Controllers

- Achieve **global** objectives with **limited information** about the network
- Objective: implement distributed controllers that are
  - Feature 1:** Robust to adversarial intervention
  - Feature 2:** Robust to large modeling uncertainties
- Approach:** Use a **game-theoretic** framework which allows for various models of agents and yields **robust** strategies



## Proof of Concept: Virus Spread Control

- Two states per node: **healthy** or **infected**
- Curing:  $\text{Poi}(\delta_i)$ ; Infection:  $\text{Poi}(\beta_i)$



- Prob. of infection:  $p_i(t) \in [0, 1]$ . Graph adjacency matrix:  $A$   
 $\dot{p}(t) = (AB - D - U(t))p(t) - P(t)ABp(t)$   
 $D = \text{diag}(\delta_1, \dots, \delta_n), \quad B = \text{diag}(\beta_1, \dots, \beta_n)$   
 $P = \text{diag}(p_1, \dots, p_n), \quad U = \text{diag}(u_1, \dots, u_n)$
- Control curing rates of a **limited subset** of nodes; uncontrolled nodes depend on their **arbitrarily small immunity**  
 $\delta_i = \epsilon \iff u_i = 0$   
 $\delta_i = 0 \iff u_i > 0$

## Proof of Concept: Robust Multi-Agent Systems

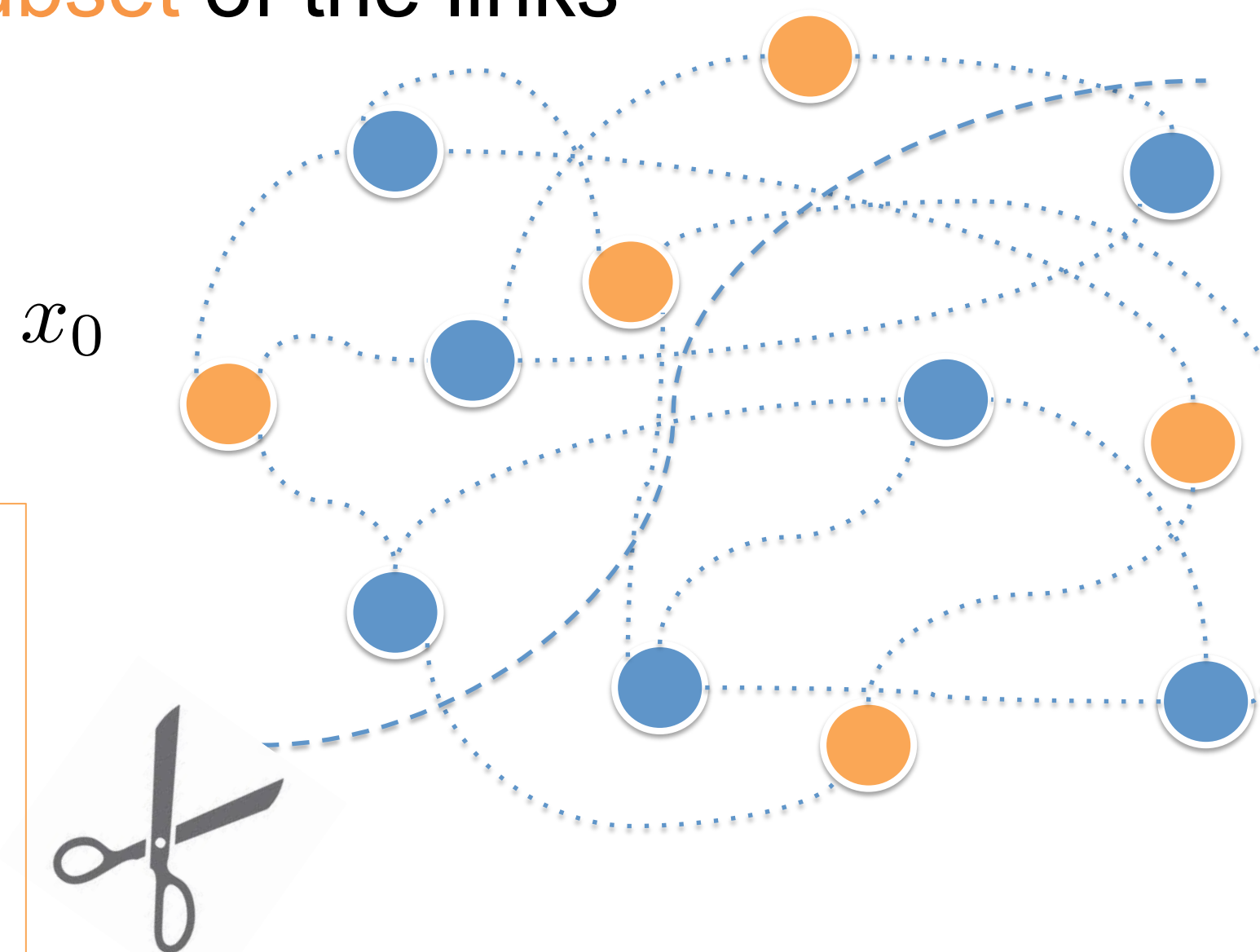
- Centralized worst-case attack to **disrupt** distributed computation
- Adversary is allowed to **break a subset** of the links

$$\max_{u \in \mathcal{U}} \int_0^T k(t) |x - \bar{x}|^2 dt$$

subject to  $\dot{x} = A(u)x, \quad x(0) = x_0$   
 $\|u(t)\|_1 \leq \ell$

### Theorem

The optimal strategy at time  $t$  is to break  $\ell$  links with **maximum**  $w_{ij}(t) = a_{ij}(x_i(t) - x_j(t))^2$  values



- Implication:** Optimal attack depends on **local** quantities
  - Distributed** defense mechanisms can be effective