CPS: Synergy: Collaborative Research:

Efficient Traffic Management: A Formal Methods Approach

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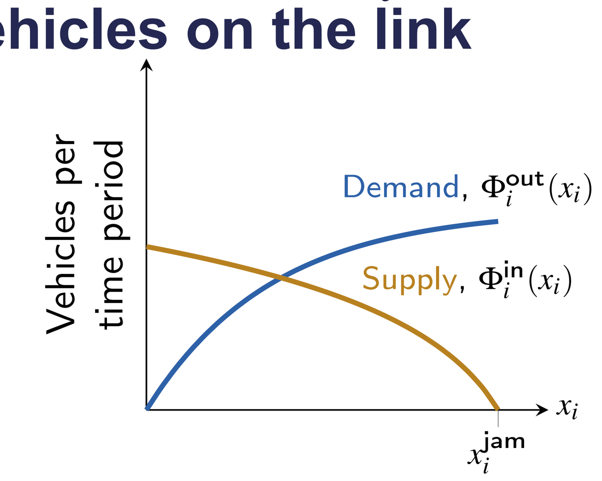
Overview

This project is bringing tools from formal methods to traffic management to meet control objectives expressed in temporal logic. This approach is being applied to signal timing and ramp metering strategies for signalized intersections and freeway traffic control. In addition to meeting temporal logic specifications we aim to incorporate optimality criteria, such as total travel time, throughput, and vehicle miles traveled.

Traffic Network Model

- For each link $\ell \in \mathcal{L}$, the state $x_\ell[t] \in [0, x_\ell^{\mathrm{cap}}]$ represents the number of vehicles on the link
- Each link has:
  - Demand $\Phi_\ell^{\mathrm{out}}(x_\ell)$ to move downstream
  - Supply $\Phi_\ell^{\mathrm{in}}(x_\ell)$ to accept upstream flow

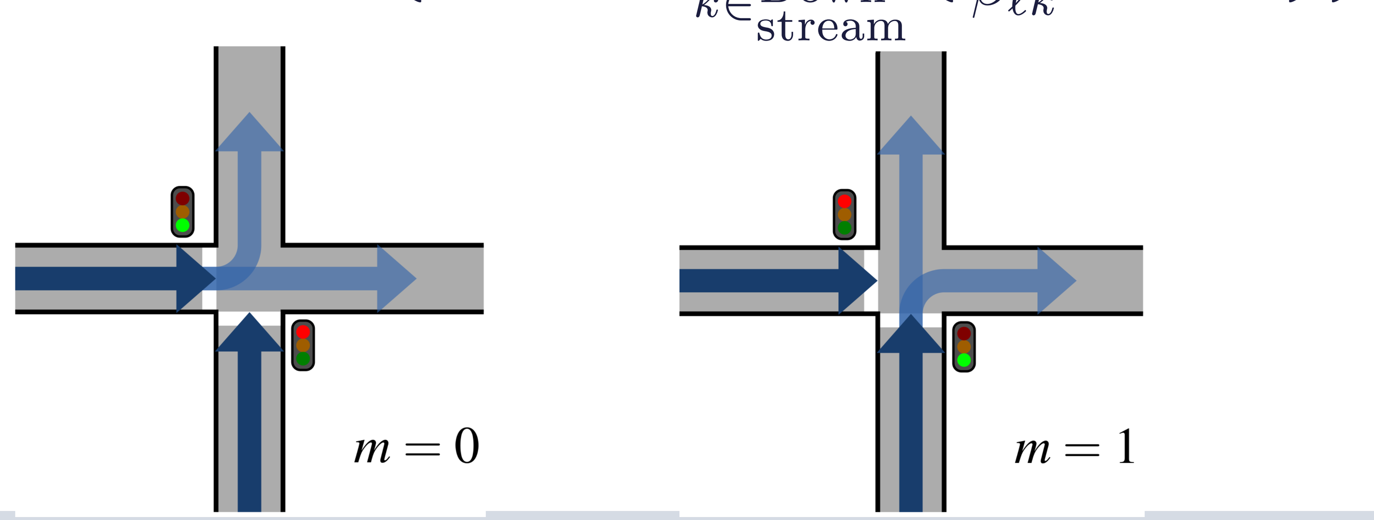


Dynamics:

$$x_\ell^+ = x_\ell + f_\ell^{\mathrm{in}}(x) - f_\ell^{\mathrm{out}}(x) =: F_\ell(x)$$

- Turn ratios $\beta_{\ell k}$ divide demand among downstream links and supply ratios $\alpha_{\ell k}$ divide supply among upstream links
- Signal variable $s_\ell \in \{0,1\}$ indicates if link ℓ is active

$$f_\ell^{\mathrm{out}}(x) = s_\ell \cdot \min\left\{\Phi_\ell^{\mathrm{out}}(x_\ell), \min_{k \in \text{Down-stream}} \left\{\frac{\alpha_{\ell k}}{\beta_{\ell k}} \Phi_k^{\mathrm{in}}(x_k)\right\}\right\}$$



Mixed Monotonicity

- Traffic networks are *mixed monotone* systems:

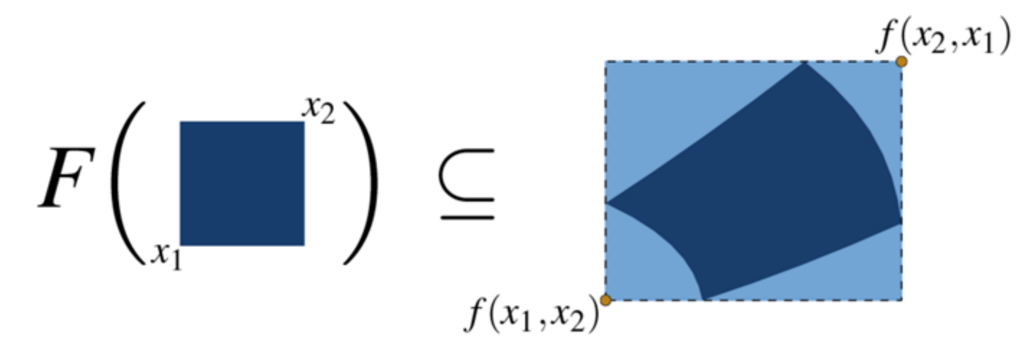
$$\exists \delta_{\ell k} \in \{-1, 1\} \quad \text{s.t.} \quad \delta_{\ell k} \frac{\partial F_\ell(x,d)}{\partial x_k} \geq 0 \quad \forall \ell, k$$

- Increasing and decreasing components
- Decomposition function $f(x,y,d)$
- Congestion causes nonmonotone behavior

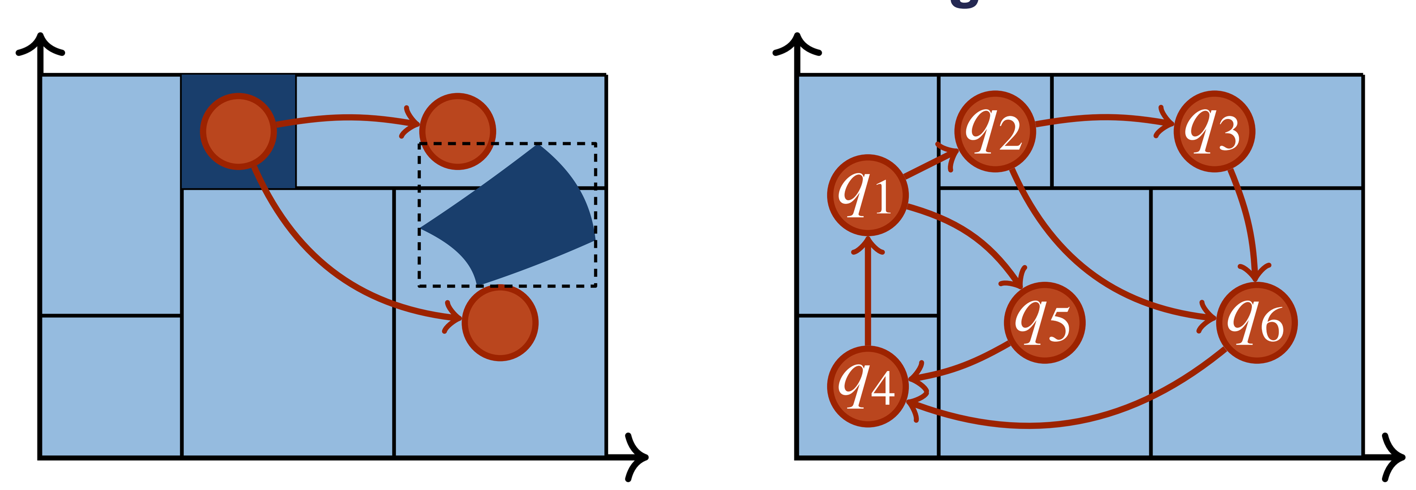
Abstraction

- Mixed Monotonicity leads to efficient abstraction:

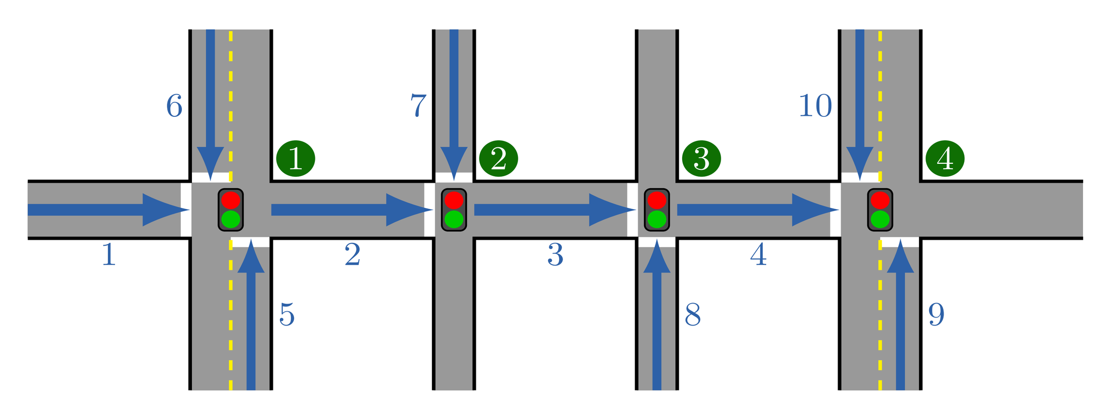
$$(\underline{x} \leq x \leq \overline{x}) \implies (f(\underline{x}, \overline{x}, d) \leq F(x,d) \leq f(\overline{x}, \underline{x}, d))$$



- The one-step reachable set from a box of initial conditions is tightly over-approximated by computing the decomposition function at only two points
- This allows a scalable abstraction algorithm

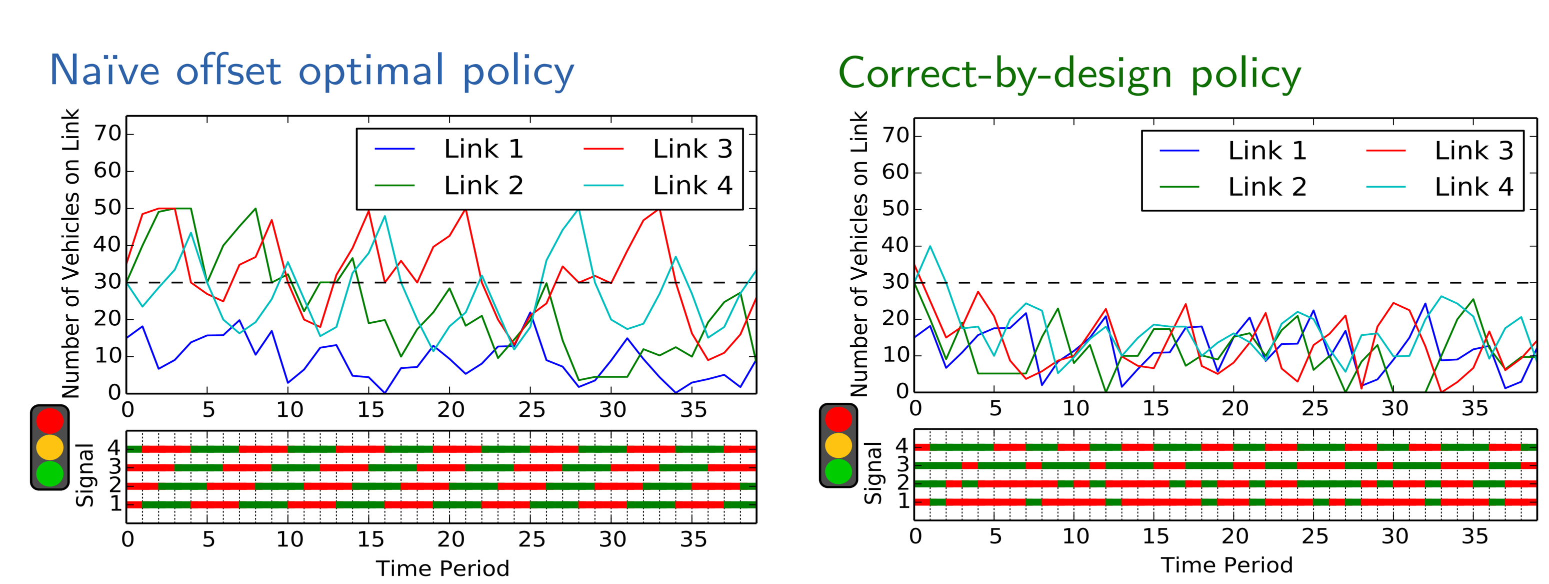


Example



LTL Specification:

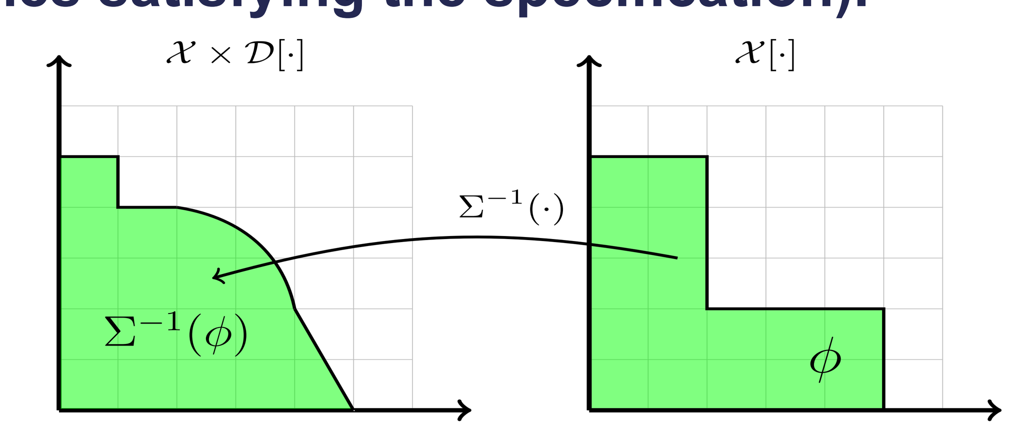
- Each signal actuates cross street traffic infinitely often
- Eventually, links 1, 2, 3, and 4 have fewer than 30 vehicles on each link and this remains true for all time
- The signal at junction 4 must actuate cross street traffic for at least two sequential time-steps



Directed Specifications

Directed specifications describe upper or lower sets in a partially ordered signal space. They arise naturally in traffic control to encourage lower occupancy. Directed specifications paired with monotone systems allow:

- Sparse abstractions that prevent an exponential blowup in the number of finite state transitions
- Compositional controller synthesis
- Systematic assumption mining (identifying the set of input signals, e.g., traffic demand, that generate trajectories satisfying the specification).



Current Research

- Adding optimality criteria to specifications
- Compositionality via assume/guarantee contracts
- Freeway onramp and arterial signaling coordination
- Validation with hybrid freeway / arterial simulation

Publications

- Coogan, Gol, Arcak, Belta, "Traffic network control from temporal logic specifications" IEEE Transactions on Control of Network Systems, 2016.
- Sadra Sadraddini, Calin Belta, "A provably correct MPC approach to safety control of urban traffic networks" ACC 2016.
- Samuel Coogan, Murat Arcak, Calin Belta, "Finite state abstraction and formal methods for traffic flow networks" ACC 2016.
- Eric Kim, Murat Arcak, Sanjit Seshia, "Directed specifications and assumption mining for monotone dynamical systems" HSCC 2016.
- Eric Kim, Murat Arcak, Sanjit Seshia "Compositional controller synthesis for vehicular traffic networks" CDC 2015.
- Coogan and Arcak, "Efficient finite abstraction of mixed monotone systems" HSCC 2015.