CPS Medium Collaborative Research

Smart Power Systems of the Future:

Foundations for Understanding Volatility and Improving Operational Reliability

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Emulating Batteries with Flexible Electricity Loads

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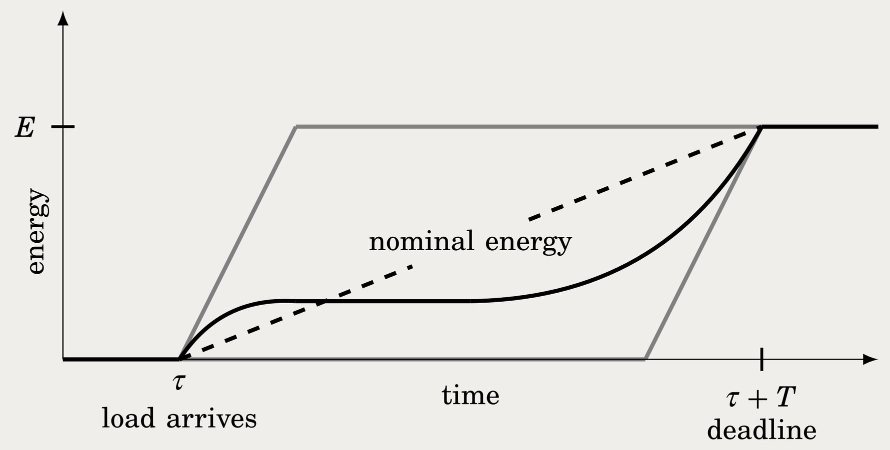
Introduction

Standard storage technologies, such as batteries and flywheels, are *reliable* and *controllable*, but too *expensive* for large scale deployment.

Meanwhile, a substantial part of electricity consumption is flexible.

Can we coordinate flexible loads in order to *emulate* conventional energy storage?

Single deferrable load



Gray lines delimit the set of feasible energy trajectories. Dashed line shows nominal energy consumption

- E Energy demand
- T Service period
- $\overline{P}$ Maximum consumption rate ($0 \le p \le \overline{P}$)
- P_0 Nominal consumption rate ($P_0 = E/T$)

Flexibility: ability to deviate from nominal consumption.

Aggregate flexibility

$\mathcal{A}_1$: loads are identical in terms of E, T and $\overline{P}$.

$\mathcal{A}_2$: one arrival at each point in time.

- $x_\sigma(t)$ Energy level of load that arrived σ sec ago
- $u_\sigma(t)$ Power consumption of load that arrived σ sec ago

$$\frac{1}{T}\int_0^T u_\sigma(t)d\sigma = P_0 + w(t) \qquad \text{(tracking)} \qquad (1)$$

$$\frac{\partial x_\sigma(t)}{\partial t} + \frac{\partial x_\sigma(t)}{\partial \sigma} = u_\sigma(t) \qquad \text{(dynamics)} \qquad (2)$$

$$x_0(t) = 0, \quad x_T(t) = E, \quad 0 \le u_\sigma(t) \le \overline{P} \qquad \text{(load constarints)} \qquad (3)$$

The (average) *aggregate flexibility* under a *causal* policy $\mu : w \to u$ is

$$\mathbb{W}(\mu) = \{w : u = \mu(w) \text{ satisfies (1)–(3)}\}$$

Battery emulation

An ideal battery with parameters $\phi = [C\ \overline{W}\ \underline{W}]$ is characterized by

$$\mathbb{B}(\phi) = \left\{ w : -\frac{C}{2} \le \int_{-\infty}^{t} w(\theta)d\theta \le \frac{C}{2}, -\underline{W} \le w(t) \le \overline{W},\ t \in \mathbb{R} \right\}$$

- C Energy storage capacity
- $\overline{W}$ Maximum charge rate
- $\underline{W}$ Maximum discharge rate

Given load parameters E, T, and $\overline{P}$, what is the set of (ϕ, μ) such that

$$\mathbb{B}(\phi) \subset \mathbb{W}(\mu)$$

Individual battery parameter bounds

Let $\phi_{\max} = [C_{\max}\ \overline{W}\ \underline{W}]$, where

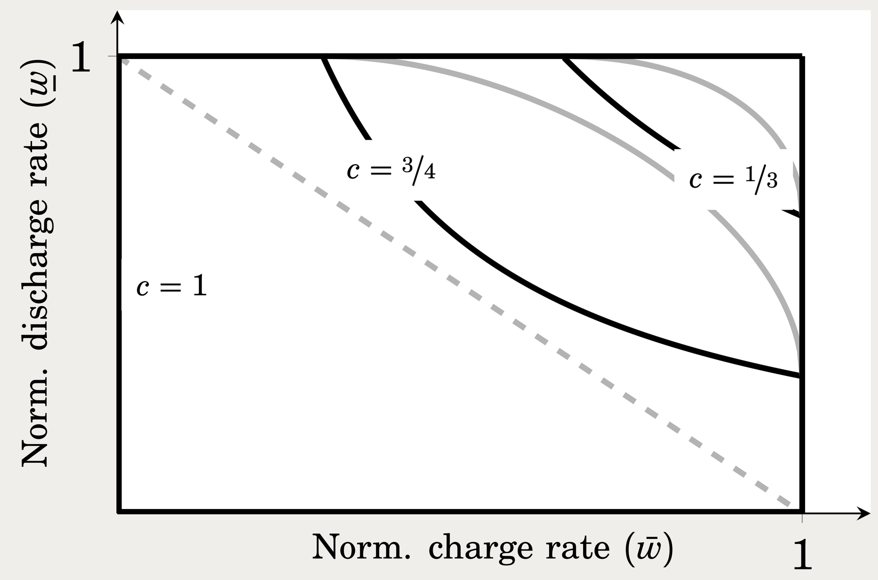
$$C_{max} = (1 - \frac{P_0}{\overline{P}})E \qquad \overline{W}_{\max} = (\overline{P} - P_0) \qquad \underline{W}_{\max} = P_0.$$

Theorem

- $B(\phi_{\max})$ is the smallest battery that contains all realizable $\mathbb{B}(\phi)$.
- There is μ, such that $\mathbb{B}(\phi_{\max}) \subset \mathbb{W}(\mu)$, if and only if $\overline{P} = \infty$.

| There is a trade-off between the batteries that can be emulated.

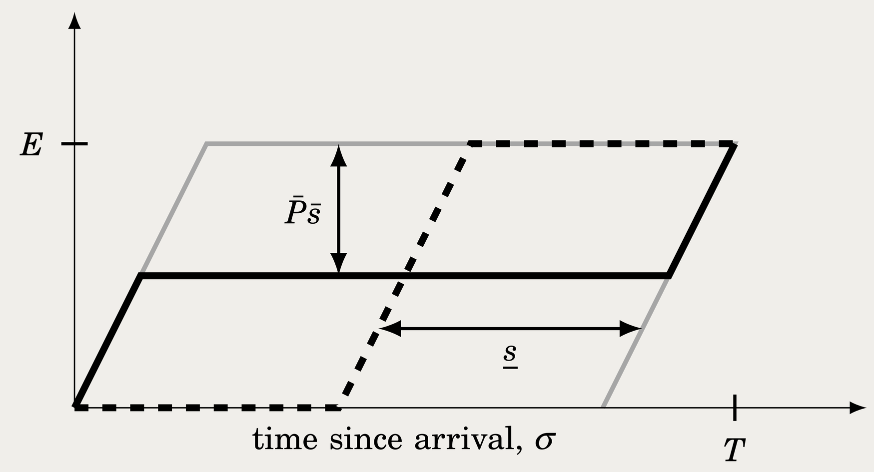
Battery capacity trade-offs



General upper bounds (gray) on normalized charge/discharge rates that can be attained for different normalized energy storage capacities $c = C/C_{\max}$. The black lines show the rates that can be attained with mixed-slack policies.

| Substantial trade-off between battery parameters.

Reason for trade-off

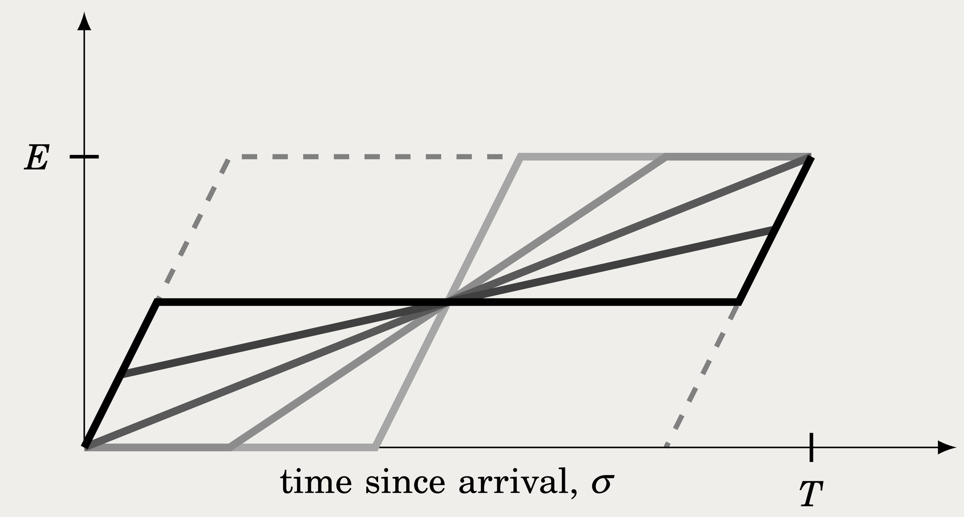


The solid/dashed energy allocation allows the loads to absorb/release the largest volume of energy possible at the highest attainable rate.

- $\overline{s}_\sigma$ Time that load can maintain maximum consumption rate, $\overline{P}$.
- $\underline{s}_\tau$ Time that load can maintain minimum consumption rate, i.e. 0.

| The initial energy allocations (states) that allow load to absorb/release energy at high rates are conflicting.

Mixed-slack policies



Equilibrium energy allocation, $x(t)$, under μ^η. Lower η (darker) gives a better ability to absorb energy at a high rate.

Mixed slack:

$$s^\eta_\sigma(t) = \eta \underline{s}_\sigma(t) + (1-\eta)\left(\frac{E}{\overline{P}} - \overline{s}_\sigma(t)\right), \qquad \eta \in [0,1].$$

A *mixed-slack policy*, μ^η, prioritizes power allocation to small s^η_σ.

Theorem Suppose $\phi \le \phi_{\max}$, $\overline{P} < \infty$ and

$$\left(\frac{\overline{W}}{\overline{W}_{\max}}\right)\left(\frac{\underline{W}}{\underline{W}_{\max}}\right) + \frac{C}{C_{\max}} \le 1. \qquad (4)$$

Then $\mathbb{B}(\phi) \subset \mathbb{W}(\mu^\eta)$, where $\eta = \frac{1}{1+\overline{W}/\underline{W}}$. Moreover, if either $C = C_{\max}$, $\overline{W} = \overline{W}_{\max}$, or $\underline{W} = \underline{W}_{\max}$, then (4) is also necessary for $\mathbb{B}(\phi) \subset \mathbb{W}(\mu)$.

- Mixed-slack policies balance ability to absorb/release energy
- μ^1 is the standard and well-known *least-laxity-first* policy.

Related work

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A. Subramanian, M. Garcia, A. Dominguez-Garcia, D. Callaway, K. Poolla, and P. Varaiya, "Real-time scheduling of deferrable electric loads", In *Proc. of ACC*, 2012

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