Event-triggered and self-triggered control design with guaranteed performance

T.M.P. Gommans W.P.M.H. Heemels M.C.F. Donkers P. Tabuada

I. Introduction

In digital control applications, controllers are typically implemented in a time-triggered fashion, in which control tasks are executed periodically. This design choice is motivated by the fact that it enables the use of a well-developed theory on sampled-data systems. This design choice, however, leads to over-utilization of the available communication resources, and/or a limited lifetime of battery-powered devices, as it might not be necessary to execute the control task every period to guarantee certain closed-loop performance. In fact, this observation leads to the *fundamental* problem of determining the optimal sampling and communication strategies, where optimality needs to reflect both communication cost as well as control performance.

It is expected that the solution to such problems results in control strategies that abandon the time-triggered periodic control paradigm. Two approaches that abandon the periodic communication pattern are event-triggered control (ETC), and self-triggered control (STC). In ETC and STC, the control law consists of two elements: namely, a feedback controller that computes the control input and a triggering mechanism that determines when the control input has to be updated. The difference between event-triggered control and self-triggered control is that in the former the triggering consists of verifying a specific condition continuously and when it becomes true, the control task is triggered, while in the latter at a control update time the next update time is pre-computed.

Currently, ETC and STC form popular research areas. However, three important issues have only received marginal attention: (i) the co-design of both the feedback law and the triggering mechanism, (ii) providing performance guarantees by design, and (iii) dealing with constraints on control inputs and states. In this presentation, we propose self-triggered and event-triggered approaches addressing the mentioned issues and allowing to trade guaranteed performance levels with utilization of communication resources. We will consider discrete-time linear systems and the performance will be measured in terms of a standard LQR-type of cost. The methods are such that an LQR cost can be chosen *a priori*, and the design approach directly guarantees this performance level for the self-triggered and event-triggered controllers. We will present three novel strategies that aim at reducing the use of communication resources, at the price of obtaining a lower but guaranteed sub-optimal level of control performance. The first two methods are still of an emulation-based nature, while the third approach will be a co-design method. Although the focus will be on a discrete-time setup, the results can easily be applied to continuous-time linear systems as well. In the continuous-time case, the control schemes lead to so-called *periodic event-triggered controllers* with performance guarantees in terms of *continuous-time* LQR cost.

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Maurice Heemels and Tom Gommans are with the Hybrid and Networked Systems group of the Department of Mechanical Engineering, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, the Netherlands, {m.heemels,t.m.p.gommans}@tue.nl. Tijs Donkers is with the Powertrains department of TNO Transport and Mobility, PO Box 756, 5700 AT Helmond, Netherlands, tijs.donkers@tno.nl. Paulo Tabuada is with the Dept. of Electrical Engineering, University of California, Los Angeles, CA, USA, tabuada@ee.ucla.edu

II. Problem formulation

In this presentation, we consider the regulation of

$$x_{t+1} = Ax_t + Bu_t, \qquad t \in \mathbb{N}, \tag{1}$$

in which $x_t \in \mathbb{R}^{n_x}$ is the state and $u_t \in \mathbb{R}^{n_u}$ is the input, respectively, at discrete time $t \in \mathbb{N}$. In particular, we are interested in control strategies that guarantee certain control performance in terms of an infinite horizon cost function

$$J(x_0, \mathbf{u}) = \textstyle\sum_{t=0}^{\infty} \left(x_t^\top Q x_t + 2x_t^\top S u_t + u_t^\top R u_t\right), \tag{2}$$

based on the weighting matrices Q, R and S. Here, $\mathbf{u} = (u_0, u_1, \ldots)$. When transmission of measured states and updates of input can occur, for each $t \in \mathbb{N}$, it is well known that the optimal cost for initial state x_0 is given by

$$\begin{aligned}&\min_{\mathbf{u}} J(x_0, \mathbf{u}) = V(x_0) = \\ &\textstyle\sum_{t=0}^{\infty} \left(x_t^\top Q x_t + 2x_t^\top S u_t^* + (u_t^*)^\top R u_t^*\right) = x_0^\top P x_0,\end{aligned} \tag{3}$$

where P is the solution to the discrete algebraic Riccati equation and u_t^*, $t \in \mathbb{N}$, is given by the feedback policy

$$u_t^* = K^* x_t \text{ with } K^* = -\left(R + B^\top P B\right)^{-1}\left(B^\top P A + S^\top\right).$$

As already indicated, the LQR optimal control law requires the transmission of measured states and updates of control actions for each sample instant $t \in \mathbb{N}$. In this presentation, we are interested in *synthesizing* control laws that require (much) less communication between sensors, controllers, and actuators, while still providing guarantees on the quadratic performance criterion (3). More specifically, we are interested in reducing the number of times the input is updated, while still satisfying a sub-optimality condition of the form

$$\textstyle\sum_{t=0}^{\infty} \left(x_t^\top Q x_t + 2x_t^\top S u_t + u_t^\top R u_t\right) \leq \beta V(x_0), \tag{4}$$

where $V(x_0)$ denotes the optimal LQR cost as in (3) and $\beta \geq 1$ can be chosen to balance the reduction in communications and the degree of sub-optimality.

To address this problem we propose three strategies: A predictive approach, an event-triggered approach and a self-triggered approach. The former two approaches are based on emulation, in the sense that the control values are designed irrespective of the eventual predictive or event-triggered implementation, whereas the third strategy solves the co-design problem of simultaneously synthesizing the next update time and the next corresponding control value. Because space limitations do not allow all three approaches to be discussed, we only briefly describe the latter approach.

III. Self-triggered LQR control

The self-triggered strategy is based on holding the current input value as long as possible while still guaranteeing (4) for a suitably selected $\beta \geq 1$. In fact, the control strategy will have the structure

$$\begin{cases} t_{l+1} &= t_l + M(x_{t_l}) \\ u_t &= \bar{u}_l \in \mathcal{U}(x_{t_l}), \quad t \in \mathbb{N}_{[t_l, t_{l+1})}. \end{cases} \tag{5}$$

with $t_0 := 0$, $M : \mathbb{R}^{n_x} \to \mathbb{N}$ and $\mathcal{U} : \mathbb{R}^{n_x} \rightrightarrows \mathbb{R}^{n_u}$. Here $M(x)$ denotes the "sleep" time between two transmissions when being in state x and $\mathcal{U}(x)$ denotes the set of possible control values.

Instrumental in the co-design of the mappings M and $\mathcal{U}$ will be the dissipation-like inequality

$$\sum_{t=t_l}^{t_{l+1}-1} \left(x_t^\top Q x_t + 2x_t^\top S \bar{u}_l\right) + (t_{l+1}-t_l)\bar{u}_l^\top R \bar{u}_l + \beta V(x_{t_{l+1}}) \leq \beta V(x_{t_l}) \tag{6}$$

for update time t_l, $l \in \mathbb{N}$, which can be shown to guarantee the sub-optimality condition (4). In fact, at update time t_l, $l \in \mathbb{N}$, with state x_{t_l} we aim at finding a maximal value for t_{l+1} (which results in $M(x_{t_l}) = t_{l+1} - t_l$) and a corresponding value $\bar{u}_l \in \mathcal{U}(x_{t_l})$ such that (6) is satisfied.

This idea results in a self-triggered control algorithm as in (5) with $M(x) = \sup\{M \in \mathbb{N} \mid x^\top S_M x \leq 0\}$ and $\mathcal{U}(x) = \{K_{M(x)} x\}$, where $S_M \in \mathbb{R}^{n_x \times n_x}$ and $K_M \in \mathbb{R}^{n_u \times n_x}$, $M \in \mathbb{N}$ are predefined matrices that can be determined off-line. This control law is well defined and satisfies the sub-optimality condition (4).

IV. Illustrative example

To illustrate the self-triggered approach, consider a double integrator, whose continuous-time dynamics is given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u =: A_c x + B_c u. \tag{7}$$

By exact discretization with sampling period h, assuming a zero-order hold input between two sampling instants, we obtain a discrete-time LTI system of the form (1), where

$$A = e^{A_c h} \quad \text{and} \quad B = \int_0^h e^{A_c s} ds B_c.$$

The control performance is measured by a continuous-time infinite horizon cost function of the form

$$J_c(x_0, \mathbf{u}) = \int_0^\infty \left(x^\top(s) Q_c x(s) + u^\top(s) R_c u(s)\right) ds, \tag{8}$$

where $Q_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R_c = 1$. By exact discretization of the continuous-time cost function (8), we obtain a discrete-time infinite horizon cost (2), which is exactly equal to (8) given the sampled-data implementation.

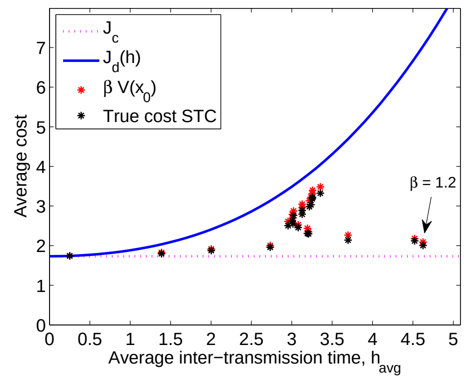

Fig. 1: Cost of implementation at (average) inter-transmission times for different control strategies, averaged over eight initial conditions.

We compare our self-triggered scheme based on $h = 0.25$ and for various values of β, with optimal LQR controllers corresponding to various values of h. Note that the latter approach with larger values of h results in less usage of communication resources as well, and is, in fact, a standard discrete-time LQR problem. In Fig. 1 the results are plotted. This figure shows the performance with respect to the average sampling time h_{avg}. To make a fair comparison, the results presented are obtained by averaging over eight initial conditions on the unit disc. Fig. 1 shows the cost J_c of the continuous-time controller and the cost $J_d(h)$ of the optimal periodic LQR controller for various sampling times h. Moreover, for $\beta \in [1,\ 1.05, \ldots,\ 1.95,\ 2]$, Fig. 1 shows the upper-bound of the cost for the self-triggered implementation $\beta V(x_0)$, and the true cost of both the self-triggered and event-triggered approach computed by using simulations over a finite, but sufficiently large, horizon. From Fig. 1 we can see that the self-triggered approach for $\beta = 1.2$ results in $h_{avg} = 4.6$ with true performance 2, whereas periodic LQR control with cost 2 requires sampling at $h = 1.3$. So, on average, we can sample and transmit a factor 3.5 less by using the self-triggered approach. Interestingly, the minimal inter-event time h_{min} for the self-triggered controller is $h_{min} = 1$ for $\beta = 1.2$. Notice also that the cost of a periodic LQR controller at $h = 4.6$ is 6.9, which is three times larger than the cost of the self-triggered approach at $h_{avg} = 4.6$. These observations demonstrate that self-triggered control offers possibilities that cannot be achieved by sticking to the periodic time-triggered control paradigm.

V. Extensions and outlook

This short summary discussed only the unconstrained case, even though extensions of the presented ideas towards constrained linear systems are possible, see [1]. This results in self-triggered MPC laws. Currently, we are extending the work also in the direction of nonlinear systems and using other performance measures.

References

[1] J. D. J. Barradas Berglind, T. M. P. Gommans and W. P. M. H. Heemels, *Self-triggered MPC for constrained linear systems and quadratic costs*, IFAC Conference on Nonlinear Model Predictive Control 2012, Netherlands, 2012