Event-triggered coordination of networked cyber-physical systems

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ABSTRACT

In this work we are interested in exploring how to design algorithms to control networked cyber-physical systems that are asynchronous, robust, and efficient. An important question that must be addressed to design efficient operation is how much information do different subsystems really need to complete a given task? Periodic implementations essentially ignore this question by simply giving agents as much information as possible which is a generally inefficient use of sensors, actuators, and/or wireless communication. By using event-triggered coordination, we give agents in a network a sufficient level of autonomy such that they can decide for themselves when information should be shared or control signals should be updated. Distributed event-triggered coordination is appealing not only in the sense that it allows for naturally asynchronous executions, but it also aligns the need for information and control with a prespecified goal.

More specifically, we consider the well-known multi-agent consensus problem. There are many different algorithms that achieve the desired consensus state, but a vast majority of them assume continuous or periodic and synchronous updates of information and control signals. There are also many recent papers that apply event-triggered control to this problem; however, event-triggered control alone still requires the continuous or periodic availability of information to check the triggering condition. In these setups, agents only update their control signal when the error between their neighboring agents' current states have changed since the last control update by some predefined threshold. While this is a good first step in that control signals are not updated continuously, the availability of continuous or periodic communication is still unrealistic when considering networked cyber-physical systems that must acquire this information through digital sensing and/or wireless communication. Thus we are interested in applying the event-triggered idea not only to control, but to communication as well.

Introduction and motivation

As computer-controlled systems become more and more ubiquitous in today's world, the physical challenges that must be overcome to ensure their reliable and efficient operation become extremely important. In general, controllers are designed assuming perfect information is available at all times and actuators can be updated continuously. When resources such as actuator power or energy consumption are not factors, this is not a problem because the system can take samples and update the control signals fast enough so that the system behaves as close to ideally as possible. When designing cyber-physical systems, such as an autonomously driving car, all these things must be taken into consideration at the design level. This means that controllers should be designed already accounting for possible uncertainties in sensor measurements, and sensors should be placed strategically to measure information that is most pertinent to the controller.

Clearly, the above problem is already an immensely difficult challenge. If we instead want to consider networks of cyber-physical systems that must cooperate with one another while communicating through wireless channels, an extra layer of difficulty is introduced to the already complicated system. In these types of networked systems, the integration of computation, control, and communication at the design level cannot be overemphasized. A network of mobile sensors tracking the motion of chemical pollutants, a camera network monitoring a busy intersection, and a group of UAVs providing force protection are only a small subset of a myriad of applications that are currently implemented or under development [2].

Figure 1 : Cyber-Physical Systems.

A common assumption in problems considering networked systems is the continuous, or at least periodic, availability of information to the various subsystems through communication and/or sensing. Furthermore, many such works assume synchronous operations meaning all agents of a network obtain new information and recompute control signals simultaneously. Unfortunately, these are quite unrealistic assumptions when considering systems with real physical constraints, especially as the number of agents in a network gets large. We are thus interested in asynchronous coordination algorithms that are not only reliable in completing the desired task, but also efficient in terms of energy and required communication.

Problem formulation

We consider the multi-agent average consensus problem on a network of N agents. The dynamics of each agent $x_i \in \mathbb{R}^d$ is given by

$$\dot{x}_i(t) = u_i(t).$$

We let $\mathcal{G}$ denote the graph in which an edge $(i,j) \in E$ means that agent i can receive information from agent j. It is known that the distributed continuous control law

$$u_i(t) = -\sum_{j \in \mathcal{N}_i^{\text{in}}} (x_i(t) - x_j(t)) \tag{1}$$

asymptotically drives all the agents to consensus. The problem with this controller is that, in order to be implemented, this control law requires agents to have continuous information about their neighbors and continuously update their control signals at all times.

State of the art

Here we show how the unrealistic requirements of the controller (1) can be removed to allow for an algorithm that can be implemented on networked cyber-physical systems. Instead of (1), we use the control law

$$u_i(t) = -\sum_{j \in \mathcal{N}_i^{\text{in}}} (\hat{x}_i(t) - \hat{x}_j(t)) \tag{2}$$

where $\hat{x}_i$ is the last state broadcast by agent i. We then seek to solve the following problem.

> **Problem 1. Event-triggered coordination for consensus on undirected graphs:** Given the control law (2) over some undirected graph $\mathcal{G}$, design a distributed mechanism for when each agent i should broadcast its state x_i to its neighbors $j \in \mathcal{N}_i$ such that all agents reach the consensus state and Zeno behavior is avoided.

This problem has only recently been solved [4, 1, 3].

Solution to Problem 1: Each agent i should broadcast its current state x_i to its neighbors when

$$e_i^2 \geq \sigma \frac{a(1-a|\mathcal{N}_i|)}{|\mathcal{N}_i|} \left(\sum_{j \in \mathcal{N}_i} \hat{x}_i - \hat{x}_j \right)^2, \tag{3}$$

where $e_i = (x_i - \hat{x}_i)$ is the error between agent i's true state and last broadcast state and $\sigma \in (0,1)$ and $a < \frac{1}{|\mathcal{N}_i|}$ are design parameters.

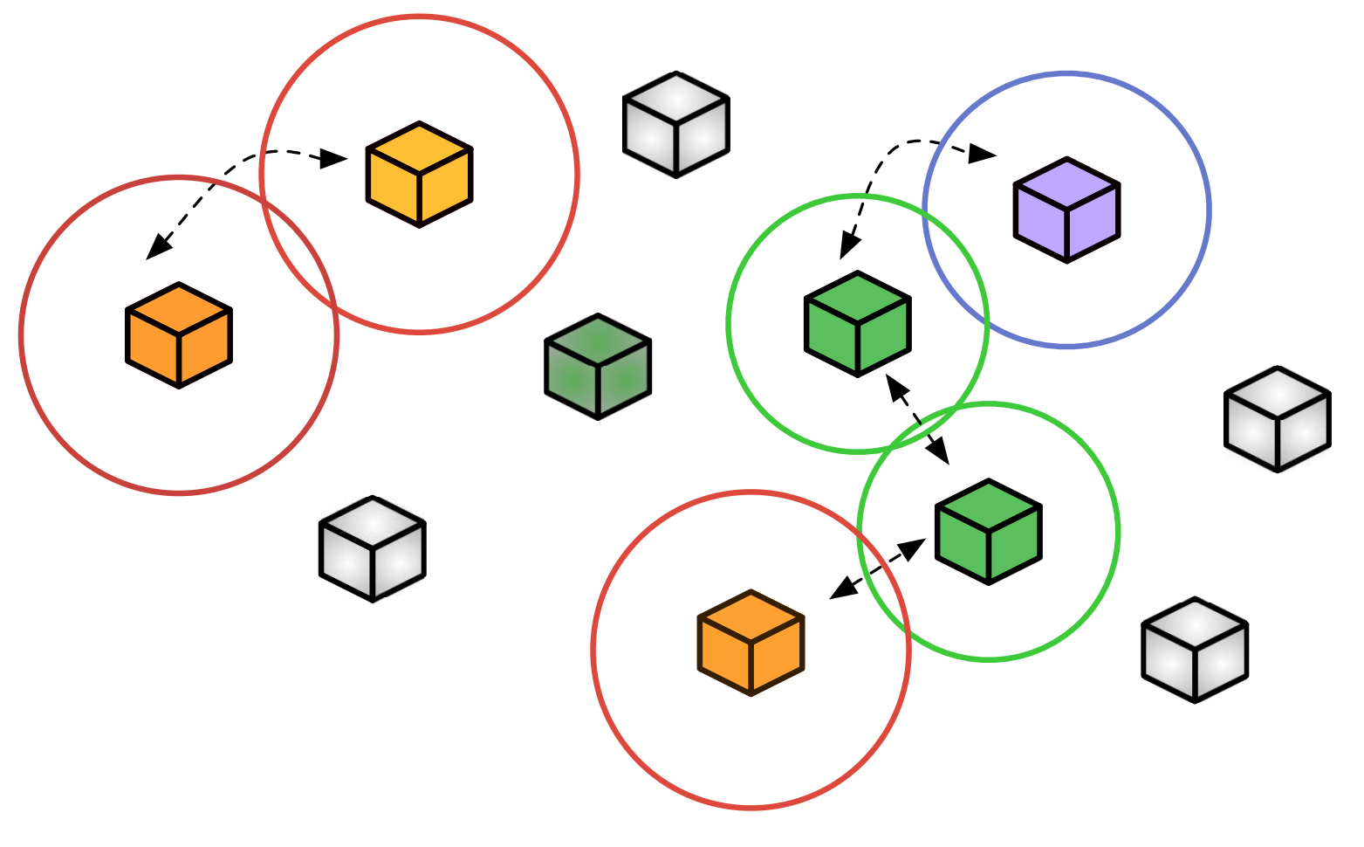

Figure 2 : Distributed and asynchronous coordination.

Correctness analysis

Consider the candidate Lyapunov function

$$V(x) = \frac{1}{2} x^T L x. \tag{4}$$

The time derivative of V under the control law (2) is

$$\begin{aligned} \dot{V} &= x^T L \dot{x} = -x^T L(L\hat{x}) \\ \dot{V} &= -(\hat{x}^T - e^T) L L \hat{x} \\ &= -\|L\hat{x}\|^2 + (L\hat{x})^T L e. \end{aligned}$$

Letting $\hat{z} = L\hat{x} = (\hat{z}_1, \dots, \hat{z}_N)$,

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^N \hat{z}_i^2 + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \hat{z}_i (e_i - e_j) \\ &= -\sum_{i=1}^N \hat{z}_i^2 + \sum_{i=1}^N |\mathcal{N}_i| \hat{z}_i e_i - \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \hat{z}_i e_j. \end{aligned}$$

Using the fact that $\mathcal{G}$ is undirected, we are able to bound

$$\dot{V} \leq \sum_{i=1}^N \left((a|\mathcal{N}_i| - 1) \hat{z}_i^2 + \frac{|\mathcal{N}_i|}{a} e_i^2 \right). \tag{5}$$

From this we can see that if

$$e_i^2 < \frac{a(1-a|\mathcal{N}_i|)}{|\mathcal{N}_i|} \left(\sum_{j \in \mathcal{N}_i} \hat{x}_i - \hat{x}_j \right)^2,$$

at all times for all i, then $\dot{V} < 0$ unless $L\hat{x} = 0$. By having each agent i broadcast its state whenever (3) is satisfied (which resets $e_i = 0$), we are able to guarantee that

$$\dot{V} \leq \sum_{i=1}^N (\sigma - 1)(1 - a|\mathcal{N}_i|) \hat{z}_i^2 < 0 \tag{6}$$

at all times until the consensus state is reached.

Future work

Due to the symmetry of the adjacency matrix of an undirected graph, we were able to take advantage of this in bounding $\dot{V}$ by (5). If instead we consider directed graphs, it becomes far less trivial to design an event-triggering law in a distributed way. We are thus interested in solving this same problem for directed graphs as stated below.

> **Problem 2. Event-triggered coordination for consensus on directed graphs:** Given the control law (2) over some *directed* graph $\mathcal{G}$, design a distributed mechanism for when each agent i should broadcast its state x_i to its neighbors $j \in \mathcal{N}_i^{\text{out}}$ such that all agents reach the consensus state and Zeno behavior is avoided.

Simulations

All simulations here are done on the same undirected graph with 5 nodes using the same initial condition $x(0) = [16, 1, 1, 1, 1]^T$.
Figure 3 shows that choosing a small period is wasteful both in terms of the required amount of communication, and even in the performance of the algorithm. In Figure 4 we see that by choosing a suitable period T we can get similar performance of the algorithms both in terms of performance and required communication. In Figure 5 we see that by choosing the period too large, we are drastically reducing the performance of the algorithm, and in general the stability guarantee can be lost as well. Figure 6 shows the trajectories of the network for the simulation done in Figure 5. Here we can directly see the negative effect that choosing a large period can have on the network.

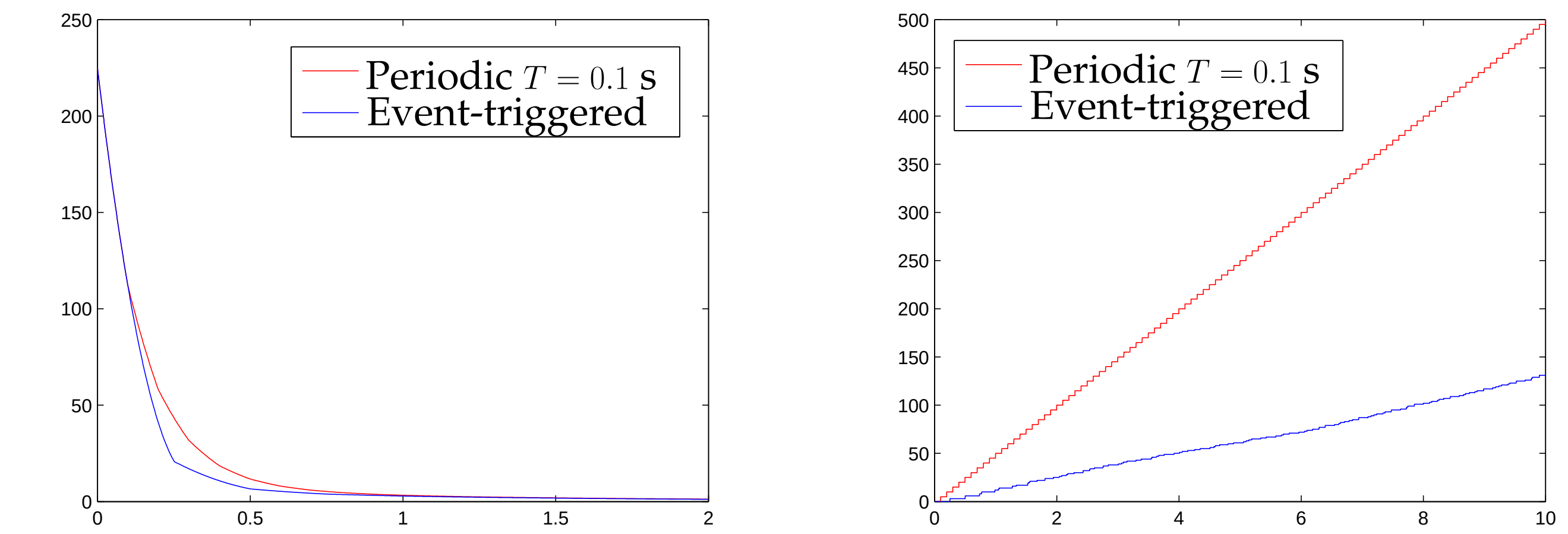

Figure 3 : Comparison of the (left) Lyapunov function and (right) number of broadcasts of the event-triggered algorithm and periodic algorithm with period 0.1 seconds.

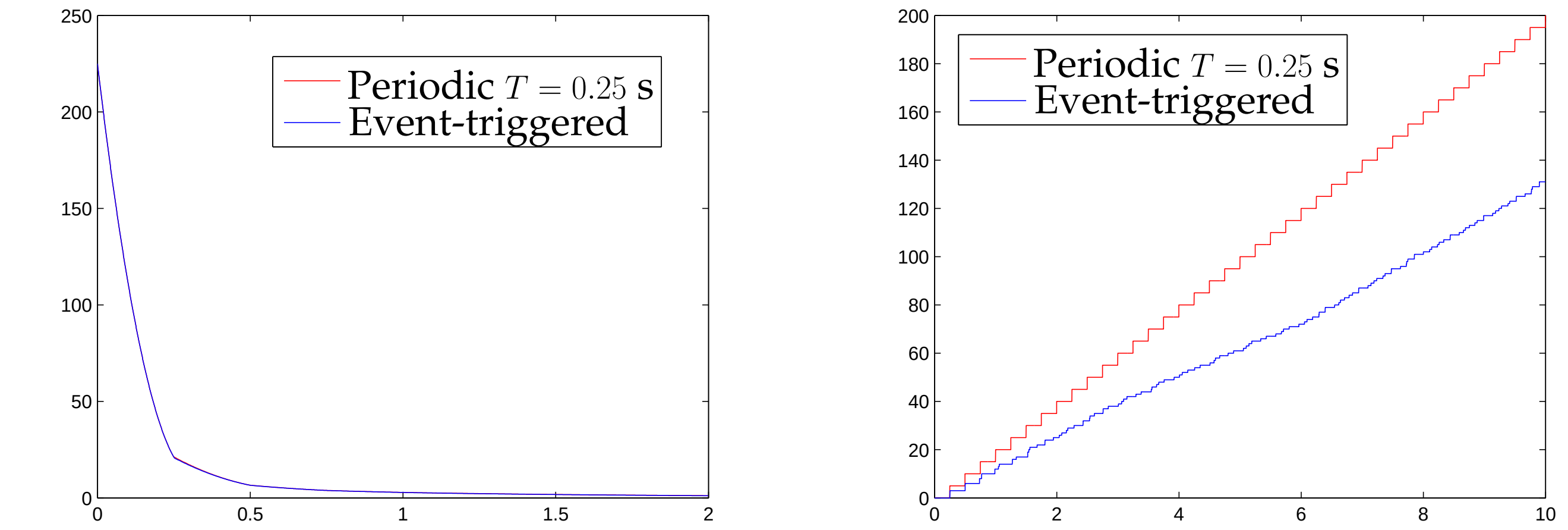

Figure 4 : Comparison of the (left) Lyapunov function and (right) number of broadcasts of the event-triggered algorithm and periodic algorithm with period 0.25 seconds.

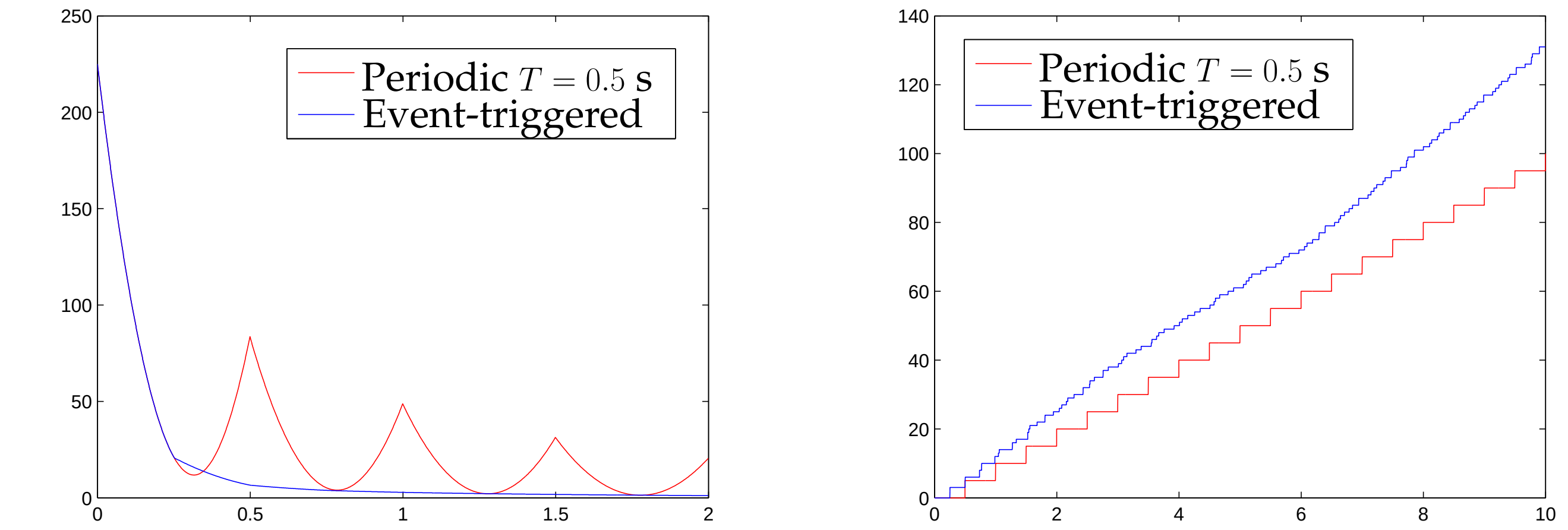

Figure 5 : Comparison of the (left) Lyapunov function and (right) number of broadcasts of the event-triggered algorithm and periodic algorithm with period 0.5 seconds.

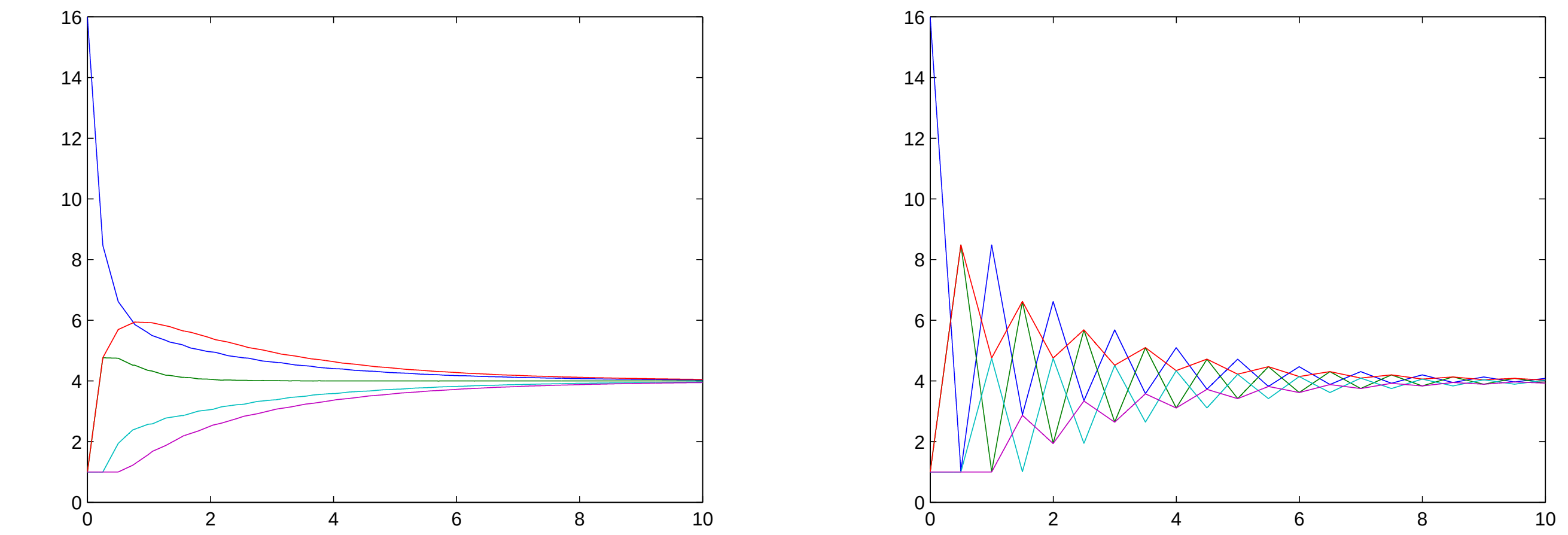

Figure 6 : Trajectories of states for the (left) event-triggered algorithm and (right) periodic algorithm with period 0.5 seconds.

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