2014 NSF Young Professionals Workshop On
Exploring New Frontiers
In Cyber-Physical Systems

Cornell University
Department of Computer Science
Department of Mechanical & Aerospace Engineering

Massively Distributed Computational Intelligence For A Self-organizing Power Grid

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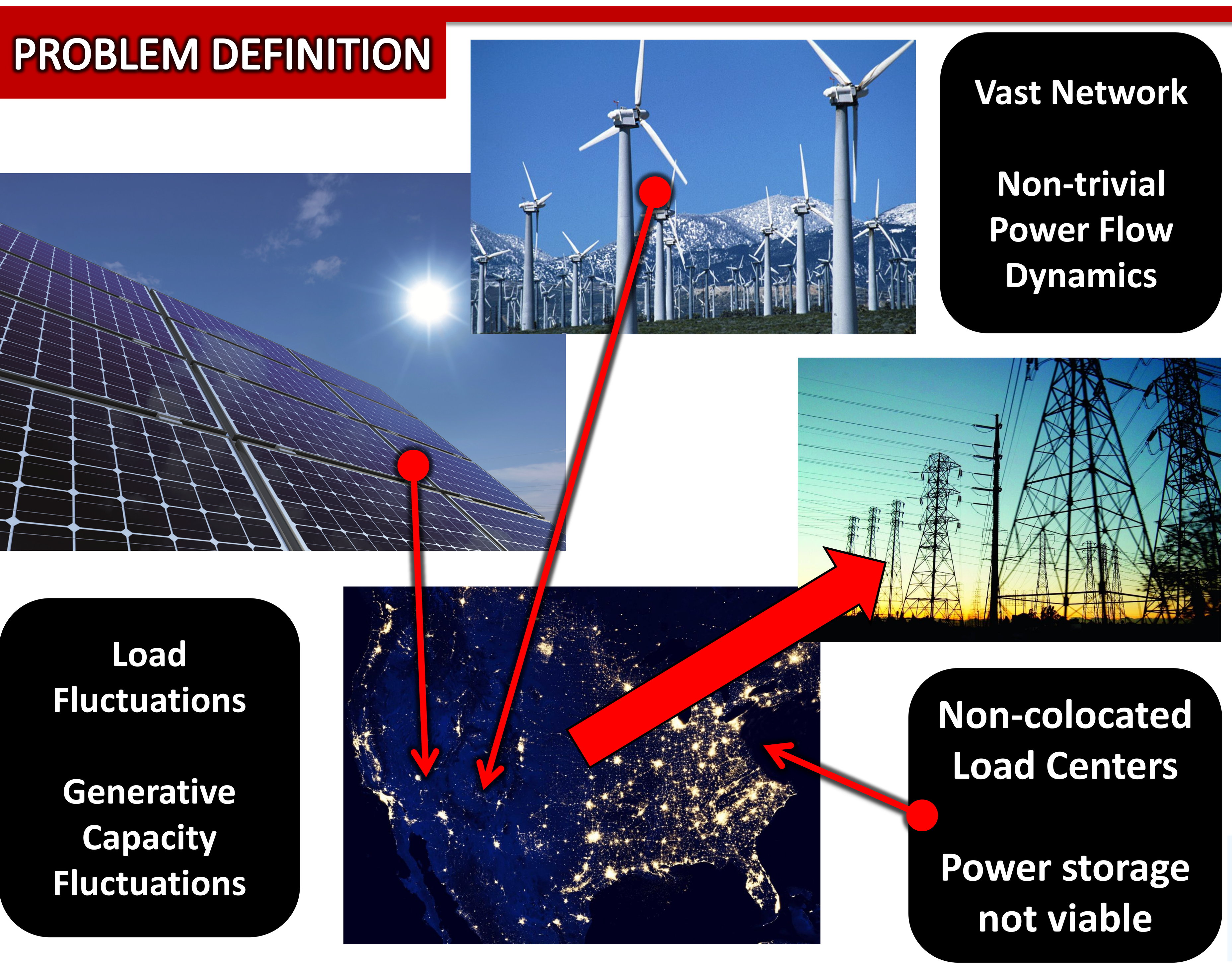
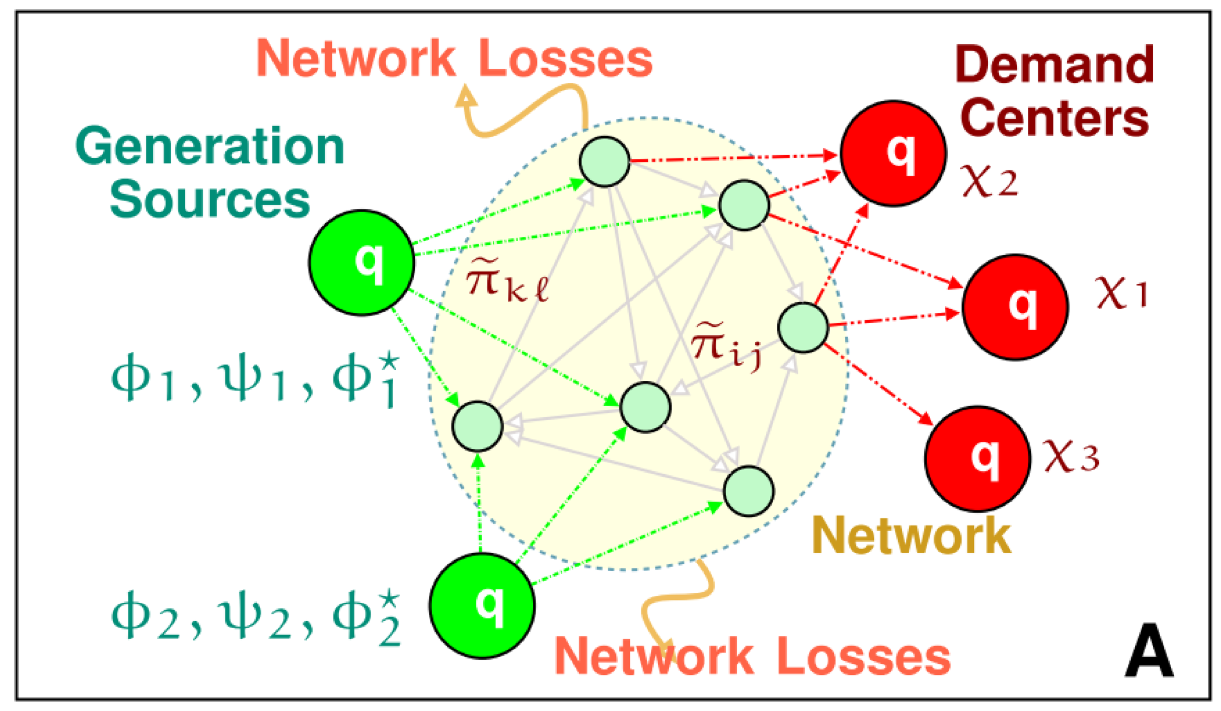
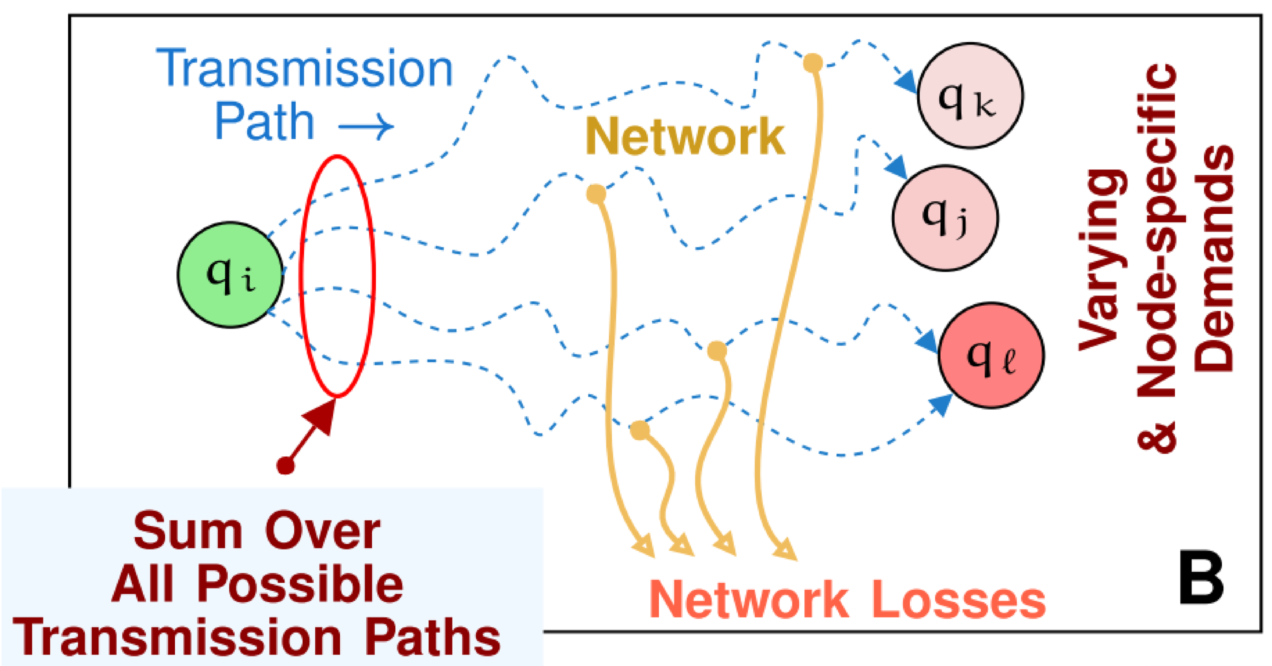
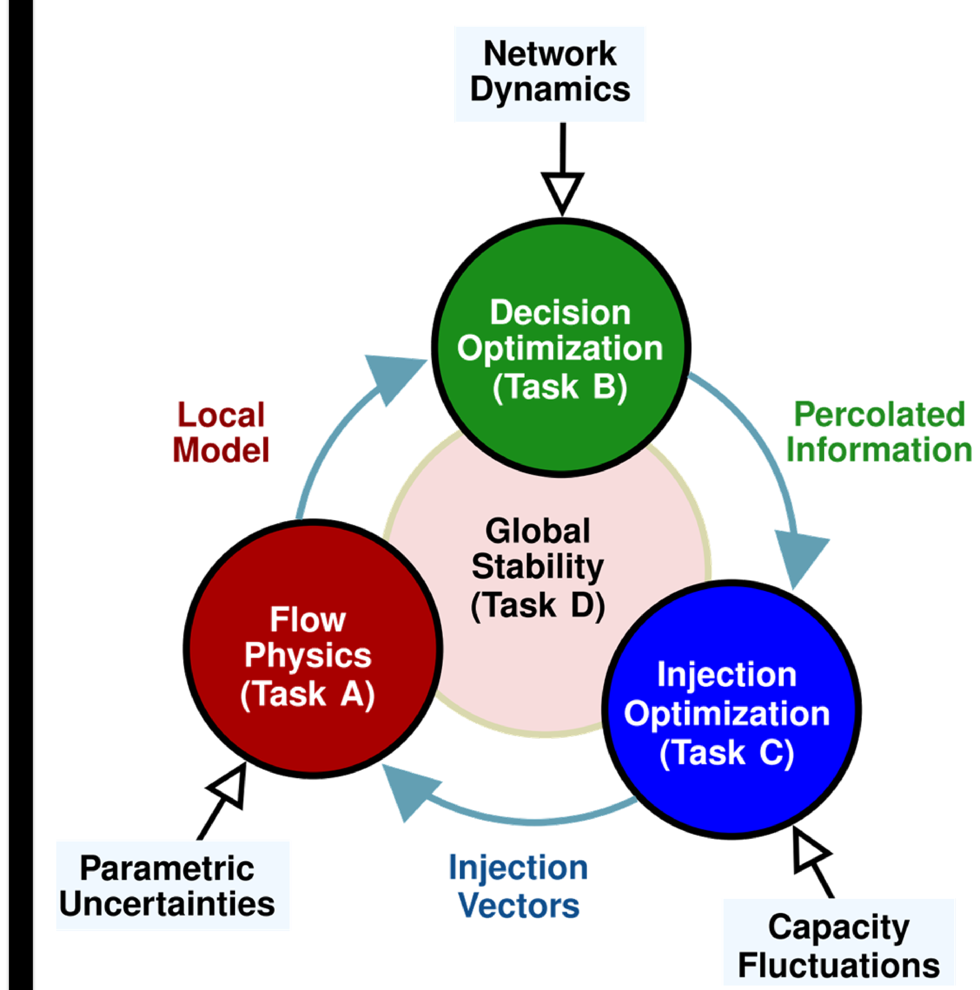
PROBLEM DEFINITION



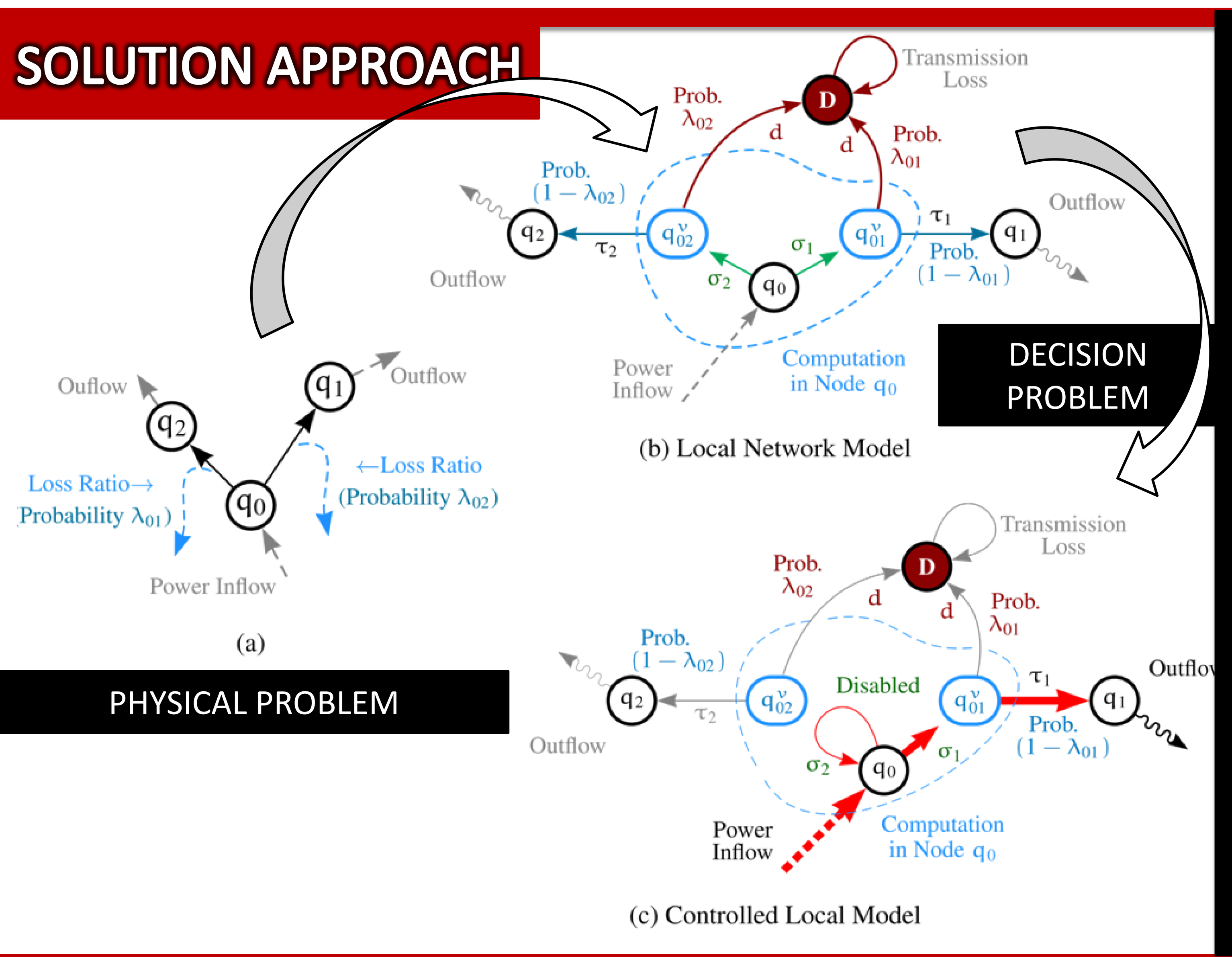






CYBER-PHYSICAL CONTROL PROBLEM

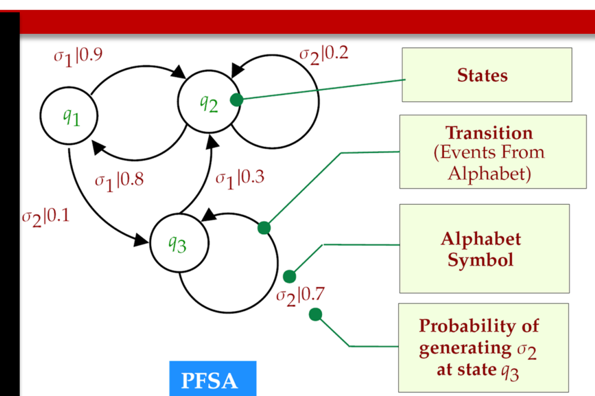
SOLUTION APPROACH



(a)

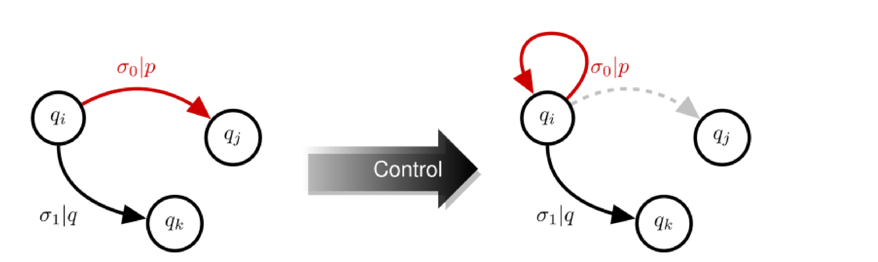
(b) Local Network Model

(c) Controlled Local Model



Parameter	Definition	Example
States:	Graph Nodes	$Q = \{q_1, q_2, q_3\}$
Alphabet:	Edge labels	$\Sigma = \{\sigma_1, \sigma_2\}$
Transition Map:	$\delta : Q \times \Sigma \rightarrow Q$	$\delta(q_1, \sigma_1) = q_2$ $\delta(q_3, \sigma_2) = q_3$
Probability Morph:	$\widetilde{\Pi} : Q \times \Sigma \rightarrow [0,1]$	$\widetilde{\Pi}(q_1, \sigma_1) = 0.9$ $\widetilde{\Pi}(q_3, \sigma_2) = 0.7$

Definition 1 (**Control Philosophy**): If $\delta(q_i, \sigma) = q_k$, then the *disabling* of σ at q_i prevents the state transition from q_i to q_k. by replacing the original transition with a self-loop with identical occurrence probability.



Definition 2 (**Generated Language**): The language $L(q_i)$ generated by a PFSA G initialized at the state $q_i \in Q$:

$$L(q_i) = \{s \in \Sigma^* \mid \delta(q_i, s) \in Q\}$$

For every $q_j \in Q$, $L(q_i, q_j)$ denotes the set of all strings that, starting from the state q_i, terminate at the state q_j, i.e.,

$$L(q_i, q_j) = \{s \in \Sigma^* \mid \delta(q_i, s) = q_j \in Q\}$$

Definition 3 (**State Transition Matrix**): The state transition matrix $\Pi \in [0,1]^{\text{CARD}(Q)\times\text{CARD}(Q)}$, for a given PFSA:

$$\forall q_i, q_j \in Q, \Pi_{ij} = \sum_{\sigma \in \Sigma \text{ s.t. } \delta(q_i,\sigma)=q_j} \widetilde{\Pi}(\sigma, q_i) \quad (1)$$

Π is a stochastic matrix, where Π_{ij} is the probability of the $q_i \rightarrow q_j$ transition.

A signed real measure [1] $\nu^i : 2^{L(q_i)} \rightarrow \mathbb{R} \equiv (-\infty, +\infty)$ is constructed on the σ-algebra $2^{L(q_i)}$ [2], implying that every singleton string set $\{s \in L(q_i)\}$ is a measurable set.

Definition 4 (**Language Measure**):

- Let $\omega \in L(q_i, q_j) \subseteq 2^{L(q_i)}$.
- The signed real measure ν_θ^i of every singleton set $\{\omega\}$:

$$\nu_\theta^i(\{\omega\}) \triangleq \theta(1-\theta)^{|\omega|}\widetilde{\Pi}(q_i, \omega)\chi(q_j)$$

- For any $\theta \in (0,1)$, the signed real measure of a sublanguage $L(q_i, q_j) \subseteq L(q_i)$ is defined as:

$$\nu_\theta^i(L(q_i, q_j)) \triangleq \sum_{\omega \in L(q_i,q_j)} \theta(1-\theta)^{|\omega|}\widetilde{\Pi}(q_i, \omega)\chi_j$$

- The measure of $L(q_i)$, is defined as

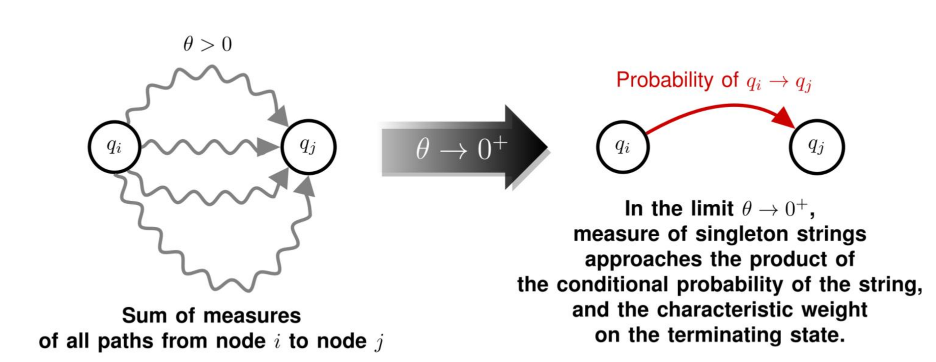
$$\nu_\theta^i(L(q_i)) \triangleq \sum_{q_j \in Q} \nu_\theta^i(L_{i,j})$$

References

[1] W. Rudin, *Real and Complex Analysis, 3rd ed.* McGraw Hill, New York, 1988.
[2] I. Chattopadhyay and A. Ray, "Language-measure-theoretic optimal control of probabilistic finite-state systems," *International Journal of Control*, vol. 80, no. 8, pp. 1271–1290, Aug. 2007.

Closed Form Representation:

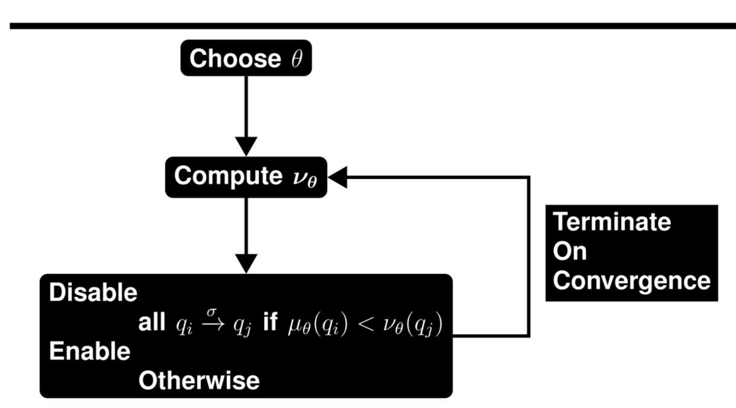
$$\nu_\theta = \theta\left[\mathbb{I} - (1-\theta)\Pi\right]^{-1}\chi$$



Sum of measures of all paths from node i to node j

In the limit $\theta \rightarrow 0^+$, measure of singleton strings approaches the product of the conditional probability of the string, and the characteristic weight on the terminating state.

Optimal Supervision Problem:
Find optimal decision to maximize language measure elementwise



- The solution obtained is optimal, unique, efficiently computable, and maximally permissive among policies with maximal performance.
- Language-measure-theoretic optimization improves the measure vector in a monotonically leading to element-wise maximization

CENTRALIZED CONTROL

Algorithm 1: Distributed Update of Node Measures

```
input : G_N = (Q, Σ, δ, Π~, χ, 𝒞), θ
begin
  Initialize ∀q_i ∈ Q, ν^_θ|_i = 0
  /* Begin Infinite Asynchronous Loop */
  while true do
    for each node q_i ∈ Q do
      if 𝒩(q_i) ≠ ∅ then
        m = CARD(𝒩(q_i))
        for each node q_j ∈ 𝒩(q_i) do
          /* (a1) Internode Communication */
          Query ν^_θ|_j & Drop Prob. λ_ij
          /* (a2) Control Adaptation */
          if ν^_θ|_j < ν^_θ|_i then
            Π_ii = Π_ii + Π_i(q^V_ij)
            Π_i(q^V_ij) = 0;   /* Disable */
          else
            if Π_i(q^V_ij) == 0 then
              Π_i(q^V_ij) = f_ij
              Π_ii = Π_ii − f_ij /* Enable */
            endif
          endif
          /* (a3) Updating Virtual Nodes */
          ν^_θ|_(q^V_ij) = (1 − θ)(1 − λ_ij) ν^_θ|_j
        endfor
      endif
      /* (a4) Updating Node */
      ν^_θ|_i = Σ_{j: q_j ∈ 𝒩(q_i)} (1 − θ) Π_i(q^V_ij) ν^_θ|_(q^V_ij)
                + (1 − θ) Π_ii ν^_θ|_i + θ χ|_i
    endfor
  endw
end
```

DE-CENTRALIZED CONTROL

Theoretical Guarantees:

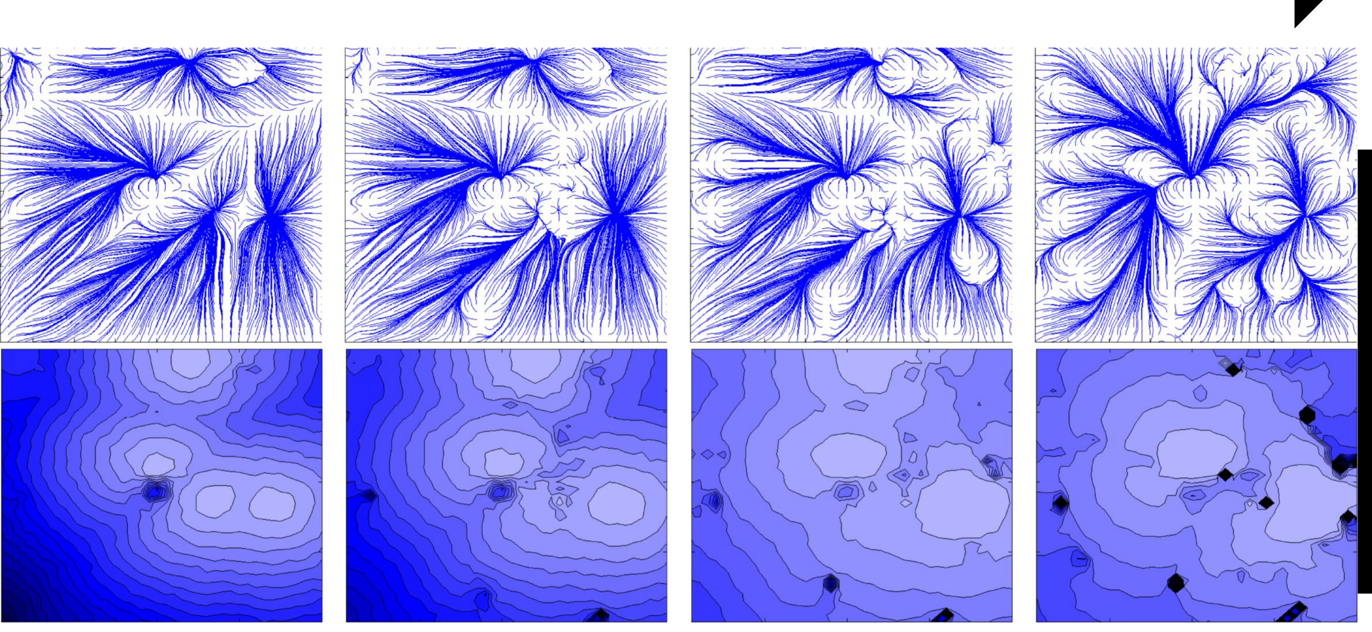
- Guaranteed convergence to **ϵ-optimal supervision policy**
- **Polynomial time-complexity** of convergence
- Guaranteed convergence to **stationary** supervision policy
- Only uses **local information** & local communication
- **Robust** to estimation uncertainties & load fluctuations

Reference

GODDeS: Globally ϵ-Optimal Routing Via Distributed Decision-theoretic Self-organization, I Chattopadhyay. ArXiv preprint arXiv:1008.1620

RESULTS

TOPOLOGY SELF-ORGANIZATION



INFORMATION PERCOLATION

FUNDING:
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