Source Localization by a Network of Imperfect Binary Sensors

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Problem

Performance of source localization by a network of imperfect binary sensors in the presence of noise

Perfect Sensors

Modeling:
Source location $y^* = \begin{pmatrix} y_1^* \\ y_2^* \end{pmatrix}$, sensor locations $x_i = \begin{pmatrix} x_{1i} \\ x_{2i} \end{pmatrix}$

distance between the source and the sensor $d_i^* = \|x_i - y^*\|$

The received signal $s(x_i)$ can be any (unknown) function strictly and monotonically decreasing in distance. Let

$$c_i = \{x \in R^2 \mid \|x - x_i\| \le r_0\}$$

and

$$1(y \in c_i) = \begin{cases} 1 & if\ y \in c_i \\ 0 & if\ y \notin c_i \end{cases}$$

represent a circle centered at sensor i with the trigger threshold $r_0 > 0$ and the indicator function. Define

$$\hat{y}_{per} = \frac{\sum_{i=1}^n x_i 1(y \in c_i)}{\sum_{i=1}^n 1(y \in c_i)} = \frac{\frac{1}{n}\sum_{i=1}^n x_i 1(y \in c_i)}{\frac{1}{n}\sum_{i=1}^n 1(y \in c_i)}$$

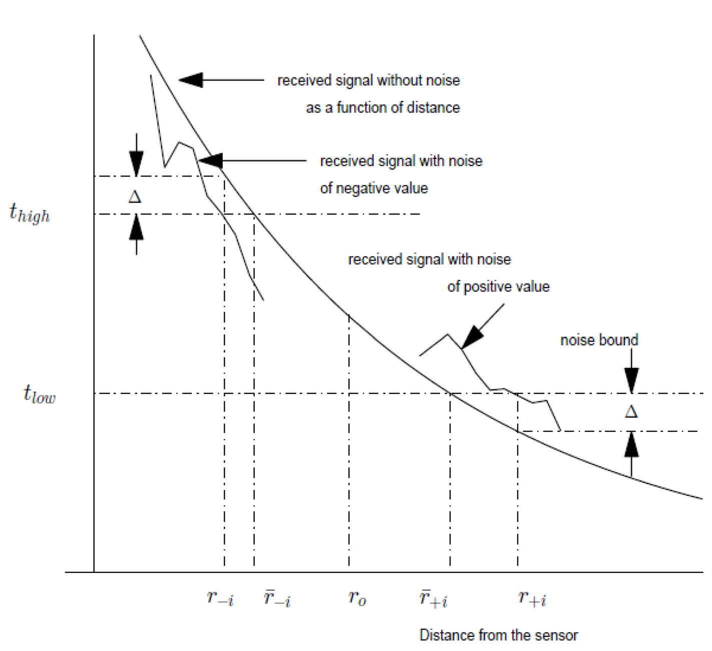
Results:

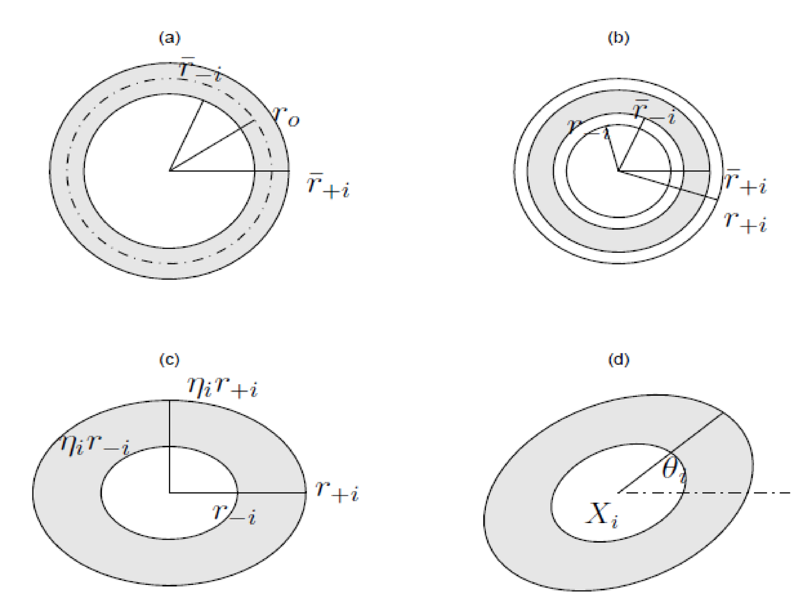
$$\sqrt{n}(\hat{y}_{per} - y) \to \mathcal{N}(0, \frac{ab}{4\pi}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix})$$

Imperfect Sensors

Non-operational sensor and sensor failure: (1) with probability p, a sensor is operational. (2) a sensor is always on.

Non-uniformity in the trigger thresholds:





Each sensor has its own threshold that is random in some interval

$$[\bar{r}_{-i}, \bar{r}_{+i}] = [r_0 - \Delta_{-i}, r_0 + \Delta_{+i}]$$

with an unknown density $\bar{\rho}_i(r)$

Noise effect: The trigger threshold becomes a random variable in

$$[r_{-i}, r_{+i}]$$

with an unknown density $\rho_i(r)$

Effects of non-radial symmetry in sensing:

$$\{z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \mid \frac{(z_1 cos(\theta_i) + z_2 sin(\theta_i) - x_{i1})^2}{r_i^2} + \frac{(-z_1 sin(\theta_i) + z_2 cos(\theta_i) - x_{i2})^2}{(\eta_i r_i)^2} \le 1\}$$

Summary: The sensing range of each binary sensor has two levels. The first level is caused by non-uniformity in the trigger threshold. The second level is due to noise.

The sensing range also has three variations. The first one describes the possible non-radial symmetry property in detection with a density $\rho_\eta(\eta)$ The second variation θ captures that directions of binary sensors that are more sensitive than other directions are random but with an equal probability. The final variation p indicates that on average (1 − p)n sensors are non-operational.

Let e_i indicate the ellipsoidal that is a standard ellipsoidal with two axes rotated by the angel θ and shifted by x_i. Define

$$\hat{y} = \frac{\sum_{i=1}^n x_i 1(y \in e_i)}{\sum_{i=1}^n 1(y \in e_i)} = \frac{\frac{1}{n}\sum_{i=1}^n x_i 1(y \in e_i)}{\frac{1}{n}\sum_{i=1}^n 1(y \in e_i)}$$

Result:

$$\sqrt{n}(\hat{y} - y) \to \mathcal{N}(0, \Sigma)$$

$$\Sigma = \begin{pmatrix} \Sigma_{11}, & 0 \\ 0 & \Sigma_{22} \end{pmatrix}, \Sigma_{11} + \Sigma_{22} = \frac{ab}{2\pi p} \cdot \frac{1 + \int_{\eta_-}^1 \eta^2 \rho_\eta(\eta) d\eta}{2\int_{\eta_-}^1 \eta \rho_\eta(\eta) d\eta}$$

Median estimator: robust in the presence of outliers

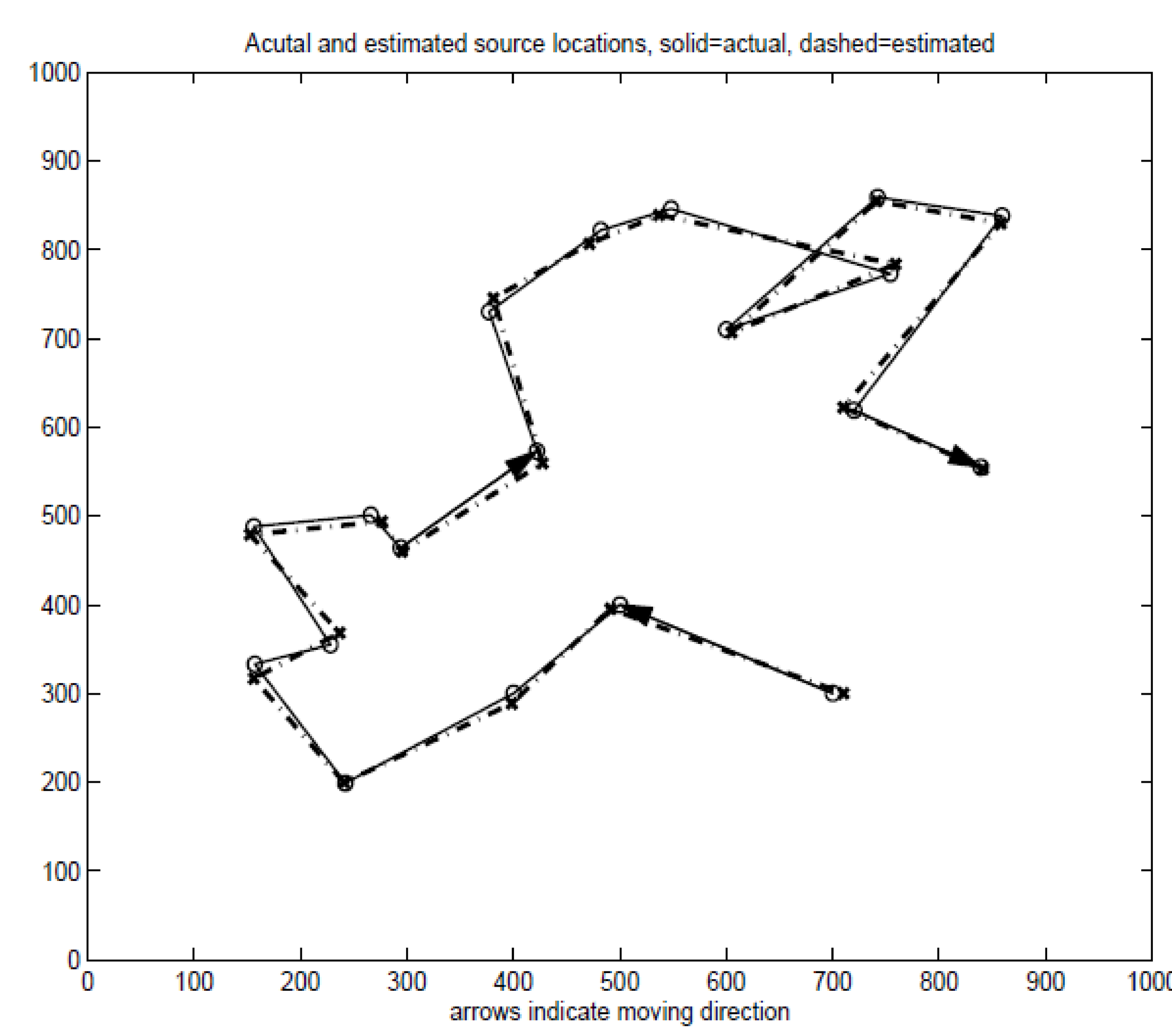
$$\hat{y}_{median} = median\{x_i\} \text{ for which } 1(y \in e_i) = 1.$$

Results: The breakdown value is 50% and

$$\hat{y}_{median} \to y.$$

Testing

Consider a radioactive source material detection problem. The area to be monitored is H = [0, 1000](meter)×[0, 1000](meter). The source propagation model is s_i = 3·10^2/d^2(i) (photon counts per sampling period). The nominal trigger threshold for an binary sensor is 12 photon counts per sampling period that is equivalently to say that the sensor i is on if and only if d(i) ≤ 50(meter). Because of non-uniformity among binary sensors, the actual trigger threshold for each binary sensor is random but between 8 to 18 counts per sampling period. Equivalently, each binary sensor detects the presence of the source between 40 and 60 meters or the uncertainty interval on the trigger threshold is [40, 60]. Now suppose the noise v is random but max |v| ≤ 4 photon counts per sampling period. This extends the uncertainty interval from [40, 60] to [36, 83]. For simulation, p = 0.95 and n = 1000. The skewness coefficient and the threshold are random, and uniformly and independently distributed in [0.9, 1] and [36, 83] respectively. The unknown source y(k) moves as a function of time. Tracking results by the mean estimate are shown.



Conclusion

The results show that most of imperfections have little or marginal effects on source localization provided that the number of sensors in the network is large.

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