Sufficient Statistics for Team Decision Problems

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Team decision problems

- finite sets Ω, X, $Y = \times_{j=1}^{n} Y_j$, $U = \times_{j=1}^{n} U_j$
- probability distribution $p : \Omega \to [0, 1]$
- random variables $x : \Omega \to X$, $y_j : \Omega \to Y_j$
- cost functional $C : X \times U \to \mathbb{R}$
- set of decentralized policies $\mathcal{D}(Y, U)$

Team decision problem [4]

optimization problem

$$\text{minimize } \mathbb{E}(C(x, \phi \circ y))$$

with variable $\phi \in \mathcal{D}(Y, U)$

- NP-hard [6]
- fundamental building block of decentralized control [1]

Sufficient statistics

- finite set $S = \times_{j=1}^{n} S_j$
- random variables $s_j : \Omega \to S_j$
- decentralized function $h \in \mathcal{D}(Y, S)$

Team sufficient statistics [7, 8]

- decentralized conditional independence, $y \perp\!\!\!\perp x \mid s$:

$$p(x, y \mid s) \in \operatorname{conv}\{p(x, g \circ s \mid s) \mid g \in \mathcal{D}(S, Y)\}$$

(reduces to conditional independence if $n = 1$)

- $h \circ y$ is team sufficient statistic if $y \perp\!\!\!\perp x \mid h \circ y$

"Weak" team sufficient statistics [2]

- decentralized simulatability, $y \mathrel{\tilde{\perp\!\!\!\perp}} x \mid s$:

$$p(x, y) \in \operatorname{conv}\{p(x, g \circ s) \mid g \in \mathcal{D}(S, Y)\}$$

(generalizes comparison of experiments [5])

- $h \circ y$ is "weak" team sufficient statistic if $y \mathrel{\tilde{\perp\!\!\!\perp}} x \mid h \circ y$

property	team sufficient	"weak" team sufficient
sufficient for optimality	yes	yes
necessary for optimality	no	yes
updatable with dynamics	yes	?

Approximately sufficient statistics

Team approximately sufficient statistics [3]

- approximate decentralized simulatability, $y \perp\!\!\!\perp_\epsilon x \mid s$:

$$\mathsf{d}_{\mathsf{TV}}(p(x, y), \operatorname{conv}\{p(x, g \circ s) \mid g \in \mathcal{D}(S, Y)\}) \leq \epsilon$$

- $h \circ y$ is team ϵ-approximately sufficient statistic if $y \perp\!\!\!\perp_\epsilon x \mid h \circ y$
- necessary and sufficient for $(2\epsilon \|C\|_\infty)$-suboptimality

Current research

- updating "weak" sufficient and approximately sufficient statistics
- algorithms to compute approximately sufficient statistics

References

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