Control improvisation in vehicle modeling and control

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Control improvisation

Given a finite alphabet $\Sigma$, find a distribution $D: \Sigma^* \rightarrow [0,1]$:

- **Hard constraint** $H$ \quad $P_{\Sigma}(\sigma) \in L(H) \wedge \sigma \in D = 1$
- **Soft constraint** $S$ \quad $P_{\Sigma}(\sigma) \in L(S) \wedge \sigma \in D \geq 1 - \xi$

\[ \forall \sigma \in L(H), \quad D(\sigma) \leq \rho \]

\[ H = \{ \text{strings of length 3 that have no consecutive 1's} \} \]

\[ S = \{ \text{strings with Hamming distance no greater than 1 from 001} \} \]

**Example:**

\[
\begin{align*}
\Sigma &= \{0, 1\}, \quad \xi = 1, \quad \rho = \frac{1}{2} \\
H &= \{ \text{strings of length 3 that have no consecutive 1's} \} \\
S &= \{ \text{strings with Hamming distance no greater than 1 from 001} \}
\end{align*}
\]

Applications:

- Composing music
- Lighting control mimicking human behavior
- Robot surveillance
- Human driving behavior
- ...

Voluntary lane-change decision-making

Train Markov traffic models using traffic data

- Statistical traffic data
- Vehicle car-following data
- Human lane-change data

Lane-change environment model for the motion of neighboring cars (Markov Decision Process)

Traffic scenario assumptions:

- Hard constraints $v \geq 0, \ h > 0$
- Soft constraints
  \[ P_v(\hat{v}(t+1) - \hat{v}(t) < \Delta v) \geq 1 - \xi_v \]
  \[ P_v(\hat{v}(t) - \hat{v}(0) < \Delta v) \geq 1 - \xi_v \]
  \[ P_h(\hat{h}(t) - \hat{h}(0) < \Delta h) \geq 1 - \xi_h \]
- Randomness

Lane-change behavior specifications:

- Hard constraints
- Satisfy safety & incentive conditions
- Soft constraints
- Number of lane changes during a period of time
  \[ P(\hat{n}_l - N_l < \Delta N_l) > 1 - \xi_n \]
- Time between two consecutive lane changes
  \[ P(\tilde{\Delta} \hat{h}_l > 1 - \xi_h) \]
- Randomness

Histogram for lane-change opportunities during stop-and-go traffic

Risk-aware motion planning

Automated vehicle (robot) $X_R$

Non-interactive vehicles (environment) $X_E$

Interactive vehicles (humans) $X_H$

Kinematic model

Motion primitives

**Maneuver automata**

**Human driving model**

- Hard constraints (stay on the road, and drive forward)
- Soft constraints (favor actions with “low enough” cost)
- Randomness (action sequences are seldom repeated)

**Optimal policy for the automated vehicle**

**Example:**

\[
\begin{align*}
\alpha^* &= \arg \min_{\alpha} \sum_{t=0}^{T} \text{Cost}(x(t), \alpha) \\
&\text{subject to} \quad x(0), x(T) \in \mathbb{X} \\
&\text{with} \quad \mathbb{X} = \{ x : x(0), x(T) \in \mathbb{X} \}
\end{align*}
\]

**References**