Private Disclosure of Information in Health Tele-monitoring

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Motivation
Example

1. Patient Bob wants to update his physician Alice about his Body Mass Index (BMI) and weight ($x$).
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2. Alice already knows the BMI category of Bob ($c$).
3. Alice and Bob want to keep the BMI category $c$ private from Eve, a passive eavesdropper, after observing the communication.
Setting and Threat Model

Setting

Disclosed Identity

The identity of the sender (s) is attached to each disclosed piece of information.
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The identity of the sender (\(s\)) is attached to each disclosed piece of information.

Intended Recipient’s Knowledge

The sender belongs to a class (\(c\)) that is known to the intended recipient.
Setting and Threat Model

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Disclosed Identity
The identity of the sender \((s)\) is attached to each disclosed piece of information.

Intended Recipient’s Knowledge
The sender belongs to a class \((c)\) that is known to the intended recipient.

Threat Model
Adversary is a passive man in the middle interested in inferring the class \(c\) of the sender \(s\) based on the disclosed information.
Idea

The sender discloses an encoded version $z$ of $x$, where the encoding depends on her class $c$. 
Objectives

Decoding Condition
The intended recipient can make full use of the sent information $z$, i.e. obtain the original message $x$ from the transmitted message $z$. 
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Hiding Class Condition
The adversary’s ability to make inference about $c$ given $s$, based on the sent information $z$ is minimized.
Some Definitions

- $S$ is the set of senders’ identities
Some Definitions

- $\mathcal{S}$ is the set of senders’ identities
- $\Sigma$ is the set of senders’ classes
Some Definitions

- $\mathcal{S}$ is the set of senders’ identities
- $\Sigma$ is the set of senders’ classes
- $\mathcal{I}$ is the set of pieces of information
The Process

The Disclosure Process

Let $R : \Sigma \rightarrow \mathcal{I}^T$ (Privacy Mapping Function)
The Disclosure Process

Let $R : \Sigma \rightarrow \mathcal{I}^\Sigma$ (Privacy Mapping Function) (Equivalent to $R : \Sigma \times \mathcal{I} \rightarrow \mathcal{I}$ being injective in the second argument)
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Let \( R : \Sigma \rightarrow \mathcal{I}^I \) (Privacy Mapping Function) (Equivalent to \( R : \Sigma \times \mathcal{I} \rightarrow \mathcal{I} \) being injective in the second argument)

Sending Information

- Sender \( s \in S \) (from class \( c \in \Sigma \)) wants to send information \( x \in \mathcal{I} \).
The Process

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Sending Information

- Sender \( s \in \mathcal{S} \) (from class \( c \in \Sigma \)) wants to send information \( x \in \mathcal{I} \).
- Let the sender encode \( z = [R(c)](x) \), and send \( z \).
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Let $R : \Sigma \rightarrow I^I$ (Privacy Mapping Function) (Equivalent to $R : \Sigma \times I \rightarrow I$ being injective in the second argument)

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- Sender $s \in S$ (from class $c \in \Sigma$) wants to send information $x \in I$.
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Receiving Information

- The intended recipient knows the identity of $s$ and her class $c$. 
## The Process

### The Disclosure Process

Let \( R : \Sigma \rightarrow \mathcal{I}^\mathcal{I} \) (Privacy Mapping Function) (Equivalent to \( R : \Sigma \times \mathcal{I} \rightarrow \mathcal{I} \) being injective in the second argument)

### Sending Information

- Sender \( s \in S \) (from class \( c \in \Sigma \)) wants to send information \( x \in \mathcal{I} \).
- Let the sender encode \( z = [R(c)](x) \), and send \( z \).

### Receiving Information

- The intended recipient knows the identity of \( s \) and her class \( c \).
- The intended recipient then can decode \( x \leftarrow [R(c)]^I(z) \).
Statistical Graphical Model

\[
P(S)
\]
Statistical Graphical Model

\[ P(C|S) \]
Statistical Graphical Model

\[ P(X|C, S) \]
Statistical Graphical Model

\[ p(Z = z | X = x, C = c) \overset{\Delta}{=} \delta(z - [R(c)](x)) \]
Statistical Graphical Model

\[ P(S) \quad P(C|S) \quad P(X|C, S) \]
Formulation of Problem

\[ \text{minimize } I(C, Z | S; R) \]
\[ \text{w.r.t } R \in (\Sigma \rightarrow \mathcal{I}^Z) \]
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1. Properties?
Formulation of Problem

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\]

1. Properties?
2. How do we learn such a privacy mapping function, \( R \)?
Theorem 1

If there exists a privacy mapping function $R$ such that $p(Z = z|C = c, S = s; R) = f(z, s)$ for all $c \in \Sigma$ then:

1. $I(C, Z|S; R) = 0$ (global optimum)
2. $p(C = c|Z = z, S = s; R) = p(C = c|S = s)$ (Bayesian updates prevented)
Intuition
Theorem 2

If $X|C = c, S = s \sim N(\mu_c, \Sigma_c)$ (Normal distribution) for every $c \in \Sigma$ and $s \in S$, then $[R(c)](x) = \Sigma_c^{-\frac{1}{2}} \cdot (x - \mu_c)$ yields $I(C, Z|S; R) = 0$ and “prevents Bayesian updates”.
Theorem 3

If \( X|C = c, S = s \sim \text{Exp}(\lambda_c) \) (Exponential distribution) for every \( c \in \Sigma \) and \( s \in S \), then \( [R(c)](x) = \lambda_c x \) yields \( I(C, Z|S; R) = 0 \) and “prevents Bayesian updates”.
Gamma Distributed Information

Theorem 4

If $X|C = c, S = s \sim \text{Gamma}(k, \theta_c)$ (Gamma distribution with shape and scale parameters) for every $c \in \Sigma$ and $s \in S$, then $[R(c)](x) = \frac{x}{\theta_c}$ yields $I(C, Z|S; R) = 0$ and “prevents Bayesian updates”.
Uniform Information

**Theorem 5**

If $X|C = c, S = s \sim U(a_c, b_c)$ (Uniform distribution) for every $c \in \Sigma$ and $S \in S$, then $[R(c)](x) = \frac{x-a_c}{b_c-a_c}$ yields $I(C, Z|S; R) = 0$ and “prevents Bayesian updates”.
The Learning Problem

Hard problem:

1. $I(C, Z|S; R)$ is non-convex in $R$. 
The Learning Problem

Hard problem:

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2. Search space is hard to compute over.
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MATLAB Implementation as a toolbox:
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1. Parametrize $R(\cdot) \rightarrow R(\cdot; \theta)$ where $\theta \in \Theta$ a (vector) of parameter(s) from a parameter space.
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1. Parametrize $R(\cdot) \rightarrow R(\cdot; \theta)$ where $\theta \in \Theta$ a (vector) of parameter(s) from a parameter space.
2. Treat all subjects as “equal”
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   - $p(C|S = s)$ is invariant in $s$. 
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3. Minimize $I(C, Z; R(\cdot; \theta))$ w.r.t. $\theta \in \Theta$
4. Non-parametric modeling of $p(X|C)$ and $p(C)$
### Information Distribution Per Weight Category

- **BMI (kg/m^2)**
- **Weight (kg)**
- **Info:**
  - Underweight
  - Healthy Weight
  - Overweight
  - Obese

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Table: Confusion Matrix. UW = Underweight, HW = Healthy Weight, OW = Overweight, OB = Obese

<table>
<thead>
<tr>
<th>Ground Truth Category</th>
<th>UW</th>
<th>HW</th>
<th>OW</th>
<th>OB</th>
</tr>
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<tbody>
<tr>
<td>UW</td>
<td>47</td>
<td>20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HW</td>
<td>14</td>
<td>1203</td>
<td>66</td>
<td>1</td>
</tr>
<tr>
<td>OW</td>
<td>0</td>
<td>45</td>
<td>194</td>
<td>47</td>
</tr>
<tr>
<td>OB</td>
<td>0</td>
<td>2</td>
<td>37</td>
<td>308</td>
</tr>
</tbody>
</table>

\[
\text{trace} (\text{Confusion Matrix}) / \text{sum} (\text{Confusion Matrix}) = 88.31\%
\]
pdi_begin

% data/information space
pdi_dimension BMI 0:2:60;
pdi_dimension weight 0:5:180;
% define classes
pdi_class underweight healthy_weight overweight obese
% provide data
pdi_datapoints underweight fv_uw
pdi_datapoints healthy_weight fv_hw
pdi_datapoints overweight fv_ow
pdi_datapoints obese fv_ob
% parameter space
pdi_var shift(pdi_nrdimensions, pdi_nrclasses);
pdi_var scale(pdi_nrdimensions, pdi_nrclasses);
% \[ z = \text{scale} \times (x - \text{shift}) \]
pdi_reference @(x, cn) bsxfun(@times, bsxfun(@minus, x, shift(:,cn)), scale(:,cn));
% such that
scale(:,1) == 1; % entry-wise
shift(:,1) == 0; % entry-wise
scale>=.1; % entry-wise
shift>=0; % entry-wise

pdi_end
Privatized Information Per Class (Top View)

- Underweight
- Healthy Weight
- Overweight
- Obese
Privatized Information Per Class (Bottom View)

- Privatized BMI
- privatized weight

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Table: Confusion Matrix After Privatizing. UW = Underweight, HW = Healthy Weight, OW = Overweight, OB = Obese

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<td>HW</td>
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<td>1217</td>
<td>276</td>
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<td>13</td>
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from 88.31%
lower bound: \#HW / sum(Confusion Matrix) = 64.01%
Future Directions

- Bounds on privacy.
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- Sensitivity analysis.
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- Study the relationships between $I(C, Z|S)$ and $I(X, Z|S)$. 
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- Sensitivity analysis.
- Relaxing the assumption of perfect classification knowledge for the intended recipient.
- Markov-type relaxation.
- Study the relationships between $I(C, Z|S)$ and $I(X, Z|S)$.
- Parametric modeling of $p(X|C)$ for learning.

Acknowledgments

- Gregorij Kurillo
- Yusuf Erol
- Arash Nourian
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