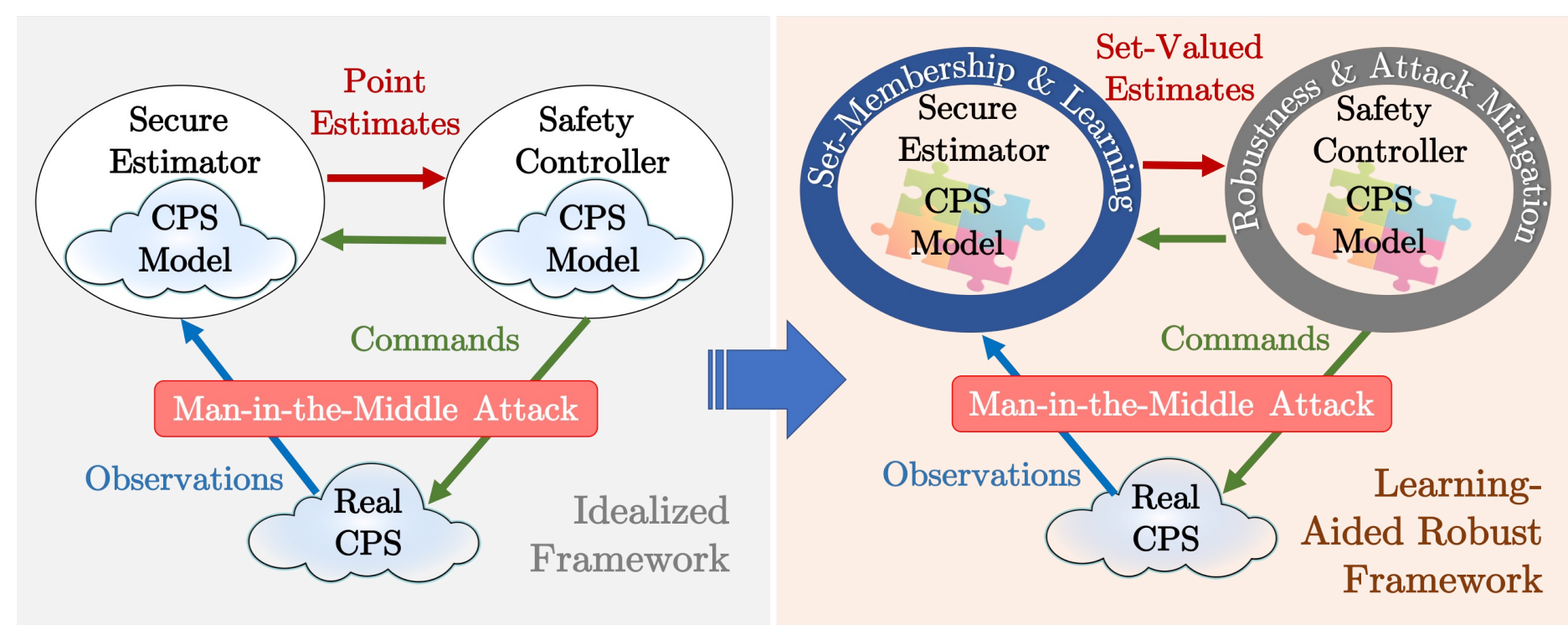


# CAREER: Towards Non-Conservative Learning-Aided Robustness for Cyber-Physical Safety and Security

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## Motivation



### Problem and Objective

- Model mismatch between real system and imperfect model jeopardize safety and security guarantees
- How to quantify and learn uncertainty using set-inclusion models?
- How to design learning-aided secure state estimator despite man-in-the-middle attacks?
- How to non-conservatively "robustify" safety control algorithms?

## Scientific Impacts

- Enable secure state estimators in the presence of set-valued uncertainties with run-time learning of attack models/strategies
- Develop safe-by-design control algorithms with attack-resilient output feedback designs with learning from run-time data
- Characterize various sources of uncertainties using inclusion models

## Broader Impacts

### Impact to Society

- Application focus: : Self-driving cars
- Improving security and safety can save lives and ensure integrity of critical infrastructures
- Broadly applicable methodology
- Can generalize to a broad class of CPS, e.g., power systems, medical devices

### Education and Outreach

- Graduate student researchers: Syed Hassaan, Mohammad Khajenejad, Zeyuan Jin, Tarun Pati
- Broadening participation in computing and engineering plan targets undergraduate and graduate students at ASU/NU, especially first-generation students and includes engagement with industry.

## Selected Publications

- Z. Jin, M. Khajenejad, M., and S.Z. Yong. "Robust Data-Driven Control Barrier Functions for Unknown Continuous Control Affine Systems." IEEE LCSS'23.
- T. Pati, S.Z. and Yong. "Robust Control Barrier Functions for Control Affine Systems with Time-Varying Parametric Uncertainties," IFAC-PapersOnLine'23.
- M. Khajenejad and S.Z. Yong. "Tight Remainder-Form Decomposition Functions with Applications to Constrained Reachability and Guaranteed State Estimation." IEEE TAC'23.
- T. Pati et al., "Interval Observers for Hybrid Dynamical Systems with Known Jump Times", IEEE CDC'23.
- M. Khajenejad et al. "Resilient State Estimation for Nonlinear Discrete-Time Systems via Input and State Interval Observer Synthesis", IEEE CDC'23.

## Methods and Results

### Robust Data-Driven Control Barrier Functions [1]:

$$\dot{x} = f(x) + g(x)u$$

- $h(x)$  is known
- $f(x)$  and  $g(x)$  are unknown but continuous

$\Rightarrow \dot{h}(x, u)$  is unknown but trajectory data is available

#### Assumption

The function  $\dot{h} : \mathcal{X} \times \mathcal{U} \rightarrow \mathbb{R}$  is

- globally Lipschitz continuous,
- globally componentwise Lipschitz continuous, or
- differentiable w.r.t.  $x$  and  $u$  with globally bounded Jacobians.

- $\dot{h} \geq \dot{h}_i - L_x \|x - x_i\|_p - L_u \|u - u_i\|_p$
- $\dot{h} \geq \dot{h}_i - L_x^\top |x - x_i| - L_u^\top |u - u_i|$
- $\dot{h} \geq \dot{h}_i + J_x \Delta x_i^+ - J_x \Delta x_i^- + J_u \Delta u_i^+ - J_u \Delta u_i^-$   
where  $\Delta x_i \triangleq x - x_i$  and  $\Delta u_i \triangleq u - u_i$ .

### Approach: Set-Membership Learning:

$$\dot{h}(x, u) \geq -\alpha(h(x))$$

Lower bounds from data?

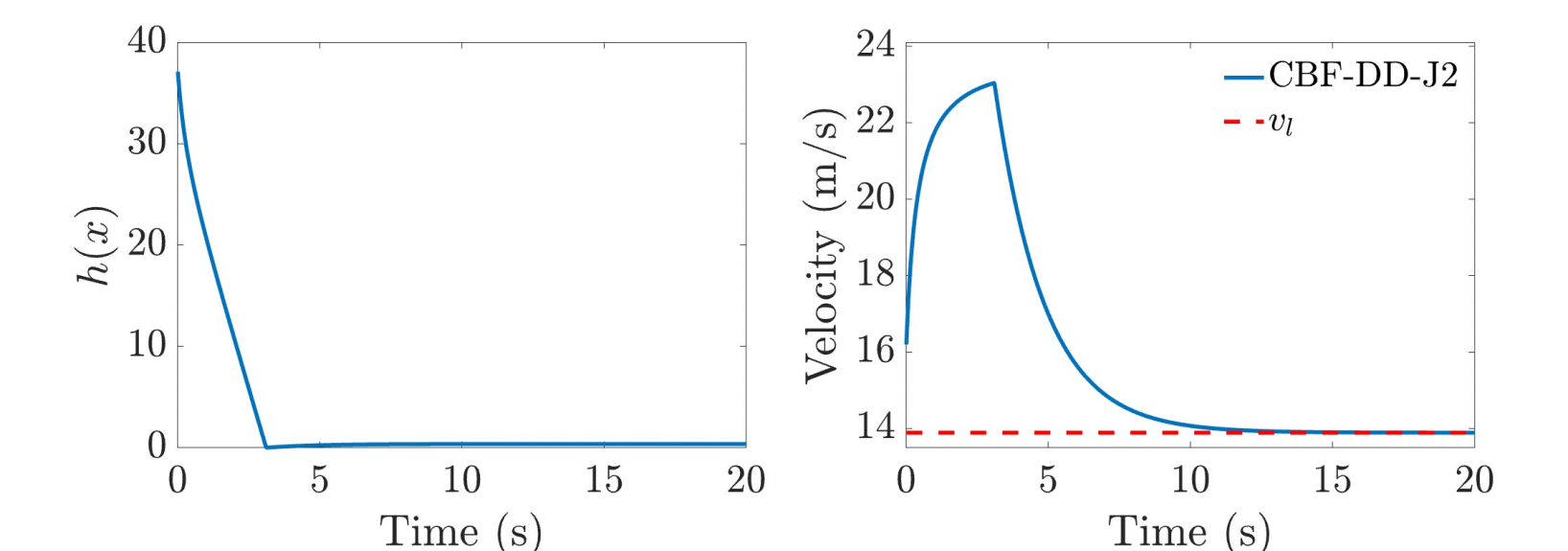


TABLE I: CPU time comparison for different methods.

Method	L	CL	J1	J2	SOCP [Taylor et al., 2021]
CPU time (s)	3029	3054	3154	2724	$2.84 \times 10^5$

### Robust CBFs with Parametric Uncertainties [2]:

$$\dot{x}(t) = f(x(t), \theta^*(t)) + g(x(t), \theta^*(t))u(t).$$

- State:  $x(t) \in \mathcal{X} \subseteq \mathbb{R}^n$ , Input:  $u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$
- Unknown parameter:  $\theta^*(t) \in \Theta \subseteq \mathbb{R}^p$ , and
- Unknown parameter variation:  $\dot{\theta}^*(t) \in \Theta_d \subseteq \mathbb{R}^p$ .

#### Method:

$$\dot{h}(x, u, \theta, \dot{\theta}) \geq -\alpha(h(x, \theta)), \quad \forall \theta \in \Theta, \dot{\theta} \in \Theta_d$$

equivalently,  $\min_{\theta, \dot{\theta}} \dot{h}(x, u, \theta, \dot{\theta}) + \alpha(h(x, \theta)) \geq 0$ ,

#### Mixed-Monotone Decomposition Functions:

- if  $\underline{\theta} \leq \theta \leq \bar{\theta} \Rightarrow f_d(\underline{\theta}, \bar{\theta}) \leq f(x, \theta) \leq f_d(\bar{\theta}, \underline{\theta})$

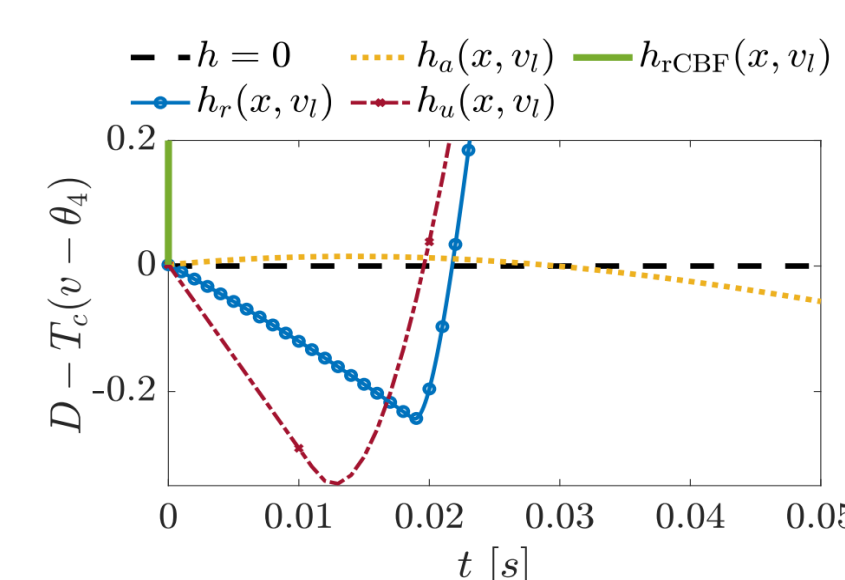
#### Concave Bounding:

- $\dot{h}(x, u, \theta, \dot{\theta}) \geq -\alpha(h(x, \theta)), \quad \forall \theta \in \text{Ver}(\Theta), \dot{\theta} \in \text{Ver}(\Theta_d)$

#### Dual LP:

$$\dot{h}(x, u, \theta, \dot{\theta}) + \alpha(h(x, \theta)) = (\phi(x) + \rho(x)u)^\top \theta + \psi(x)^\top \dot{\theta} + \sigma(x) + \tau(x)u$$

- Dual:  $\inf_{\theta, \dot{\theta}} (\phi(x) + \rho(x)u)^\top \theta + \psi(x)^\top \dot{\theta} + \sigma(x) + \tau(x)u$   
s.t.  $P_\theta \theta \leq q_\theta, P_{\dot{\theta}} \dot{\theta} \leq q_{\dot{\theta}}$

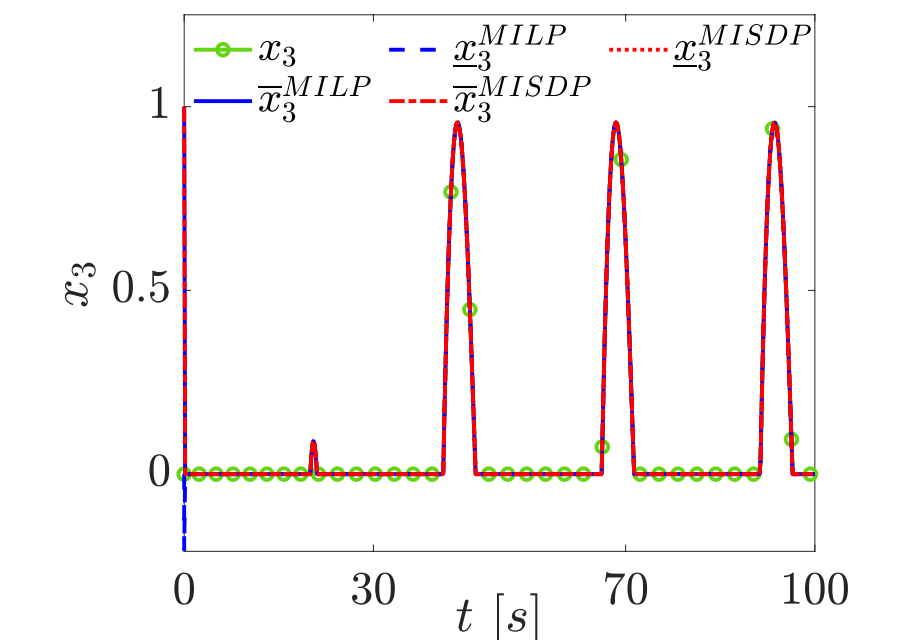


Uncertainty-dependent CBFs (e.g., time-to-collision constraint) are satisfied, while adaptive/robust approaches does not.

### Tight Mixed-Monotone Decomposition Functions [3] and Interval Observers [4]:

- Designed a remainder-form mixed-monotone decomposition with the following properties:
  - Applicable to non-smooth, semi-continuous functions
  - Tightest in the family (that includes Yang et al. 2019)
  - Tractable/Computable in closed form.

- Leveraged mixed-monotone embedding systems for interval observer designs for continuous- and discrete-time and hybrid systems with known jumps.



### Resilient State Estimation [5]:

- Designed interval observer for estimation of states and unknown inputs (with no known bounds or statistics).
- Leveraged system structure to cancel out the effects of the unknown inputs and utilized mixed-monotone decompositions to design a correct and stable observer.

