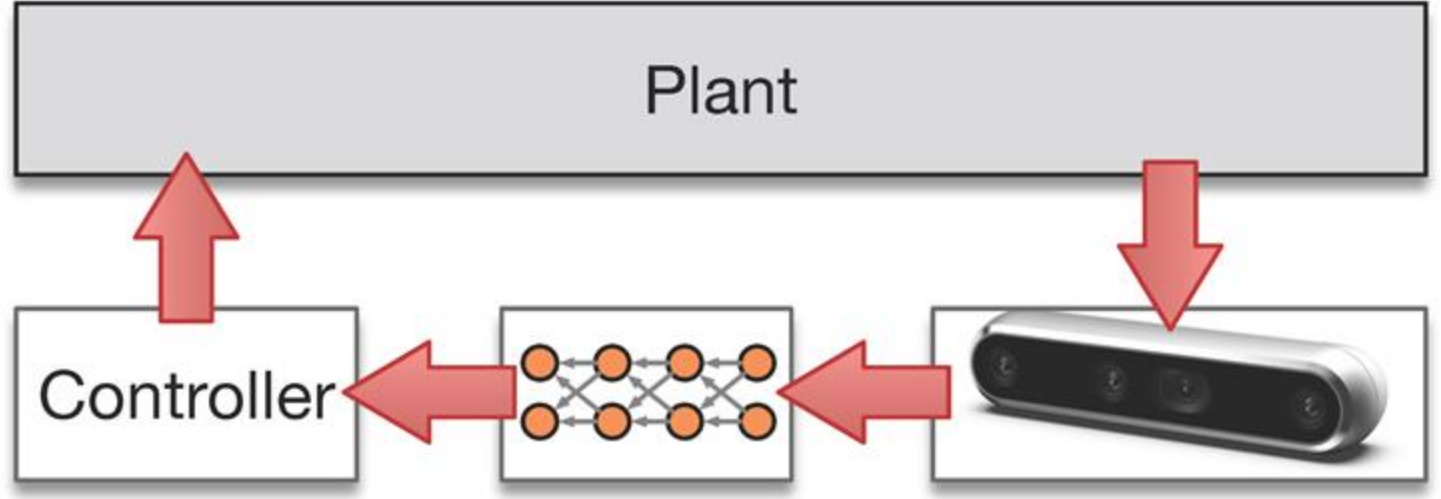


Formally Correct Deep Perception For Cyber-Physical Systems

LiDAR localization with guarantees

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Context

- The **perception** pipeline of autonomous systems increasingly relies on deep learning methods, that provide **no formal guarantees** of correctness or performance.
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- We aim to provide such guarantees, in two possible ways:
 - proving **deterministic worst-case bounds** on learning models;
 - using a **supervisor** to correct the learning method when needed
 - Meeting our goals will have major impact by:
 - boosting the adoption of **provably safe** autonomous cars, UAVs, and other autonomous platforms.
 - reducing risks** associated with existing autonomous vehicles
 - introducing a new graduate course on the **intersection of learning and control**.

Problem

- Point cloud registration** (relating two LiDAR scans from different poses) is a key step in localization/SLAM.
- We want a reliable way to perform **global** registration. I.e., no "good initialization" should be required for correctness.
- Existing algorithms are **local** and need point correspondences, which may not exist in LiDAR point clouds.
- Old PASTA algorithm requires a convex environment to provide bounds, which limits its usability.

Old PASTA

- We previously developed a registration algorithm named **PASTA** [1,2] (Provably Accurate Simple Transformation Alignment).
- It is **fast** and comes with **worst-case guarantees**.

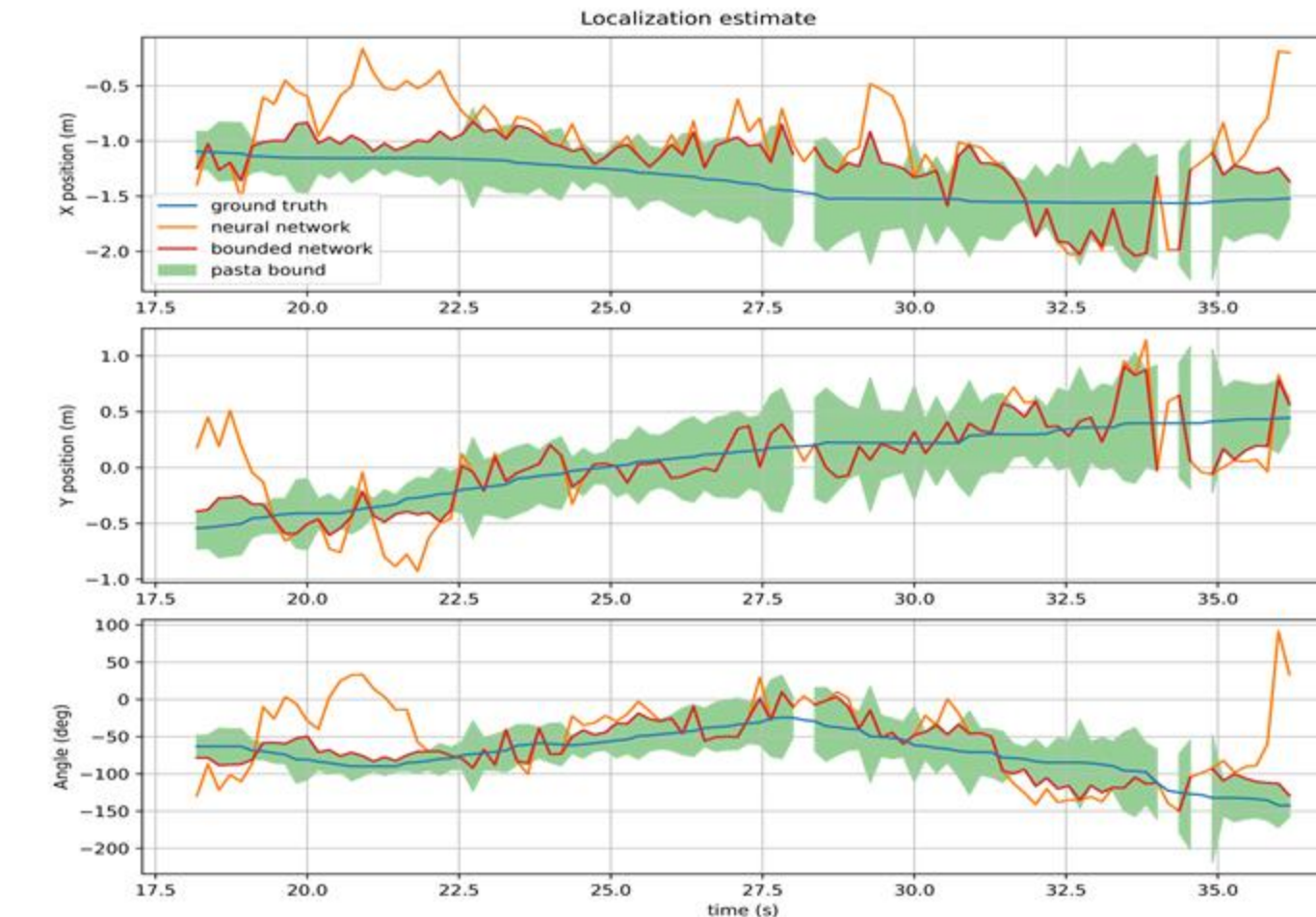
Algorithm 1 PASTA

Input: Point clouds $\{\mathbf{r}_1^{(i)}\}_{i=1}^{m_1}, \{\mathbf{r}_2^{(i)}\}_{i=1}^{m_2}$
Output: Transformation $\hat{\mathbf{R}}, \hat{\mathbf{p}}$

for each point cloud i **do**
 $H_i \leftarrow$ convex hull of $\{\mathbf{r}_i^{(j)}\}_{j=1}^{m_i}$
 $\mathbf{c}_i, \Sigma_i \leftarrow$ first and second moments of H_i
end for
 $\hat{\mathbf{R}} \leftarrow$ closed-form solution of $\Sigma_2 = \mathbf{R}\Sigma_1\mathbf{R}^T$
 $\hat{\mathbf{p}} \leftarrow$ closed-form solution of $\mathbf{c}_2 = \mathbf{R}\mathbf{c}_1 + \mathbf{p}$

Old PASTA Supervised Neural Network

- We train a **neural network** to perform localization using point clouds as input, and test it on the trajectory data we collected.
- Old PASTA and its bound act as a **supervisor** for the neural network.
- The neural network has good average performance, but sometimes behaves poorly. The **old PASTA supervisor bounds the network's output**, maintaining a limited worst-case error.



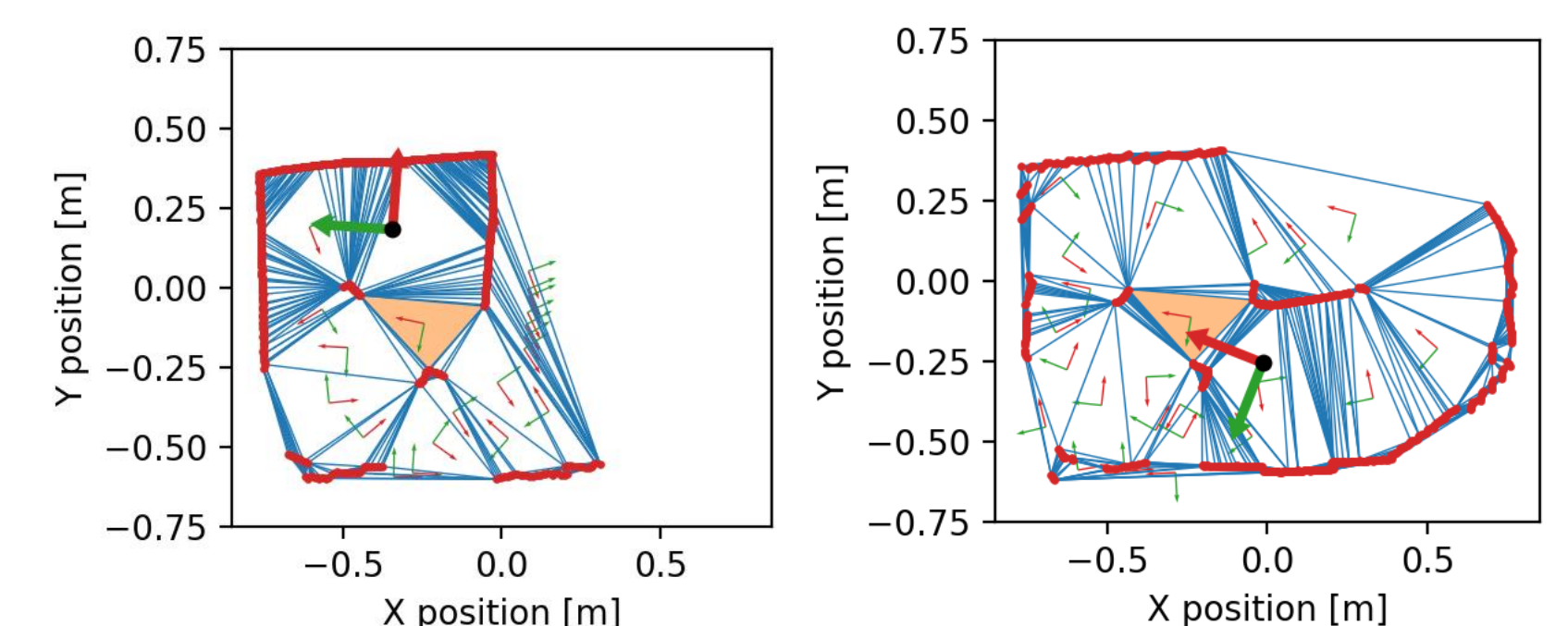
Triangle PASTA

- We developed Triangle PASTA to extend the old PASTA to non-convex environments, using triangles to find common areas in both scans (highlighted in orange).
- The overlapping areas in both scans are passed to old PASTA to estimate the roto-translation and calculate bounds error bounds

Algorithm 1 Triangle PASTA

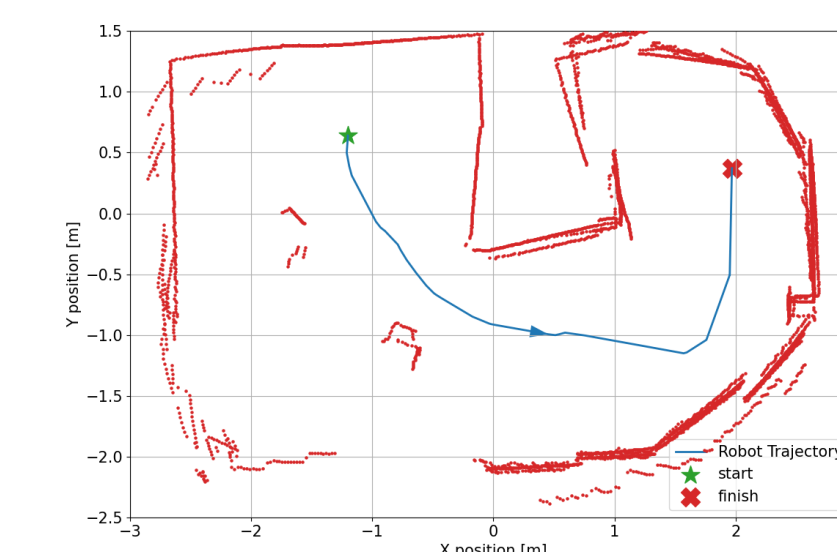
Input Point clouds $\{\mathbf{r}_1^{(i)}\}_{i=1}^{m_1}, \{\mathbf{r}_2^{(i)}\}_{i=1}^{m_2}$
Output Transformation $\hat{\mathbf{R}}, \hat{\mathbf{p}}$

$T_1 \leftarrow$ triangulation of $\{\mathbf{r}_1^{(i)}\}_{i=1}^{m_1}$
 $T_2 \leftarrow$ triangulation of $\{\mathbf{r}_2^{(i)}\}_{i=1}^{m_2}$
 $(M_1, M_2) \leftarrow$ matched triangles points
for $i=1,2$ **do**
 $H_i \leftarrow$ convex hull of M_i
 $\mathbf{c}_i, \Sigma_i \leftarrow$ first and second moments of H_i
end for
 $\hat{\mathbf{R}} \leftarrow$ closed form solution of $\Sigma_2 = \mathbf{R}\Sigma_1\mathbf{R}^T$
 $\hat{\mathbf{p}} \leftarrow$ closed form solution of $\mathbf{R}\mathbf{c}_1 + \mathbf{p}$

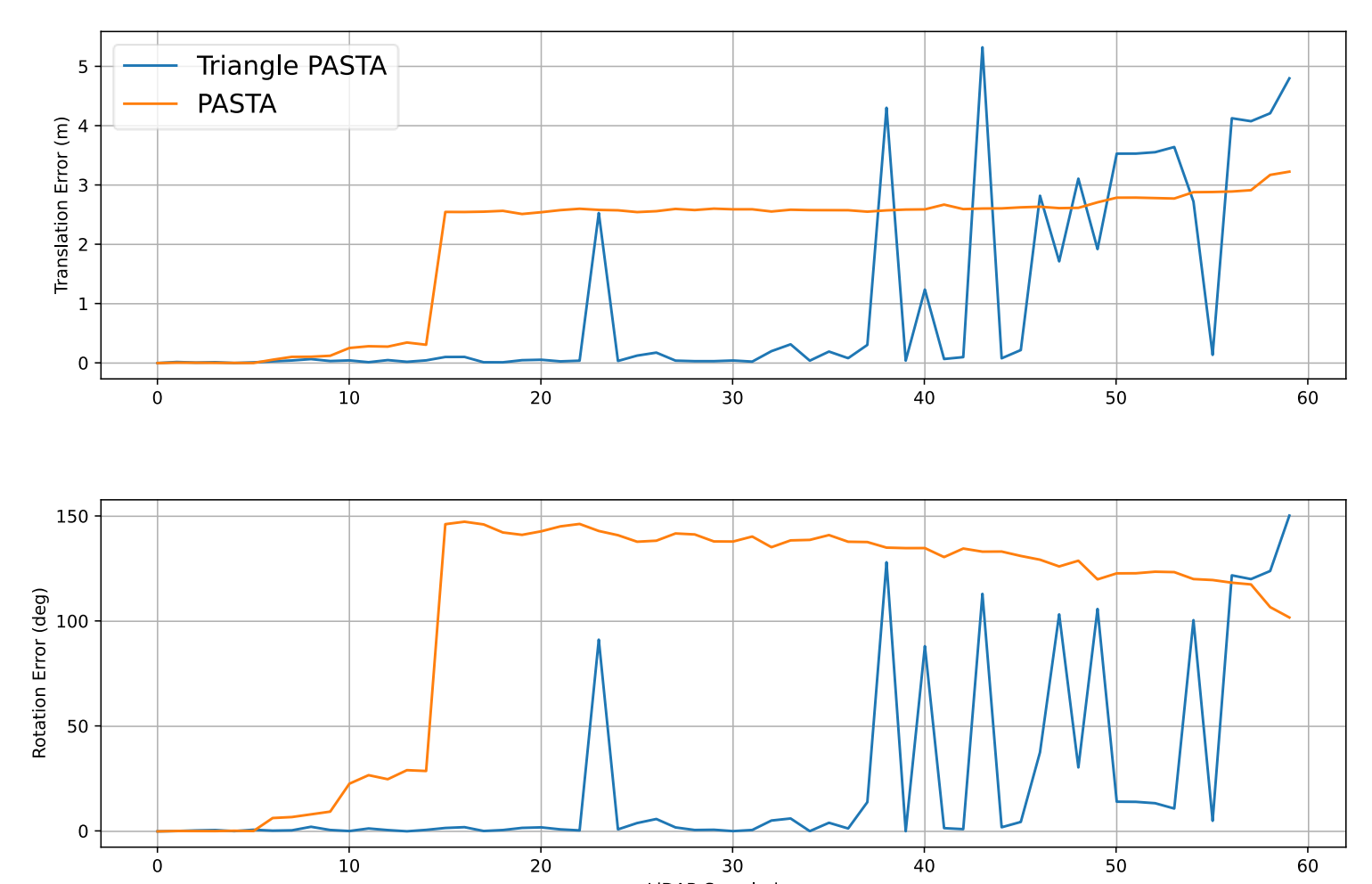


Triangle PASTA Performance

- We compare Triangle PASTA's performance against old PASTA.
- A robot moves in a non-convex environment so that its starting position is completely occluded at the end of its path while recording point clouds with a 2D LiDAR.



- We align each point cloud along the robot's trajectory with the initial point cloud.
- Triangle PASTA provides more accurate roto-translation estimates than old PASTA, especially as the overlap between point clouds shrinks.



- [1] M. Marchi, J. Bunton, B. Ghahesifard, P. Tabuada. "LiDAR Point Cloud Registration with Formal Guarantees." IEEE 61st Conference on Decision and Control. 2022.
- [2] M. Marchi, J. Bunton, Y. Gas, B. Ghahesifard, P. Tabuada. "Sharp Performance Bounds for Pasta". IEEE Control Systems Letters. 2023.