Resilience of Networks with Switching Topologies

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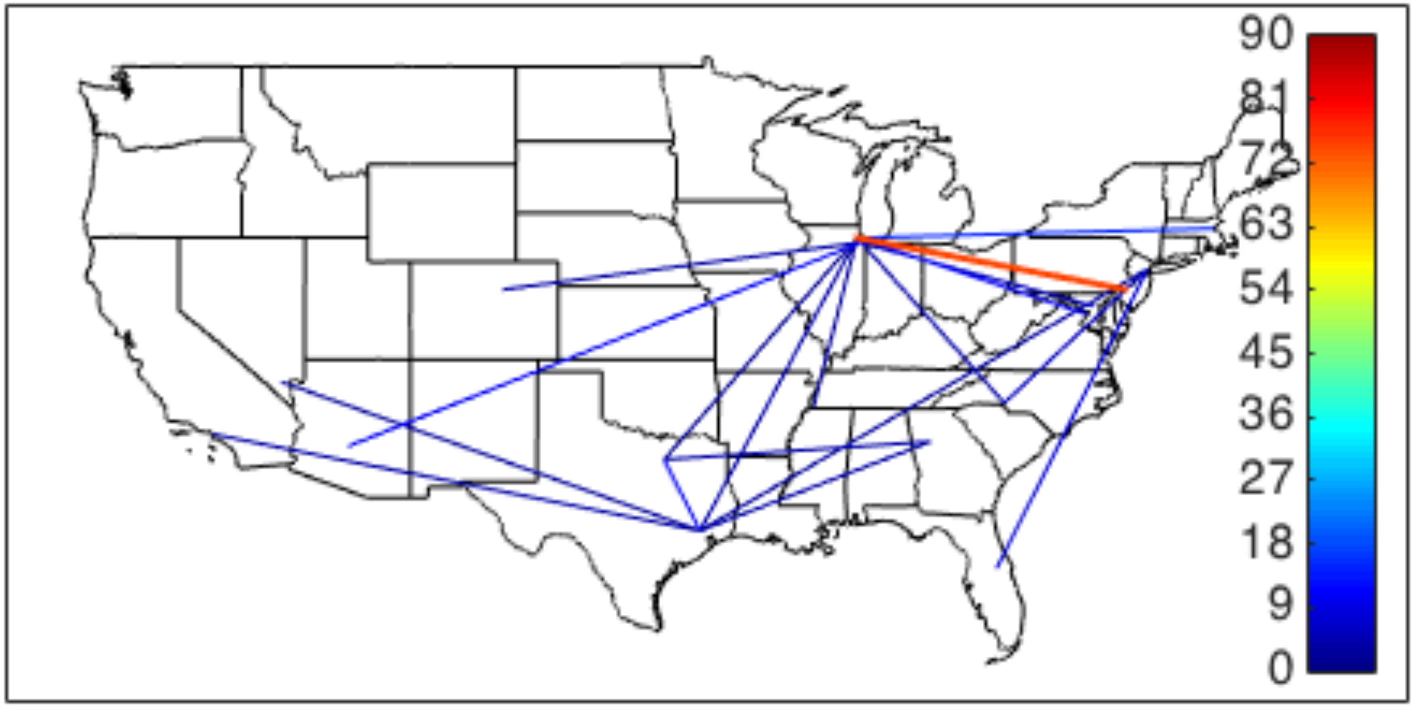
(with Karthik Gopalakrishnan & Joao Cavalcanti)

Infrastructure systems are networked, leading to propagation of impacts

* e.g, Delay propagation in air traffic networks

26 July 2012, 4 AM EST



(Average link delay in minutes)

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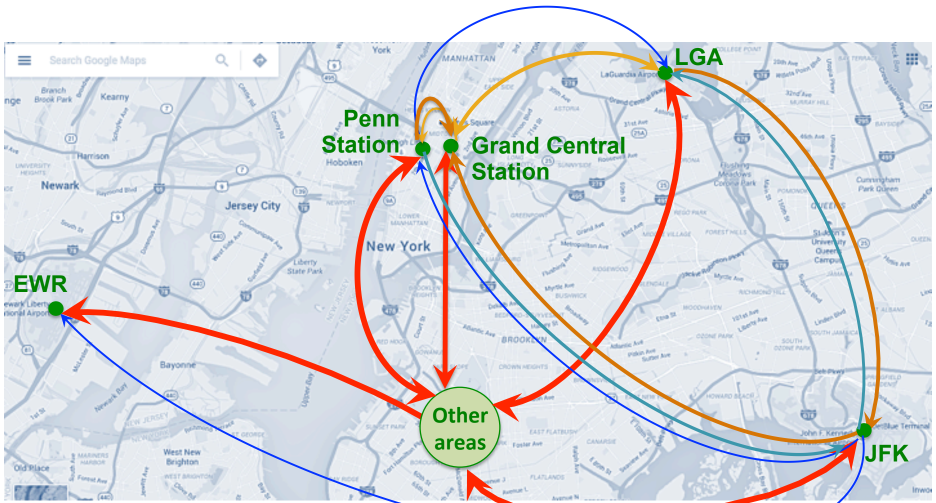
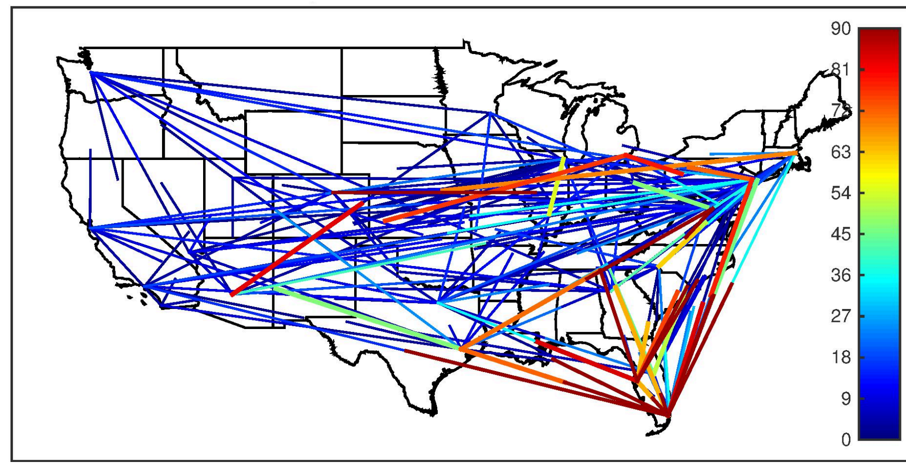
Some properties of infrastructure networks



[Traffic Data: NYC Taxi & Limousine Commission]



[Delay Data: Bureau of Transportation Statistics]

* Nodal and link states are best modeled as continuous variables
* Interactions are weighted and directed (asymmetric)
* Interactions (network topologies) vary with time

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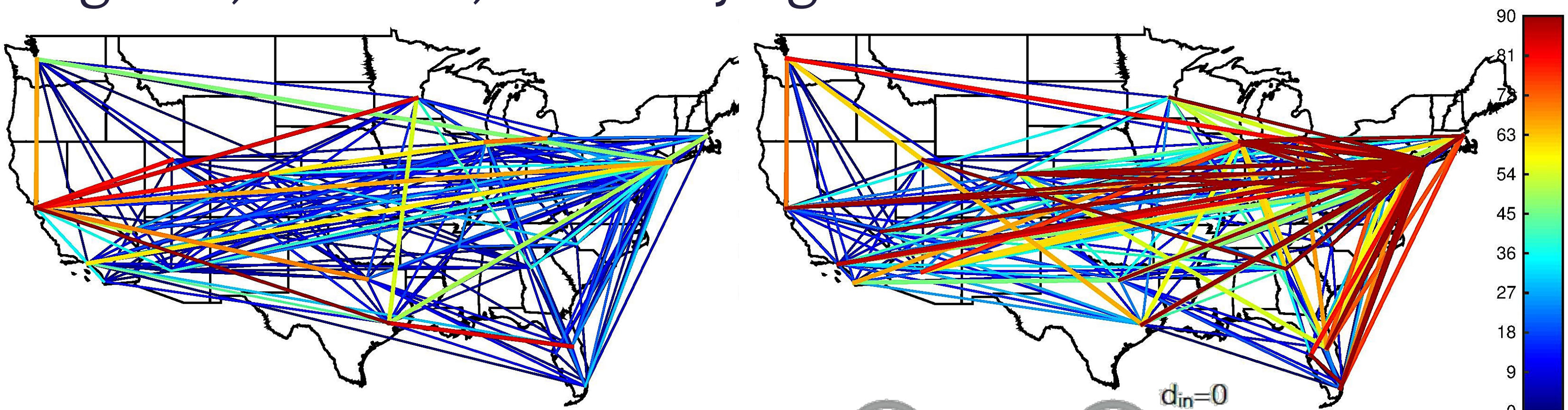
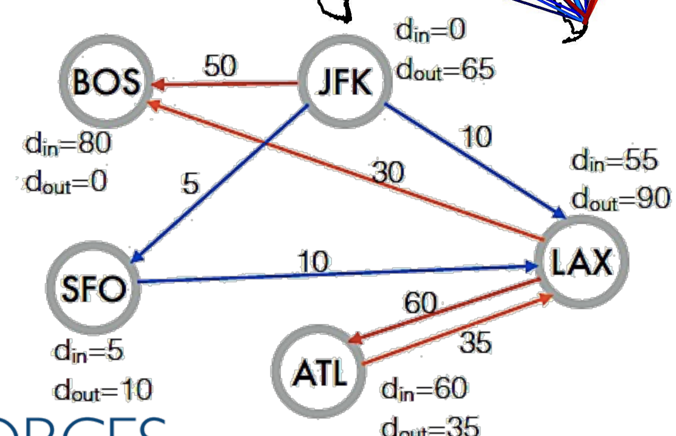
A network-centric view of air traffic delays

* For example, delay levels on edges between airports
* Weighted, directed, time-varying networks



Adjacency matrix, A:

$$a_{ij} = \begin{cases} w_{ij}, & \text{if } (i,j) \in \mathscr{E}, \\ 0, & \text{otherwise} \end{cases}$$



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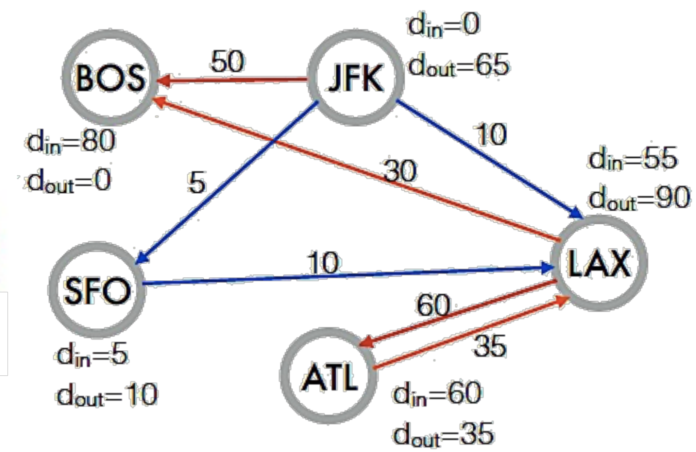
Simplistic model of delay dynamics

- Given an adjacency matrix, $A = [a_{ij}]$

$$d^i_{in}(t+1) = \alpha^i_{in} d^i_{in}(t) + \sum_j \beta^{in}_{ji} \bar{a}_{ji}(t) d^j_{out}(t)$$

$$d^i_{out}(t+1) = \alpha^i_{out} d^i_{out}(t) + \sum_j \beta^{out}_{ij} \bar{a}_{ij}(t) d^j_{in}(t)$$



- "State" of system: $\vec{x}(t) = \begin{bmatrix} \vec{d}^{\text{out}}(t) \\ \vec{d}^{\text{in}}(t) \end{bmatrix}$

- Therefore, for given network topology: $\vec{x}(t+1) = \Gamma(t)\vec{x}(t)$

  where $\Gamma(t) = [\alpha] + [\beta] \begin{bmatrix} 0 & \bar{A}(t)^T \\ \bar{A}(t) & 0 \end{bmatrix}$

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[Gopalakrishnan et al. CDC 2016]

Network topology is time-varying

* For tractability, assume that network topology belongs to a finite (known) set of possibilities
  * Results in a hybrid system
* Assume that network topology switches between different values in a Markovian manner
  * Results in a Markov Jump Linear System
* Each discrete mode has its own linear dynamics, depending on the network topology (adjacency matrix)

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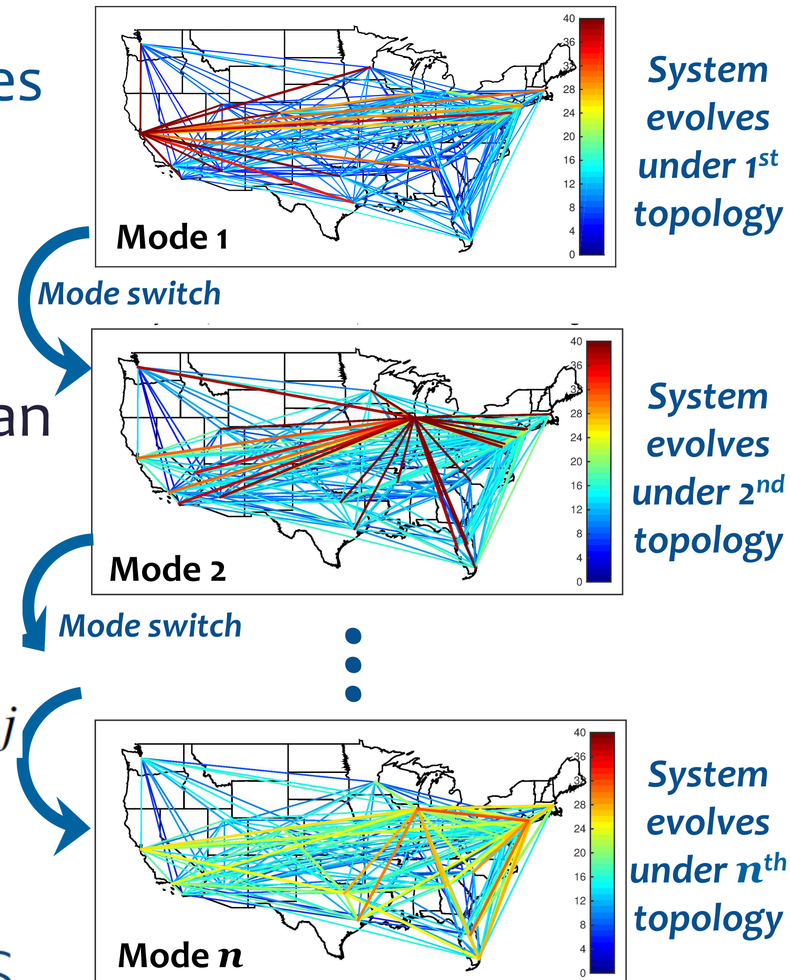
Dynamics with switching network topologies

* Identify set of characteristic topologies ("discrete modes of operation")
* Determine linear continuous state dynamics under a fixed topology
* Switched linear system with Markovian transitions:

$$
\begin{aligned}
\vec{x}(t+1) &= \Gamma_{m(t)}\vec{x}(t) \\
\pi_{ij}(t) &= \Pr[m(t+1)=j|m(t)=i] \\
\vec{x}(t+1) &= J_{ij}\Gamma_i\vec{x}(t),\ \text{if } m(t)=i \text{ and } m(t+1)=j
\end{aligned}
$$

* Markov Jump Linear System (MJLS)



[Gopalakrishnan et al. CDC 2016]

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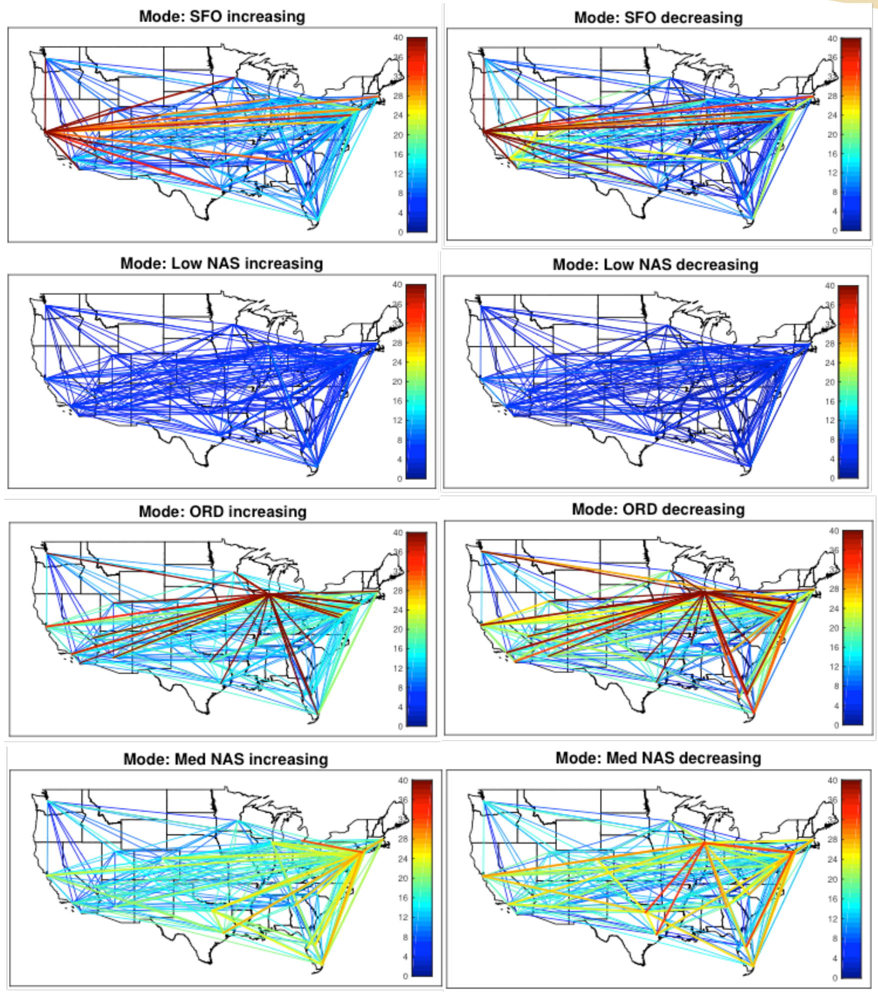
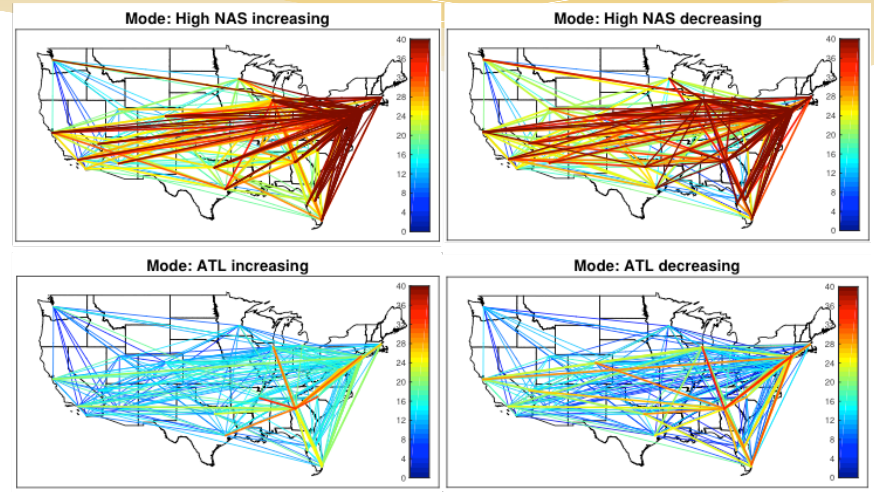
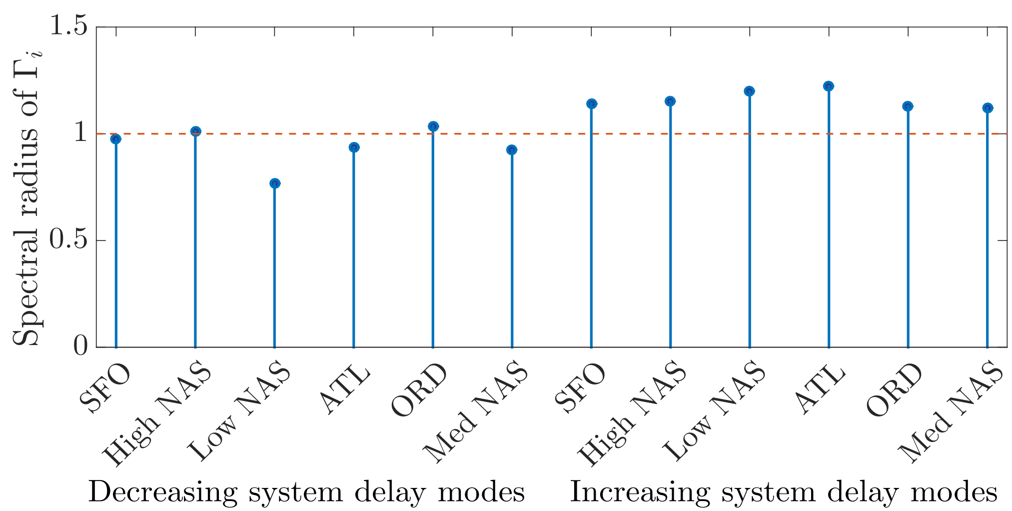
Individual discrete modes







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[Gopalakrishnan et al. CDC 2016]

Stability of MJLS models

- "Physical interpretation": Will delays increase or decrease over time (e.g., over the course of a day)?
- Mean Stability: Expected value of state tends to zero as time tends to infinity, that is, $\lim_{k\to\infty} \mathbb{E}[\|\vec{x}(k)\|] = 0$.
- **Almost-Sure Stability:** A system is said to be almost-surely stable if the state tends to zero as time tends to infinity with probability 1, that is,

$$\Pr[\lim_{k\to\infty} \|\vec{x}(k)\| = 0] = 1,$$

for any nonnegative initial condition $\vec{x}(0)$.

- Derive conditions for the stability of a discrete-time Markov Jump Linear System with time-varying transition matrices and continuous state resets (depends on Γ_i's, $\pi_{ij}(t)$ and J_{ij})

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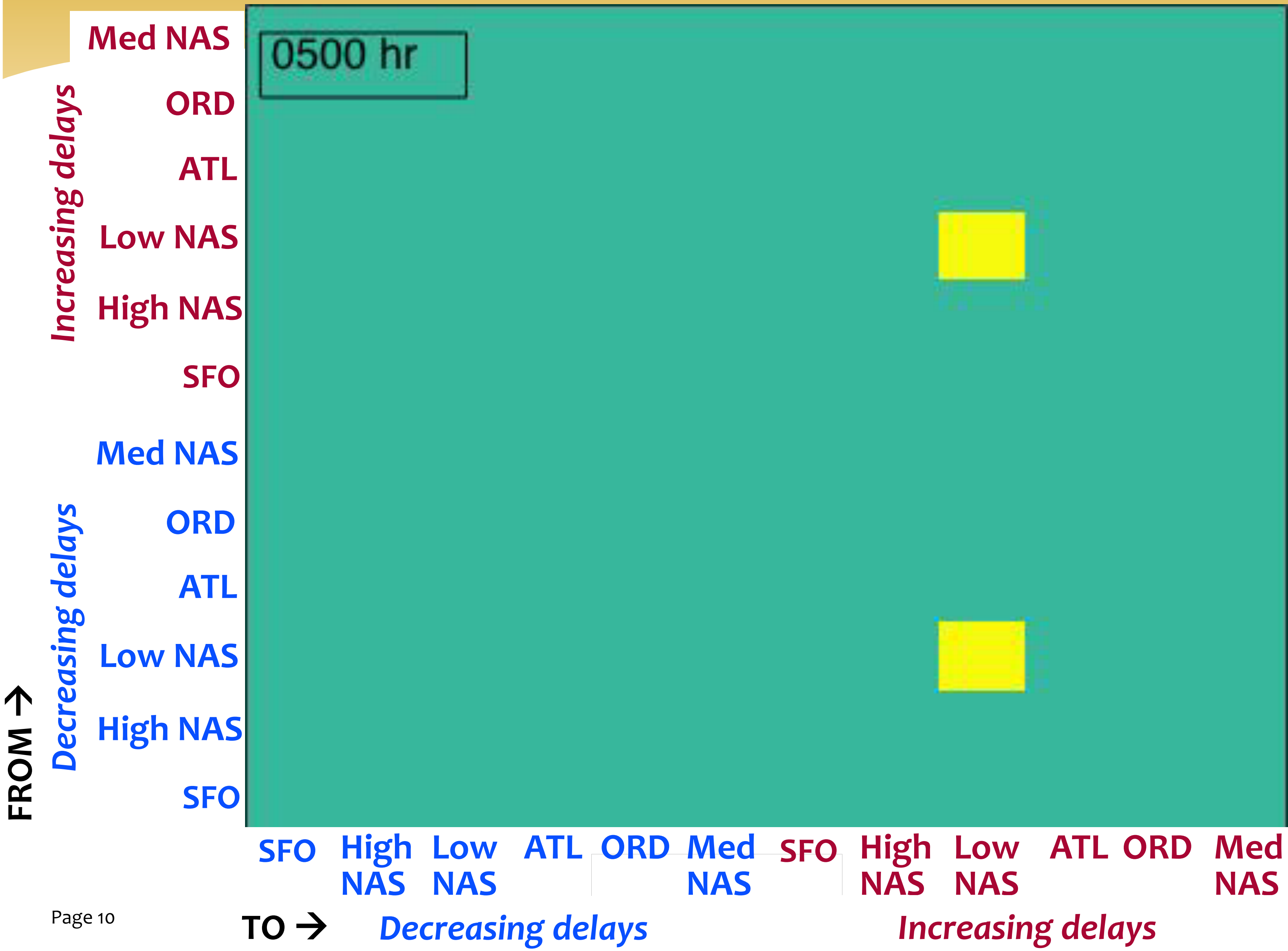
[Gopalakrishnan et al. CDC 2016]

Transition matrices exhibit temporal patterns



Stability of MJLS model

* Consider stability of MJLS model with periodic time-varying mode transition matrices (determined by hour of day)

* Resulting MJLS model shown to be mean and almost surely stable

* System appears to be stabilized by the temporal variations in the mode transition matrices

[Gopalakrishnan et al. CDC 2016]

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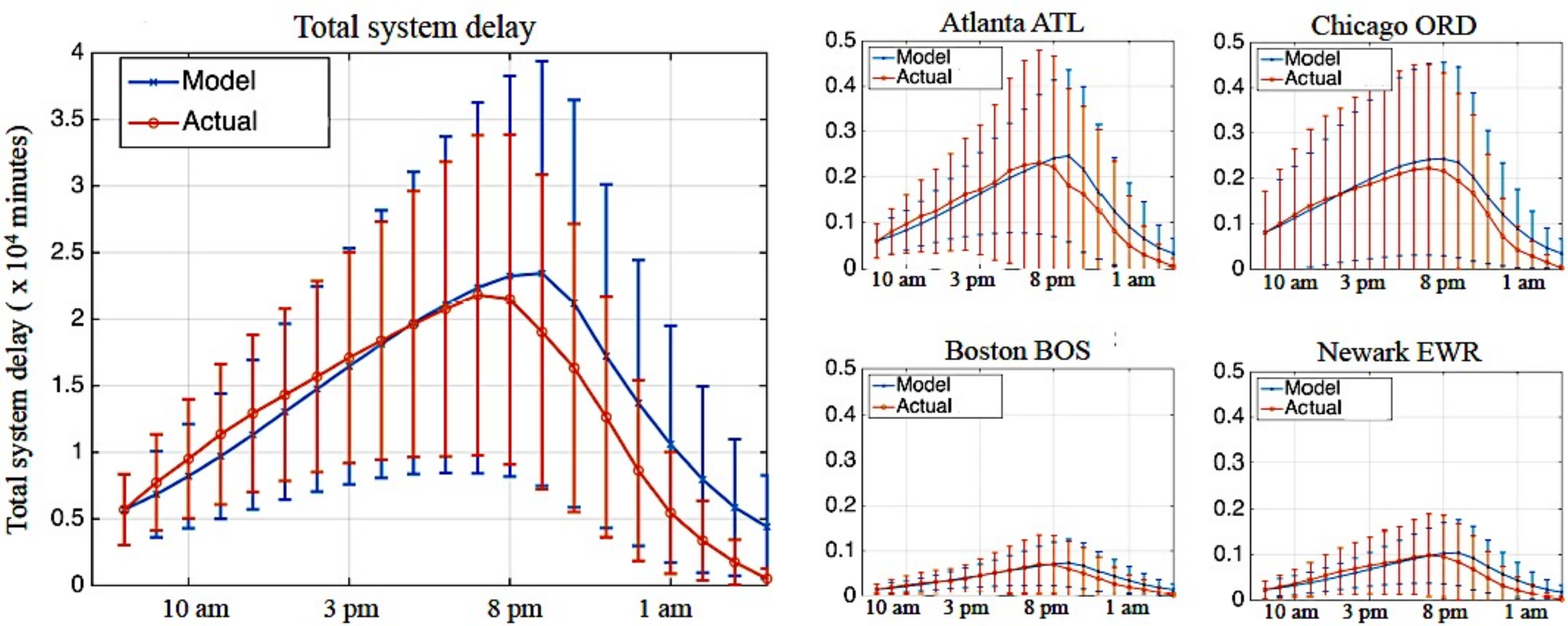
MJLS model validation

* Model learned using 2011 data; validation using 2012 data



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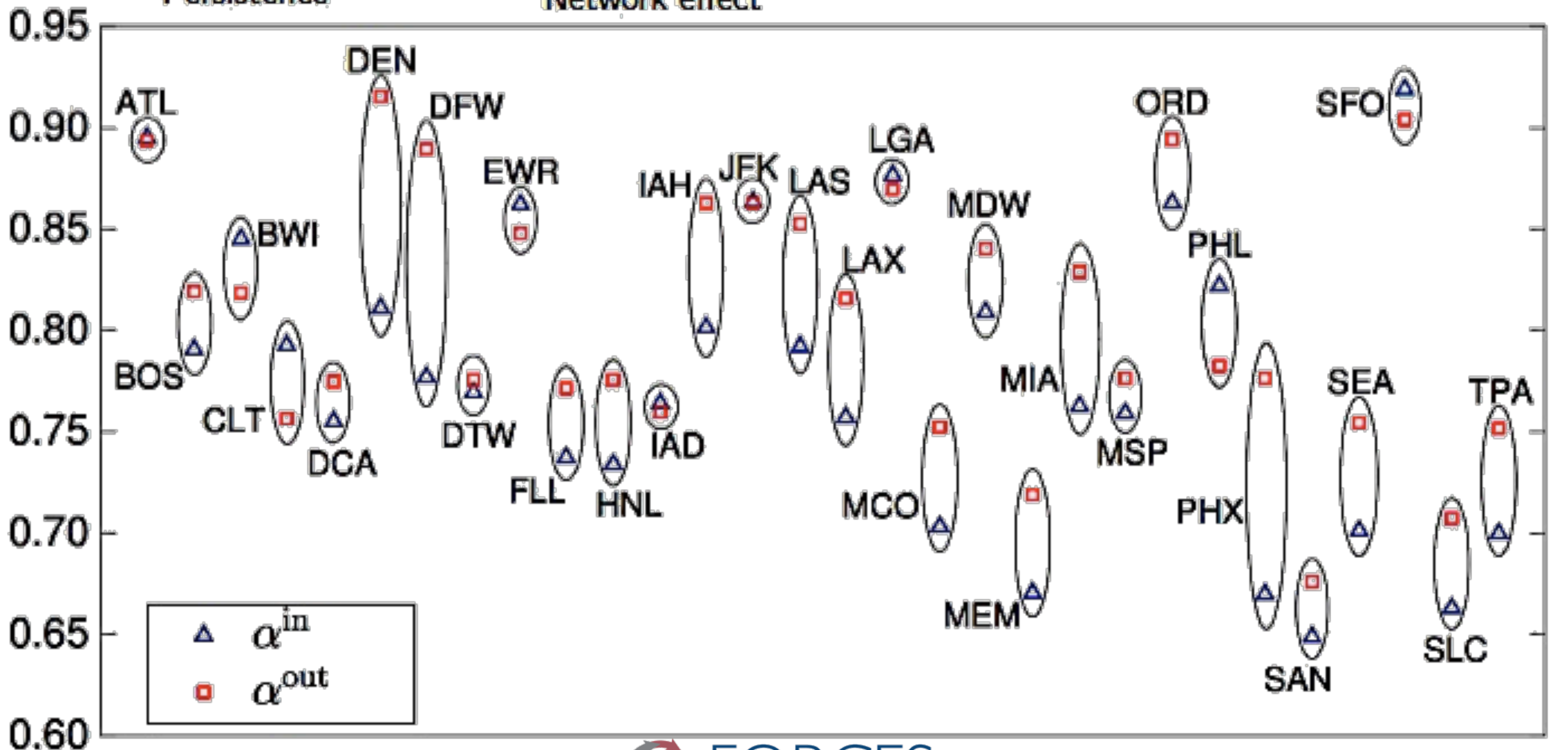
Measure of airport resilience: Delay persistence

$$d_{in}^{i}(t+1) = \underbrace{\alpha_{in}^{i} d_{in}^{i}(t)}_{\text{Persistence}} + \sum_{j} \underbrace{\beta_{ji}^{in} \bar{a}_{ji}(t) d_{out}^{j}(t)}_{\text{Network effect}}$$

$$d_{out}^{i}(t+1) = \underbrace{\alpha_{out}^{i} d_{out}^{i}(t)}_{\text{Persistence}} + \sum_{j} \underbrace{\beta_{ij}^{out} \bar{a}_{ij}(t) d_{in}^{j}(t)}_{\text{Network effect}}$$



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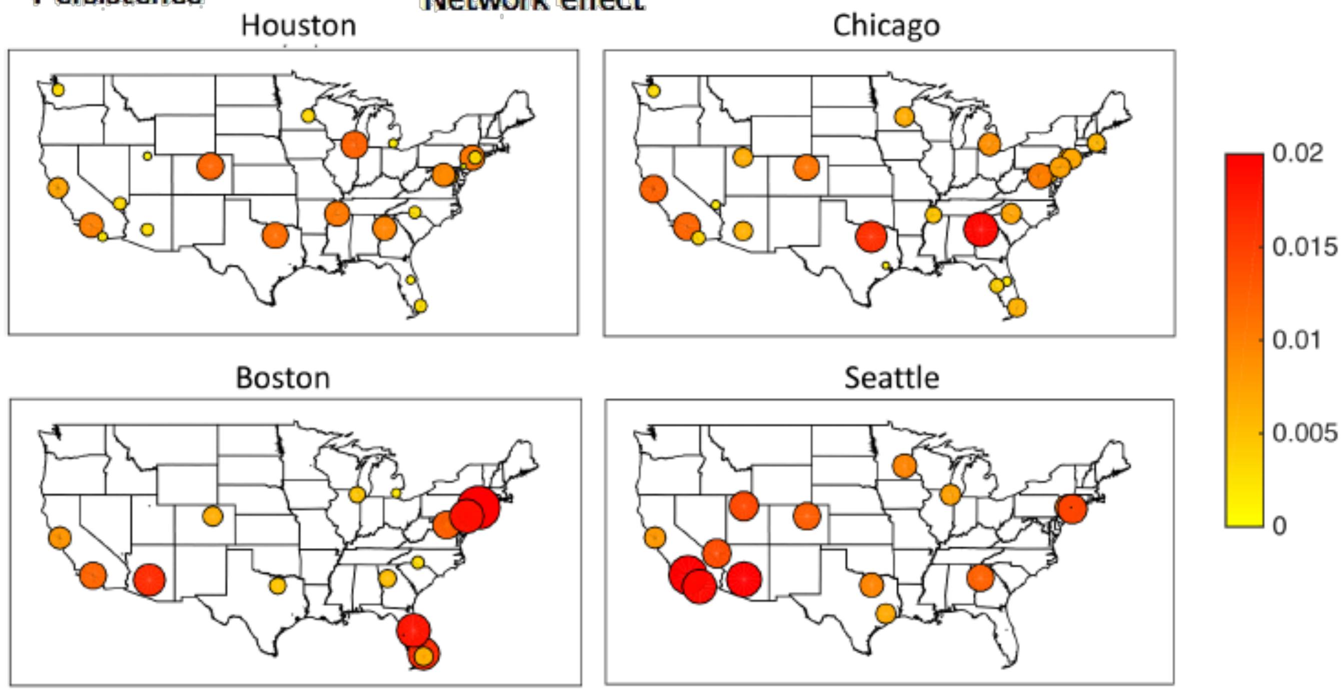
[Gopalakrishnan et al. CDC 2016]

Measure of airport resilience: Network effects

$$d^i_{in}(t+1) = \underbrace{\alpha^i_{in} d^i_{in}(t)} + \sum_j \underbrace{\beta^{in}_{ji}\overline{a}_{ji}(t) d^j_{out}(t)}$$

$$d^i_{out}(t+1) = \underbrace{\alpha^i_{out} d^i_{out}(t)}_{\text{Persistence}} + \sum_j \underbrace{\beta^{out}_{ij}\overline{a}_{ij}(t) d^j_{in}(t)}_{\text{Network effect}}$$



(Color and size of circle both denote induced inbound delay per unit delay at other airport)

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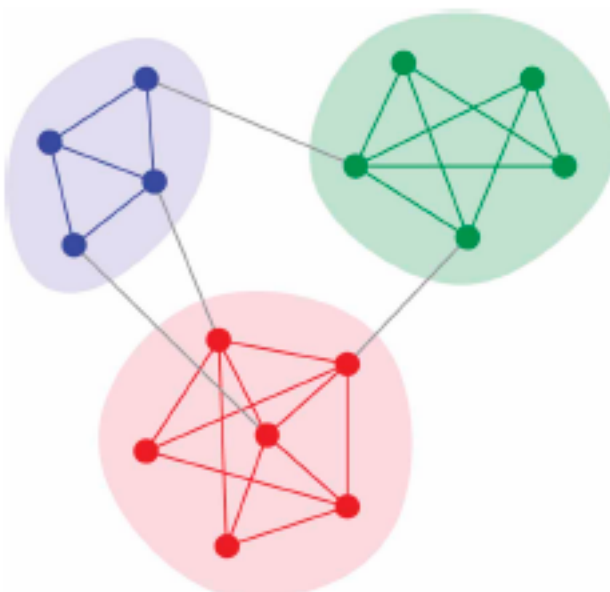
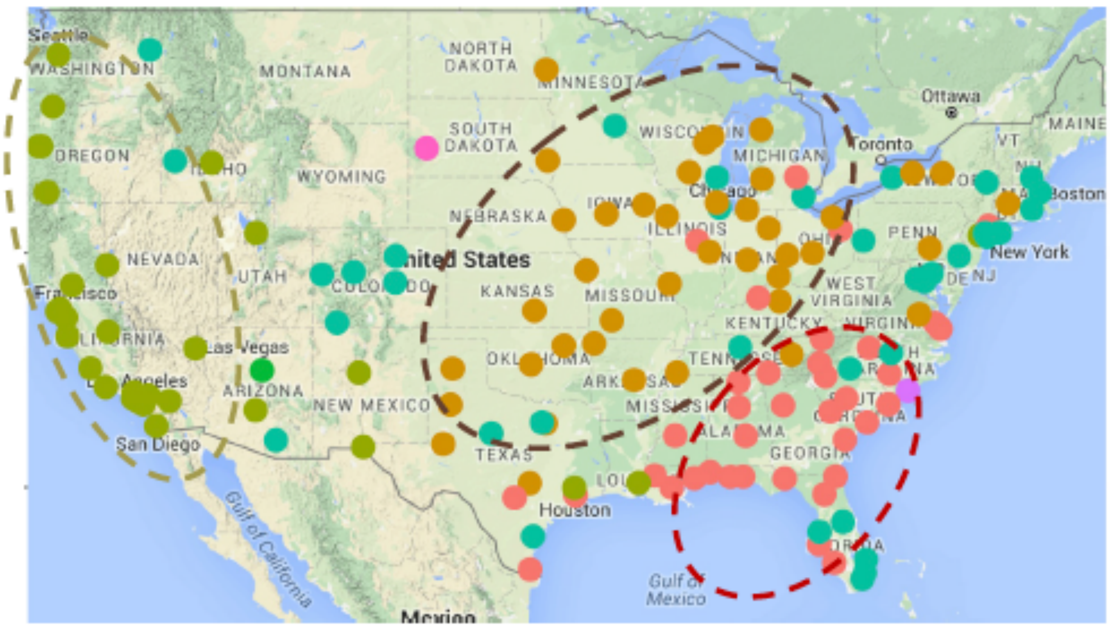
Delay communities

* Airports within a community have high delays between them





Community structure for delay network (23 March 2011)

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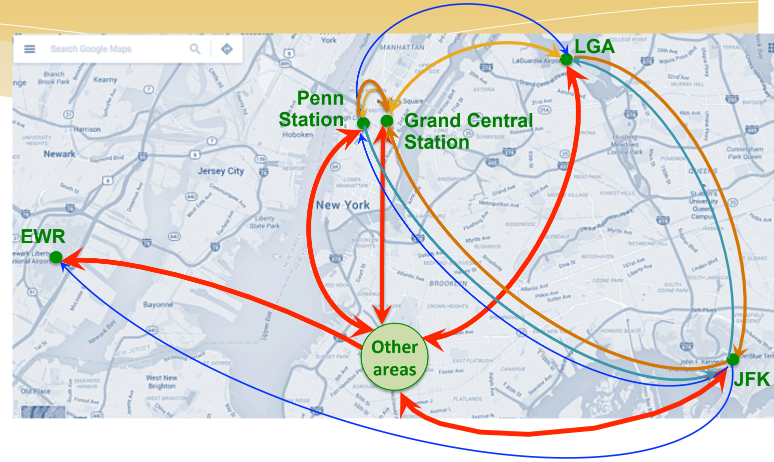
[Gopalakrishnan et al. ACC 2016]

Ongoing efforts and next steps

- **Analysis of finite-time behavior**
- Factors that trigger mode transitions
  - Weather impacts, Traffic Management Initiatives
- **Post-disruption recovery**
  - Optimal control of networks with switching
- **Prediction of future delays and delay states**
- **Multi-layer, multi-timescale networks**
  - Cancellations, **operations**, capacity impact [ICRAT 2016]
  - Interactions between networks



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