FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Cellpath: state estimation of traffic networks via convex optimization

Cathy Wu[1], Jérôme Thai[1], Steve Yadlowsky[1], Alexei Pozdnoukhov[2], Alexandre Bayen[1,2,3]

[1]Department of Electrical Engineering & Computer Sciences, UC Berkeley
[2]Department of Civil and Environmental Engineering, UC Berkeley
[3]Institute for Transportation Studies (ITS), UC Berkeley

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Outline

Route flow estimation problem and formulation

Block simplex constrained quadratic program: analysis and algorithms

Experiments and conclusions

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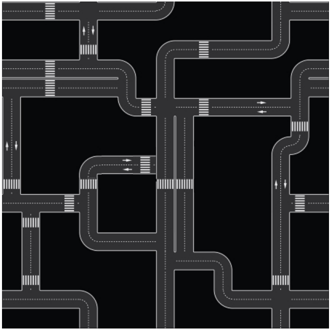
Route flow estimation problem and formulation

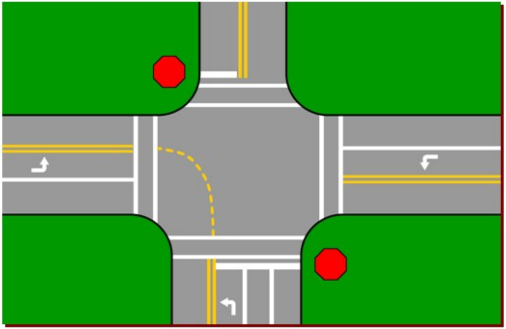
Block simplex constrained quadratic program: analysis and algorithms

Experiments and conclusions

The root of all traffic evils

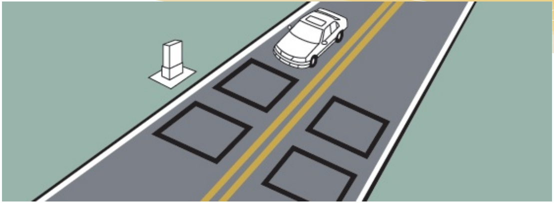
We have little information of what's going on in the road network.





Information

Traditional approaches to traffic *flow* estimation

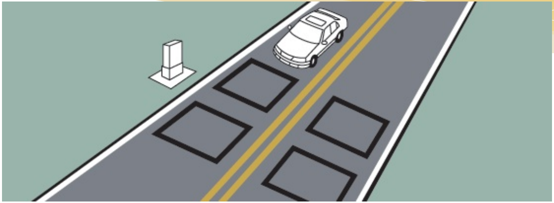


Sparse static sensors

+

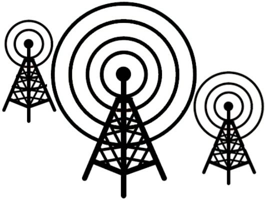
Equilibrium models

This talk: *data-driven* estimation of *route* flow



Sparse static sensors $\longrightarrow$ observed link flow

$+$



Cellular network sensors $\longrightarrow$ cellpath flow

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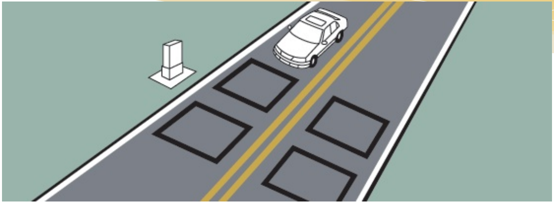

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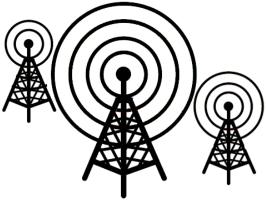
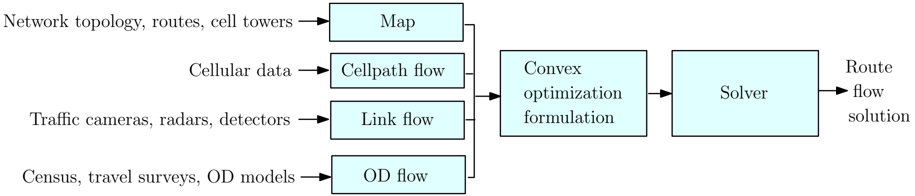


Cellular network sensors $\rightarrow$ cellpath flow

Our contributions

- Convex optimization formulation for route flow estimation problem[1,2]
- Formalize cellular network data as *cellpaths*[1]
- Formulation compatible with equilibrium concepts and other data[1]
- Projected gradient method with $O(n)$ projection step[1,3]
- 90% accuracy in numerical experiments on large-scale networks[1]
- Proposed framework: full pipeline for traffic estimation[1]



[1] C Wu, J Thai, S Yadlowsky, A Pozdnoukhov, A Bayen. "Cellpath: fusion of cellular and traffic sensor data for route flow estimation via convex optimization." *Transportation Research Part C: Emerging Technologies* (2015).

[2] C Wu, A Pozdnoukhov, A Bayen. "Block simplex signal recovery: a method comparison and an application to routing." *In review, ACM TIST*.

[3] J Thai, C Wu, A Pozdnoukhov, A Bayen. "Convex programming on the l1-ball and on the simplex via isotonic regression." *CDC* (2015).

Problem statement: route flow estimation

Route flow estimation problem

Given

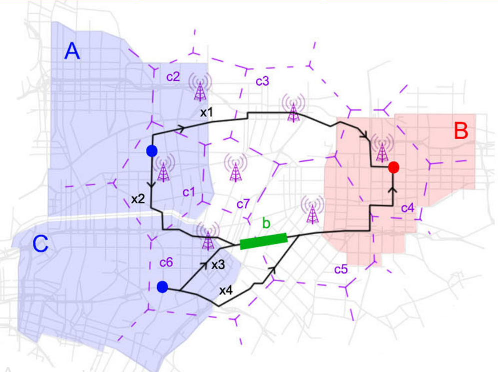
- Road network, origins, cells
- Top routes between OD pairs
- Cellpath flows, f
- OD flows, d
- Observed link flows, b

Recover

- Flow along routes, x

Cellpath flow

Flow along a sequence of cells



Assumptions

- Static, noiseless
- Cell partitioning = Voronoi
- Cellpaths contiguous
- Cellpaths well-posed

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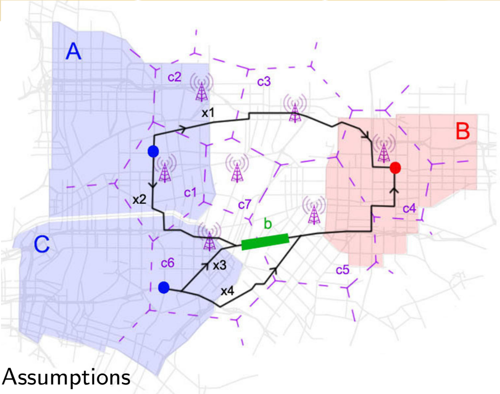

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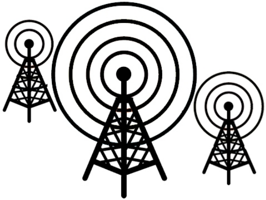
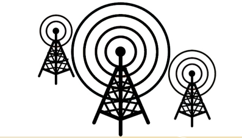
A brief note on cellular networks



- Not GPS (read-only signals)
- Cell towers spaced $\frac{1}{4} - \frac{1}{2}$ mi (urban areas) to 1-2 mi apart (suburbia)
- Cell towers collect signals as devices connect to and use the network

Related works

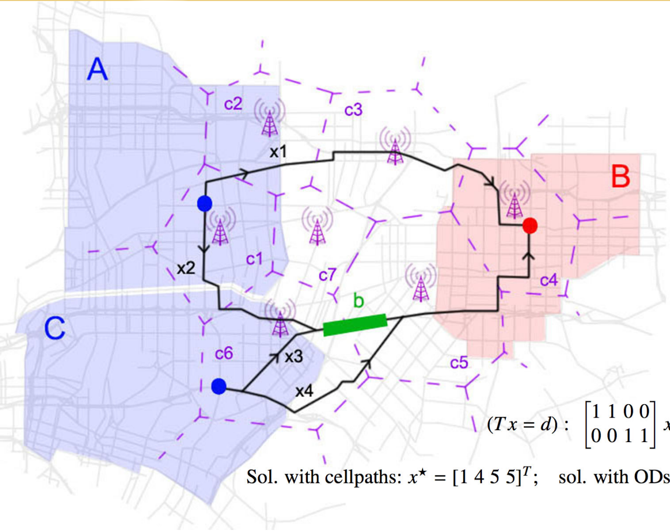


Cellular location data + transportation problems

- Excellent survey [GK09]
- OD matrix estimation (ODME) [CWB07, BHNF08, CLDLR11]
- Travel time estimation [THF06, FJS07, B07, JHVRH12]
- Congestion detection and classification [JHVRH12]
- Link density estimation [YTWPB14]
- Route choice modeling [TDV12]

This work: cellular location data + route flow estimation

Example problem setup



All flows are in 1000 vehicles/hour.

cellpath flows: $\begin{cases} f_{p1234} = 1 = x_1 \\ f_{p1654} = 4 = x_2 \\ f_{p654} = 10 = x_3 + x_4 \end{cases}$

OD demands: $\begin{cases} d_{AB} = 5 = x_1 + x_2 \\ d_{CB} = 10 = x_3 + x_4 \end{cases}$

Link flow: $b = 9 = x_2 + x_3$

$$(Ux = f): \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} f_{p1234} \\ f_{p1654} \\ f_{p654} \end{bmatrix}$$

$$(Tx = d): \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} d_{AB} \\ d_{CB} \end{bmatrix} \quad ; \quad (Ax = b): \ [0\ 1\ 1\ 0]\, x = b$$

Sol. with cellpaths: $x^\star = [1\ 4\ 5\ 5]^T$; sol. with ODs: $x = x^\star + \left[1\ -1\ 1\ -1\right]^T t, \ \forall t \in [-1, 4]$

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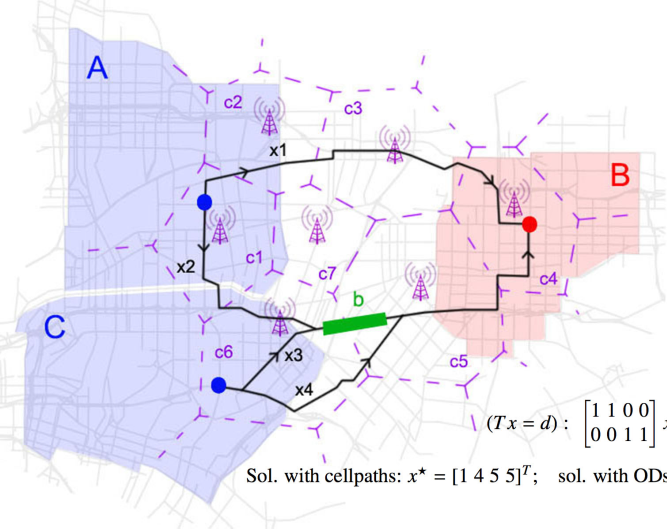

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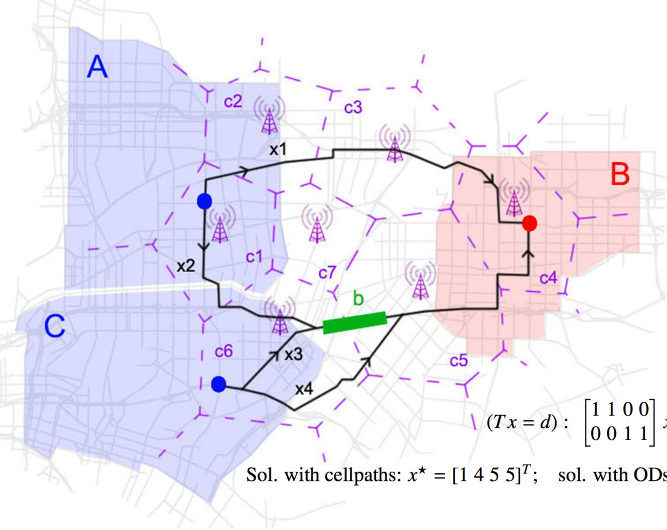

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Convex optimization formulation

Block simplex constrained quadratic program (QP):

$$\begin{array}{ll} \min & \frac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} & Ux = f,\ x \geq 0 \end{array}$$

- link-route: $A_{lr} = \begin{cases} 1 & \text{if } l \in r \\ 0 & \text{else} \end{cases}$; cellpath-route: $U_{pr} = \begin{cases} 1 & \text{if } r \in \mathcal{R}^p \\ 0 & \text{else} \end{cases}$
- $b \in \mathbb{R}_+^{|\mathcal{L}|}$ observed link flow vector, $b = (b_l)_{l \in \mathcal{L}}$
- $f \in \mathbb{R}_+^{|\mathcal{P}|}$ cellpath flow vector $f = (f_p)_{p \in \mathcal{P}}$
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- n routes
- m link flow measurements
- q cellpath flow constraints
- Separable scaled simplex constraint
- Weakly convex, underdetermined, $rk(A) \leq m \ll n, q \leq n$

Flexibility of our formulation

Cellpath + observed link flows

$$\begin{aligned} \min \quad & \tfrac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} \quad & Ux = f,\ x \geq 0 \end{aligned}$$

OD + observed link flows

$$\begin{aligned} \min \quad & \tfrac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} \quad & Tx = d,\ x \geq 0 \end{aligned}$$

Cellpath + OD + observed link flows

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And more: turning ratio data and traffic assignment problem

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Summary of analysis: block simplex quadratic program

Goal: design an efficient first-order projected descent method

General projected descent method

Algorithm 1 `Proj-descent(·)`

Require: minimizing function $f(x)$
Require: initial point x in the feasible set $\mathcal{X}$.

while stopping criteria not met **do**
  Determine a descent direction Δx
  Step in that direction: $x^+ := x + \alpha \Delta x$
  Projection: $x := \Pi_{\mathcal{X}}(x^+)$
end while
return x

1. Equivalence to separable **standard** simplex constraints
2. Transformable to order constraints via equality constraint elimination
3. Design an efficient projection under a coordinate change

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(1/3) Block standard simplex

Proposition: Equivalence of scaled and standard simplices

For C a well-posed block simplex matrix:

$$\begin{array}{ll} \min & \frac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} & Cx = \mathbb{1},\ x \geq 0 \end{array} \iff \begin{array}{ll} \min & \frac{1}{2}\|\tilde{A}\tilde{x} - b\|_2^2 \\ \text{s.t.} & C\tilde{x} = d,\ \tilde{x} \geq 0 \end{array}$$

where

$$E := \text{diag}(C^T d) \quad A := \tilde{A}E \quad x := E^{-1}\tilde{x}$$

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New interpretation: $\tilde{x}$ is route flow vector $\to$ x is route split vector

$$A \in \mathbb{R}_+^{|\mathcal{L}|\times|\mathcal{R}|} \ : \ A_{lr} = \begin{cases} f_p & \text{if } l \in r \in \mathcal{R}^p \\ 0 & \text{else} \end{cases}$$

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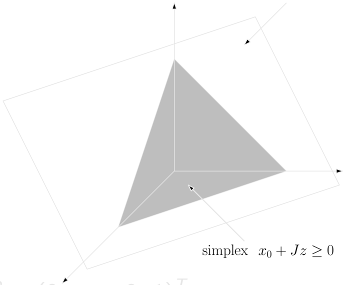
(2/3) Transforming standard simplex into ordering constraint

Proposition: Ordering constraint

$$\mathbf{1}^T x^p = 1, \quad x^p \geq 0 \quad \Longleftrightarrow \quad 0 \leq z_1^p \leq \cdots \leq z_{n_p-1}^p \leq 1$$

for an appropriate change of variables $x \to z$.



Constraint elimination trick: $x^p = x_0^p + J^p z^p$

Choose $J^p = \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & \ddots & \\ & & \ddots & \end{pmatrix} \in \mathbb{R}^{n_p \times (n_p - 1)}$ and $x_0^p = (0, \cdots, 0, 1)^T$

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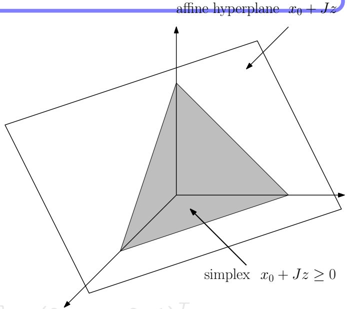

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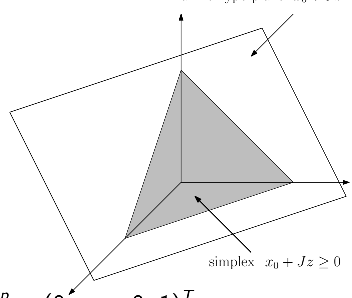

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(3/3) Isotonic regression

Related work

Isotonic regression with complete order (ABERS55, BC90, LHM09)

$$\begin{array}{rl} \text{minimize} & \sum_{i=1}^{n} w_i(y_i - x_i)^2 \\ \text{subject to} & x_1 \leq x_2 \leq \cdots \leq x_n \end{array}$$

Solvable via pool adjacent violators (PAV) algorithm in $O(n)$ (BB72, GW84)

Our work: $\Pi_\Delta(y)$

Box-constrained isotonic regression with complete order

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(3/3) Box-constrained isotonic regression – proof summary

Proposition: Optimal solution via Euclidean projection

Solution x^* to BCIR(t, u) is the Euclidean projection of the solution x^{iso} to IR onto $[t, u]^n$.

IR: isotonic regression
BCIR: block-constrained isotonic regression

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Given a solution x^{iso} to IR, if there exists k s.t. $x_k^{\text{iso}} < x_{k+1}^{\text{iso}}$,
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Lemma: Uniformly lower bound solution

Given a solution x^{iso} to IR, if $x_i^{\text{iso}} \leq t \, \forall i$, then $x_i^* = t \, \forall i$ for BCIR(t, u).

IR: isotonic regression
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Outline

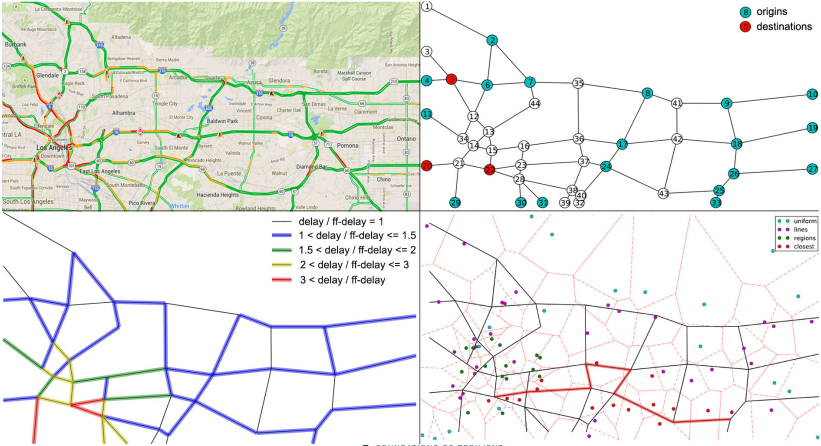
Route flow estimation problem and formulation

Block simplex constrained quadratic program: analysis and algorithms

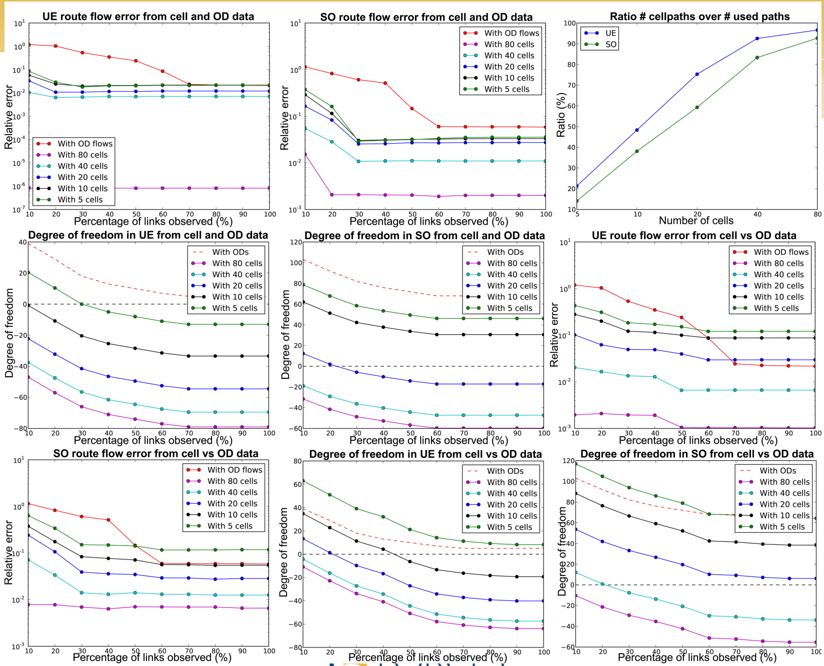
Experiments and conclusions

Experiment setup: Los Angeles highway network

- Network: 144 links, 44 nodes
- ODs: 21 origins, 3 destinations $\Longrightarrow$ 42 OD pairs
- Routes: 275 (UE), 300 (SO)
- Route flow: 91 (UE), 153 (SO) positive flows
- Link sensors: 5-100% static sensor coverage (most congested)
- Cellpath sensors (cells): $\{N^B, N^S, N^L\} \propto \{20, 40, 20\}$ (5-80 total)



Results: Los Angeles highway network



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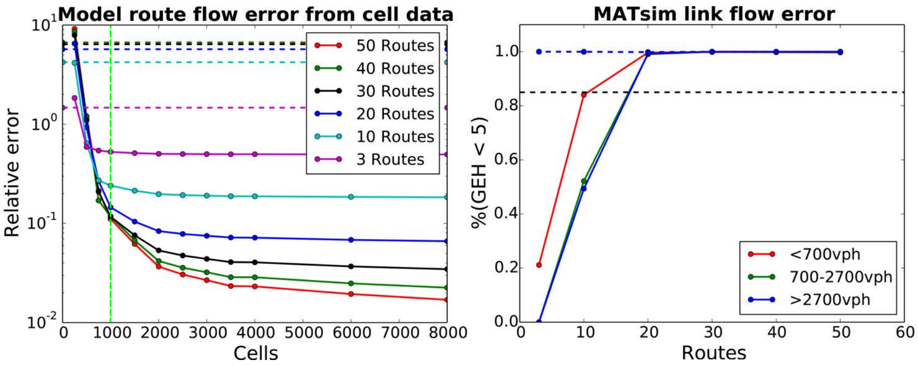
Experiment: Los Angeles full network → **90% accuracy**

- Network: 20K links, 11K nodes
- 296K routes (up to 50 routes per OD pair)
- 1K observed links (5% coverage)
- 1K cells ⟹ 203K cellpaths
- 500K agents, trajectories simulated via MATSIM
- 32K origin-destination (OD) pairs; 321 ODs

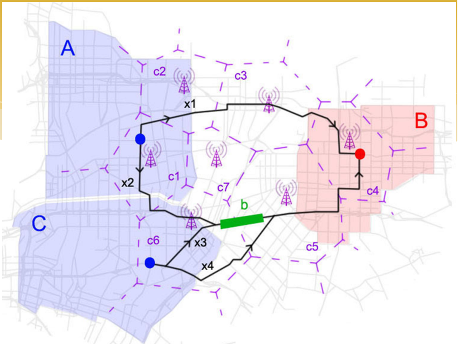
Cellpath + OD + observed link flows + model error

$$\begin{aligned} \min \quad & \frac{1}{2} \left\| \begin{bmatrix} A \\ T \end{bmatrix} x - \begin{bmatrix} b^{(50)} \\ d \end{bmatrix} \right\|_2^2 + \lambda \|x\|_2^2 \\ \text{s.t.} \quad & Ux = f, \; x \geq 0 \end{aligned}$$

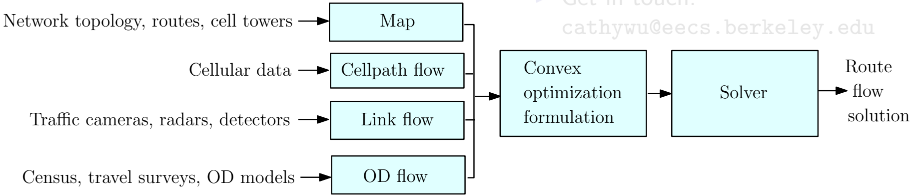


Conclusions

- Cellpath: fusion of cellular and traffic sensor data for route flow estimation via convex optimization
- Route flow estimation has received little attention due to data limitations
- Cellular data is a promising data source
- Route flow estimates will enable short time horizon applications, e.g. prediction and control
- Future work: noisy and dynamic settings, experiments with AT&T data



- Website: megacell.github.io
- Implementation: github.com/megacell
- Get in touch: cathywu@eecs.berkeley.edu



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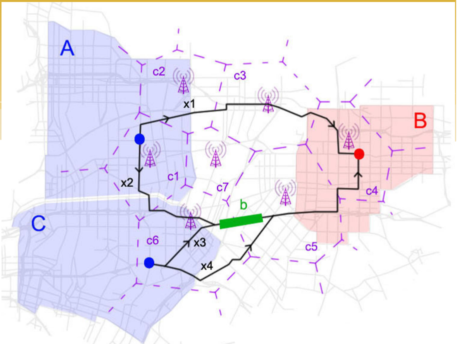

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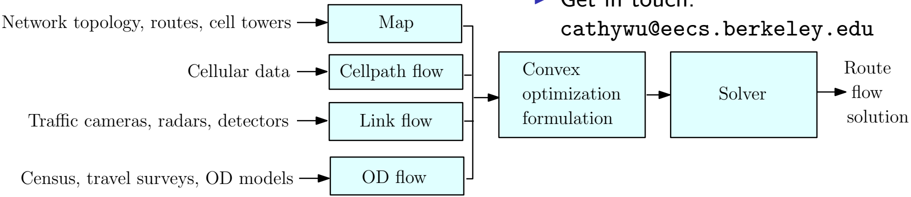

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