FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Multidimensional Contracts in Electricity Markets

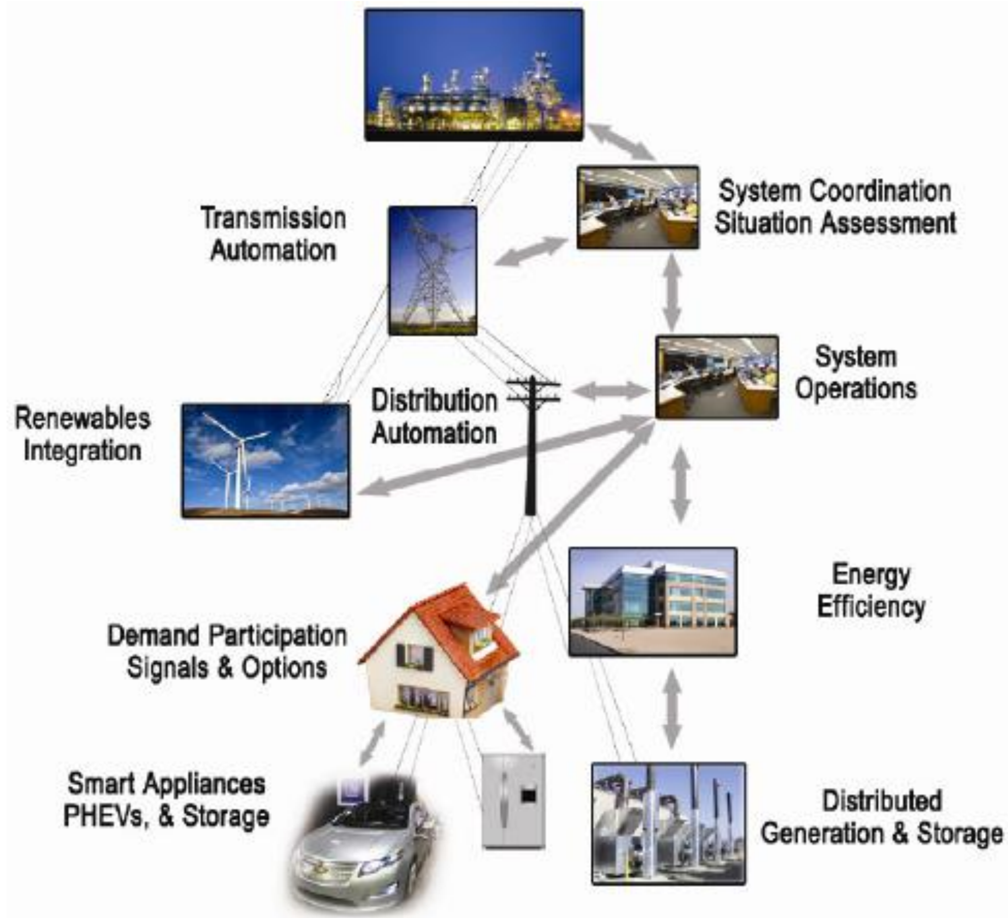
Hamidreza Tavafoghi
Demosthenis Teneketzis

June 2014

Berkeley University of California
Massachusetts Institute of Technology
NSF
Michigan
Vanderbilt University

Electricity Markets

* Pooling energy market (quantity/price bidding)
* Bilateral trade
* Price based
  * Time of use (TOU) price (offline)
  * Real time price (RTP) (online)
* Contract with Incentive Payment



Our Approach: Contract Design

* Captures:
  * Bilateral trade: An agreement between one buyer and one seller, each possessing private information
  * Contract with Incentive payments: A contract/scheme designed by an aggregator for a heterogeneous population of agents
    * Pros: simplicity, reliability
    * Cons: efficiency loss
* Give us intuition into pooling energy market

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Problem Formulation

* Model:
  * Buyer's utility: $\mathcal{V}(q) - t$
  * Seller's utility: $t - C(q, x, W)$
    * $x \in \mathcal{X} \subseteq \mathbb{R}^n$: Seller's private information
    * $W \subseteq \mathbb{R}^l$: a random variable with common prior F_W

Multi-dimensional private Information

Random utility/cost function

* Objective: design a mechanism/contract $(q: \mathcal{X} \rightarrow \mathbb{R}_+, t: \mathcal{X} \rightarrow \mathbb{R})$ so as to

$$maximize_{q,t}\ \mathbb{E}_{\mathrm{X},W}\{\mathcal{V}(q) - t\}$$

subject to the seller's voluntary participation:

$$\text{interim}: \ E_W\,\{t(x) - C(q(x), w, x)\} \geq 0 \quad \forall x \in \mathcal{X}$$
$$\text{ex-post}: t(x) - C(q(x), w, x) \geq 0 \quad \forall x \in \mathcal{X}, w$$

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Result

* **Theorem.** The optimal mechanism for the buyer is a nonlinear pricing scheme/contract $t(q)$ given by

Expected marginal utility at q

$$p(q) = \underset{\hat{p}}{\operatorname{argmax}} \left\{ P\left[x \in \mathrm{X} \mid \hat{p} \geq \mathbb{E}_W \left\{ \frac{\partial C(q,w,x)}{\partial q} \right\} \right] (\mathcal{V}'(q) - \hat{p}) \right\}$$

Probability of getting marginal quantity q

marginal utility at q

$$t(q) = \int_0^q p(l)\, dl + t_0$$

where $t_0 = \max_{x \in \mathcal{X}} \left[\{C(q(x), w, x)\} - \int_0^{q(x)} p(q')dq' \right]$

$$q(x) = argmax_{\hat{q}} \{t(\hat{q}) - \mathbb{E}_W\{C(\hat{q}, w, x)\}\}$$

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Example

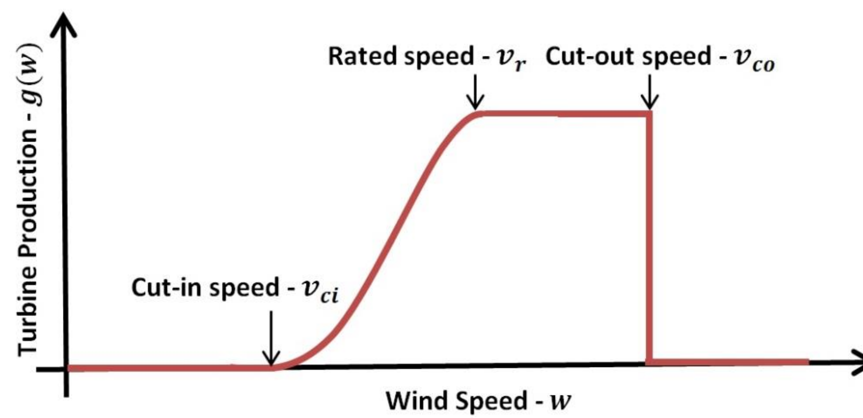
- $\mathcal{V}(q) = 2\sqrt{q}$

renewable generation — residual quantity

- $C(q, w, x) = c_0 + \theta_w \min\{q, g_x(w)\} + \theta_c \max\{q - g_w(x), 0\}$

start-up cost

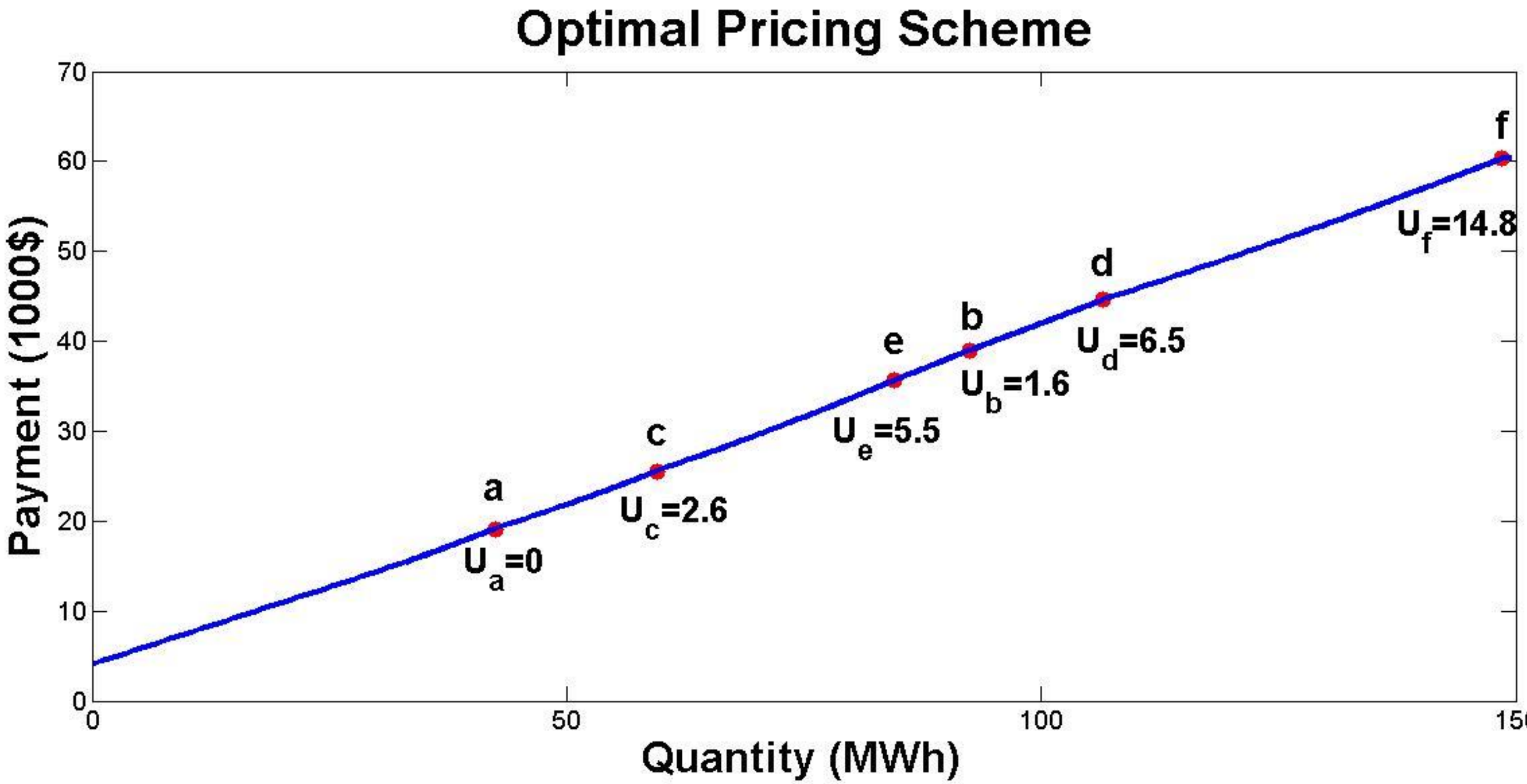
- $$g_x(w) = \begin{cases} 0 & w < v_{ci} \\ \gamma(w - v_{ci})^3 & v_{ci} \le w \le v_r \\ \gamma(v_r - v_{ci})^3 & v_r < w < v_{co} \\ 0 & w \ge v_{co} \end{cases}$$



- $The\ seller's\ type\ x = (c_0, \theta_w, \theta_c, v_{ci}, v_r, v_{co}, \gamma)$

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Example



* Marginal price varies between 33 and 45 $\frac{\$}{MWh}$

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Future Work

* Apply and customize the current result to a more detailed models
* Risk-averse agents
* Contract design for multiple goods ($q \in \mathbb{R}^m$)

H. Tavafoghi, D. Teneketzis, "Optimal energy procurement from a strategic seller with private renewable and conventional generation", arXiv.

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS