Negative Externalities of GPS-Enabled Applications: A Game Theoretical approach

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Problem motivation

Improve traffic: GPS apps optimizes road network utilization

Unintended consequences: traffic demand increase in cities bordering highways

Other side effects: complications for local taxpayers

Accelerating trend: ridesharing systems & autonomous driving rely on GPS apps

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In L.A., One Way to Beat Traffic Runs Into Backlash

Popular Waze app sends drivers to side street, riling residents

Game-theoretical framework

Problem statement:

Quantify the effect of increasing penetration of navigation apps

Approach:

Traffic Assignment framework (Wardrop equilibrium)

Multi-class traffic assignment

- **Routed users:** follow shortest routes using GPS device, i.e. cost of using a route is travel time.

- **Non-routed users:** limited knowledge of road network and of current travel times, hence favor high-capacity roads for ‘perceived’ benefits such as safety and low travel times.

Multi-class traffic assignment

- **High-capacity roads:** serve users passing through, hence maintained at state/county level. Favored by non-routed users for convenience and historical efficiency.

- **Low-capacity road segments:** residential streets for local users who live or work in the area, not meant for through traffic.

Low-capacity road segments heavily impacted by flow of routed users.

Mathematical formulation

Travel time: $t_a(x_a)$

Road segment cost for non-routed users: $$c_a^{\text{nr}}(x_a) = \begin{cases} C \cdot t_a(x_a) & \text{if } a \in \mathcal{A}^{\text{lo}} \\ t_a(x_a) & \text{if } a \in \mathcal{A}^{\text{hi}} \end{cases}$$

Route cost for non-routed users: $$\ell_p^{\text{nr}}(f) = \sum_{a \in p^{\text{hi}}} t_a(x_a) + C \sum_{a \in p^{\text{lo}}} t_a(x_a)$$

Route cost for Routed users: $$\ell_p^{\text{r}}(f) = \sum_{a \in p} t_a(x_a), \quad \forall p \in \mathcal{P}$$

Multiclass Nash equilibrium

Traffic flow between each OD pair w divided into fraction α of routed users, and fraction 1-α of non-routed users.

Eq. condition for routed users: $\forall\, p \in \mathcal{P}_w,\; f_p^{\mathrm{r}} > 0 \implies \ell_p^{\mathrm{r}}(f) = \min_{q \in \mathcal{P}_w} \ell_q^{\mathrm{r}}(f)$

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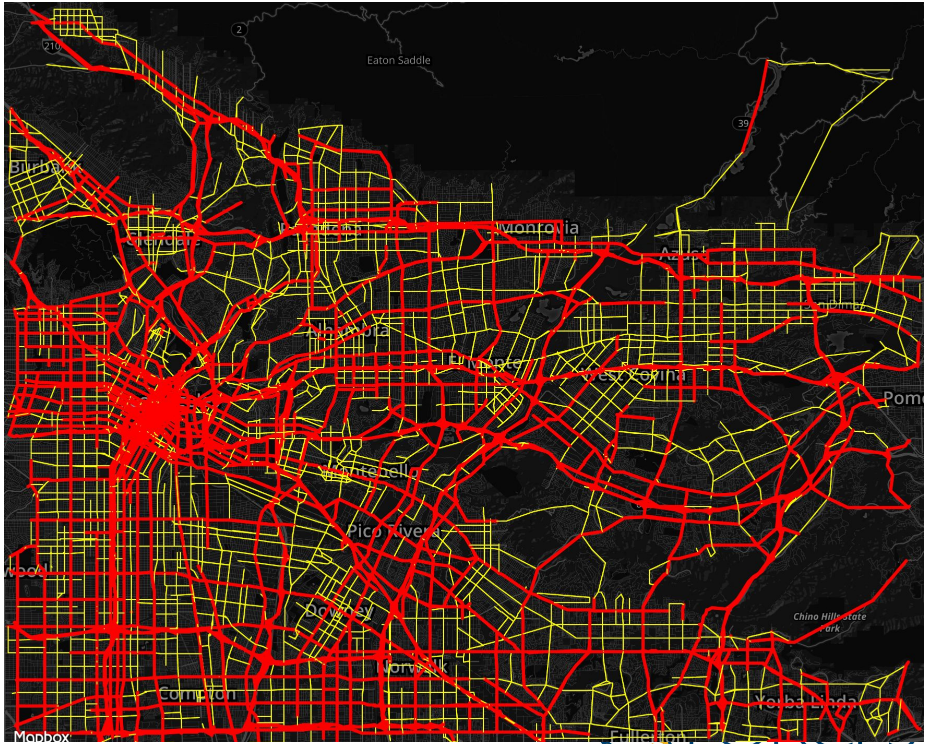
No convex optimization formulation, but can be solved using Variational Inequality theory and Frank-Wolfe algorithm.

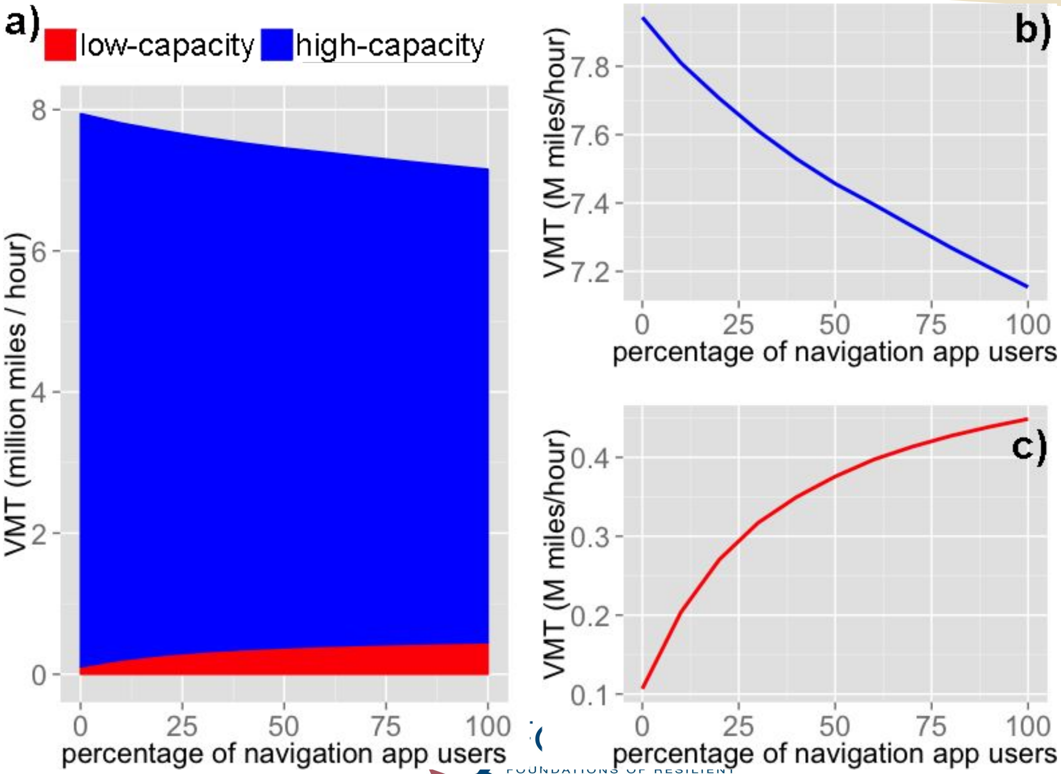
Application on the map of Los Angeles



Given:
Road map of L.A.
OD flows

Methodology:
Parametric study on the fraction α of routed users

Variation in VMT



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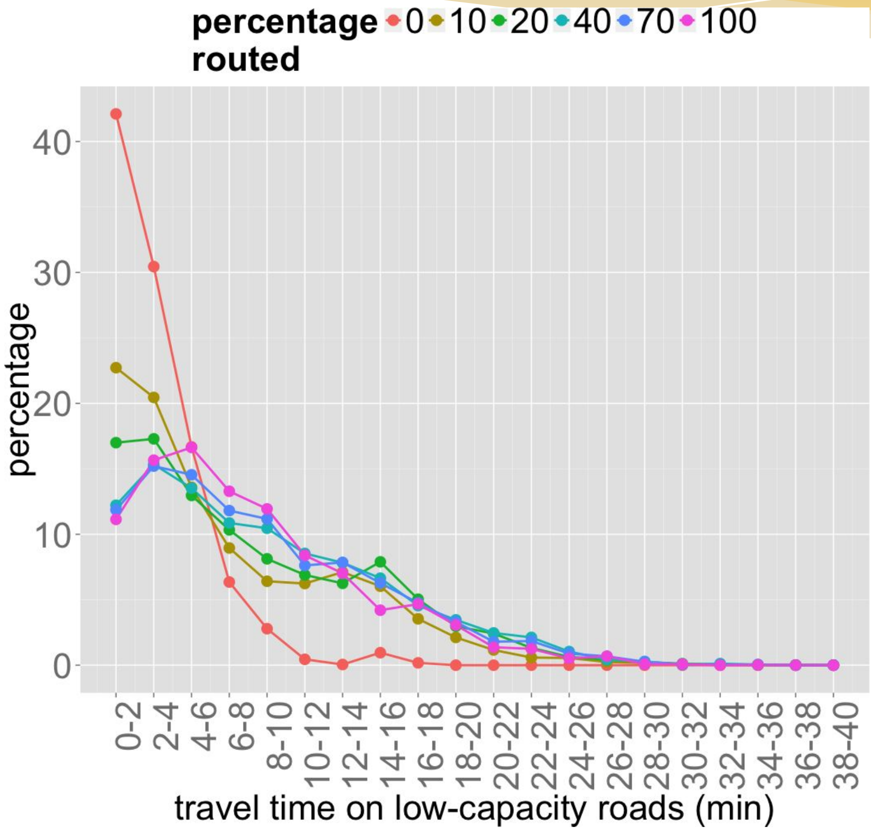
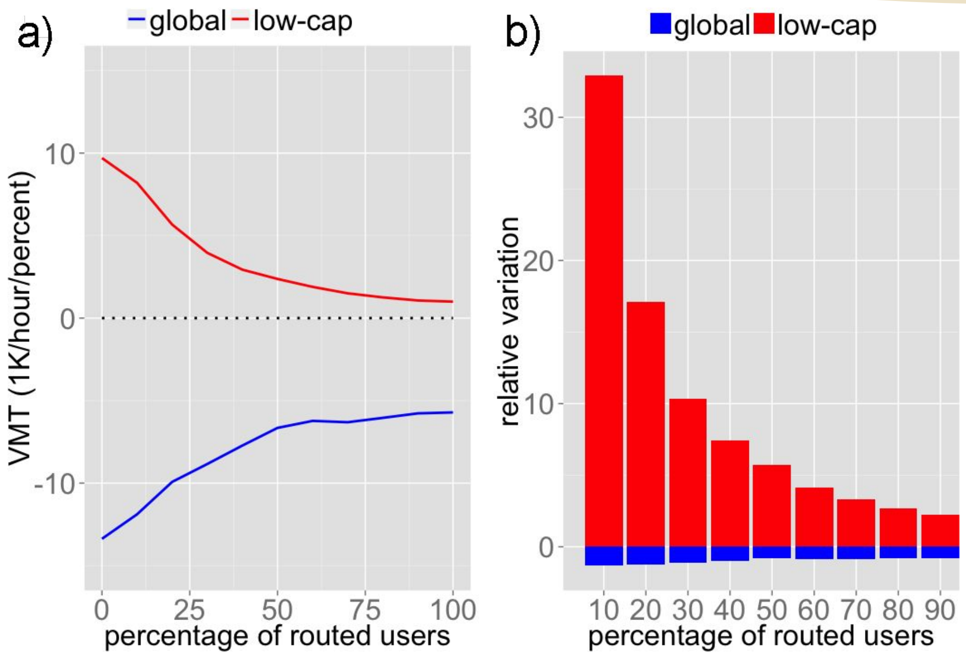
Travel distribution on local roads



Variation in VMT



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Conclusions and future work

Contributions:

- A game-theoretical framework for the impact of GPS apps
- Numerical results on the network of L.A.

Future work:

- Apply to other networks
- Prisoner's dilemma between drivers and residents

Open questions

- GPS-based tolls integrated into apps
- Collaborate with tech companies
- Higher capacity on highways
- Better public transportation

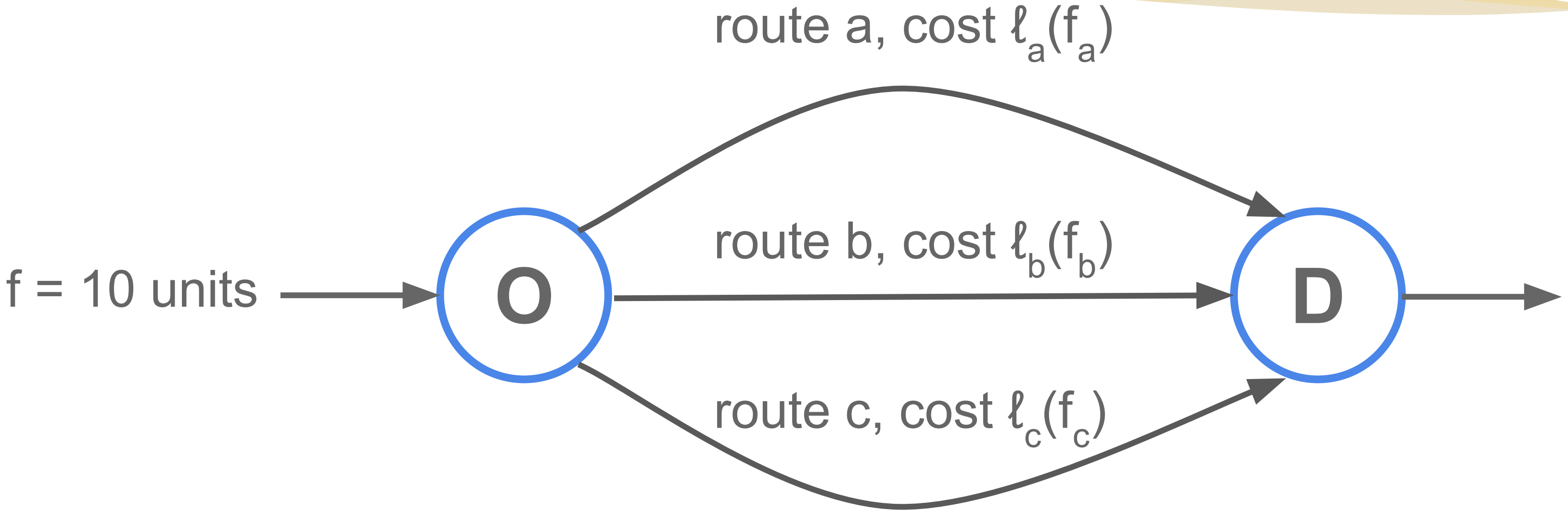

NICK STOCKTON SCIENCE 02.20.15 6:30 AM

WIRED

BOSTON IS PARTNERING WITH WAZE TO MAKE ITS ROADS LESS OF A NIGHTMARE

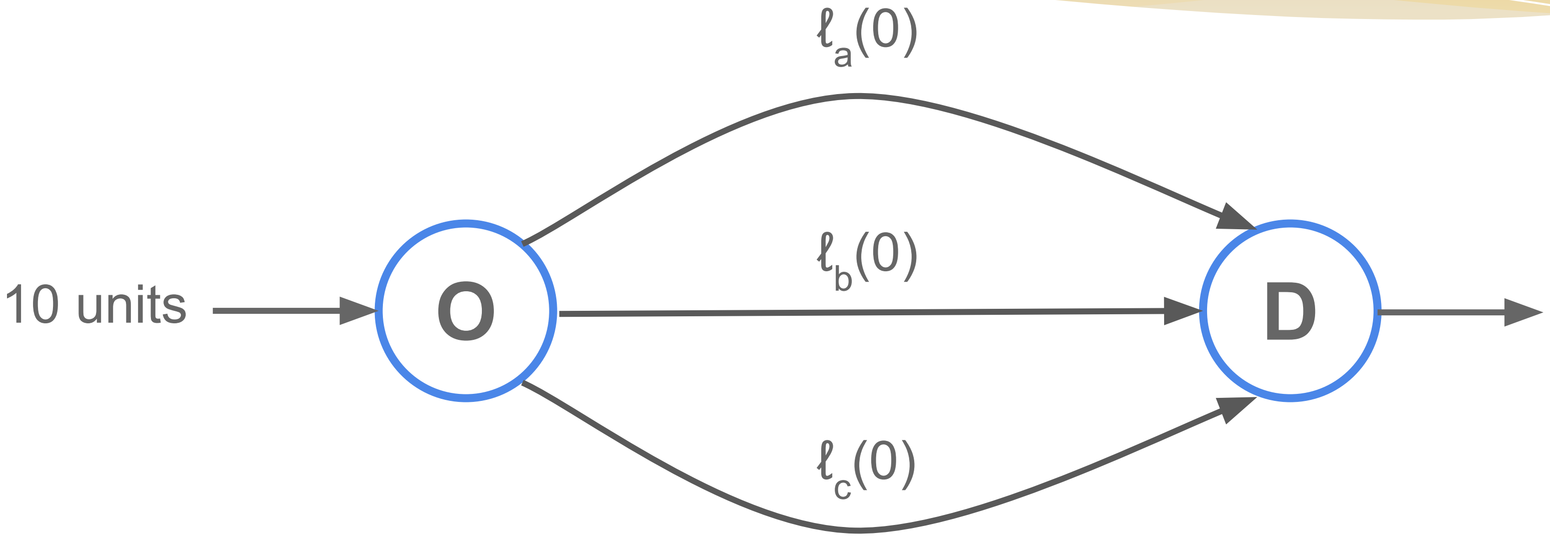
Appendix: UE framework

UE framework



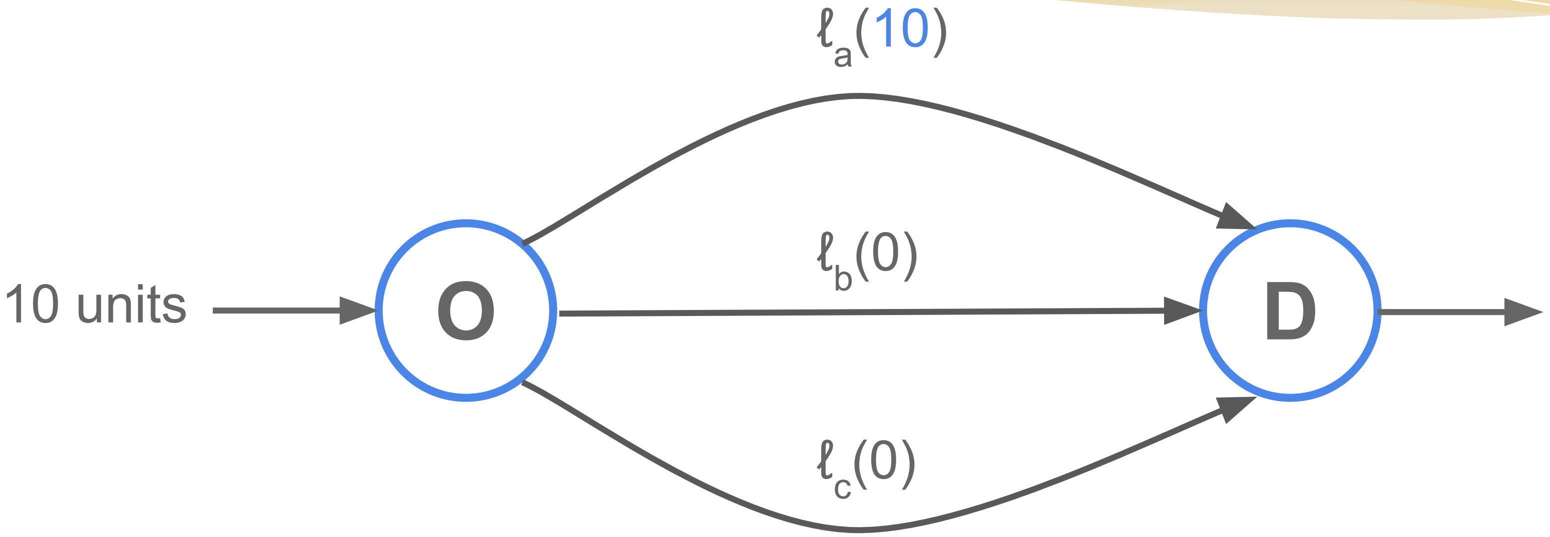
Distribute the 10 units among the 3 routes: $f_a + f_b + f_c = 10$

UE computation: step 1



Suppose $\ell_a(0) < \ell_b(0) < \ell_c(0)$: route 10 units on a.

UE computation: step 2



Now $\ell_b(0) < \ell_c(0) < \ell_a(10)$ due to congestion effect.
Re-allocate part of the 10 units on b.

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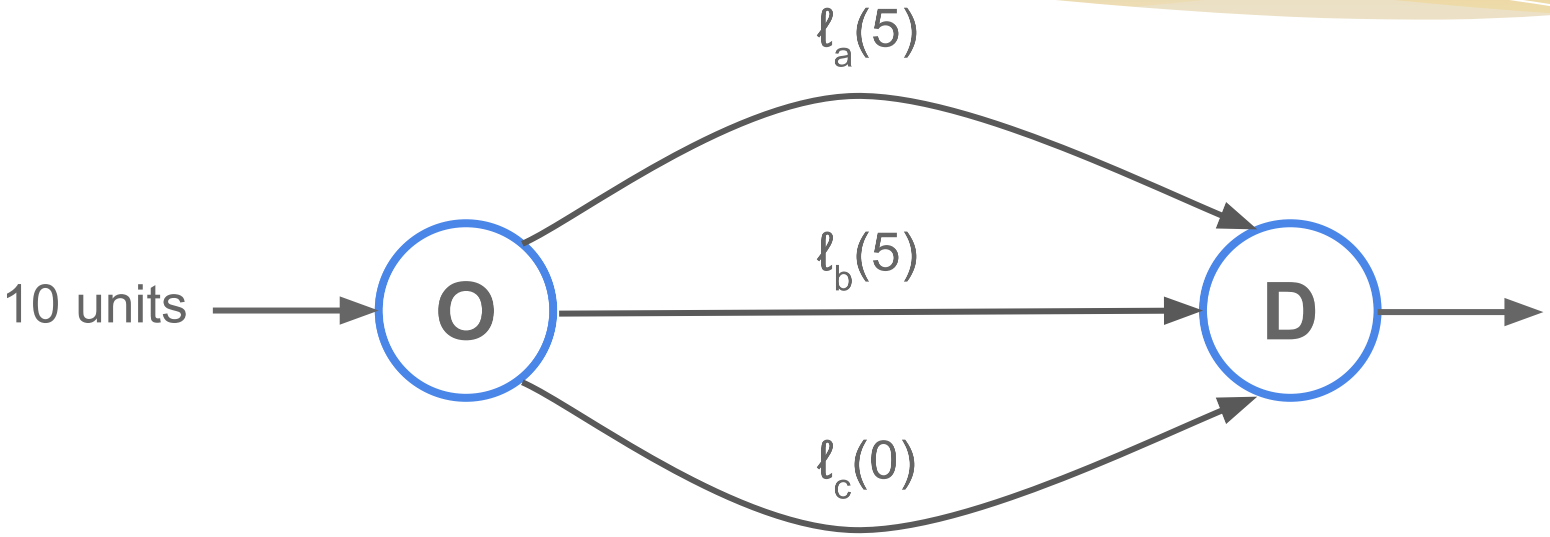
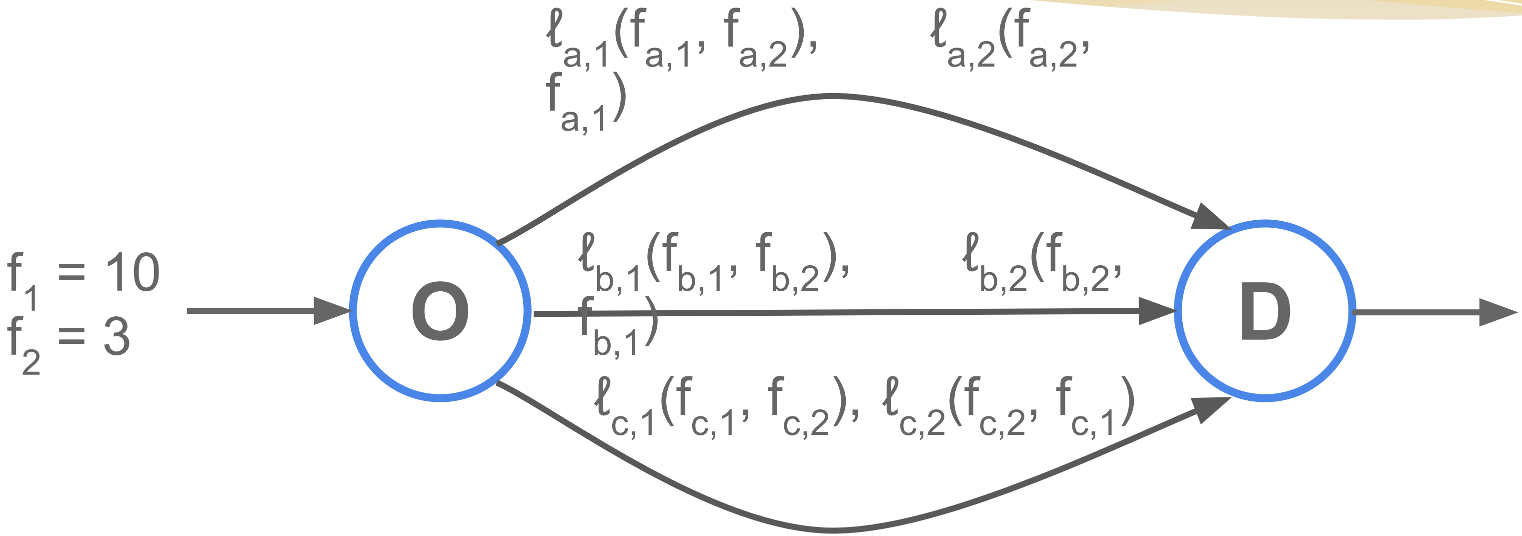
UE computation: step 3



Now $\ell_a(5) < \ell_b(5) < \ell_c(0)$, re-allocate part of the 10 units on a…

Convergence is guaranteed for smooth increasing costs.

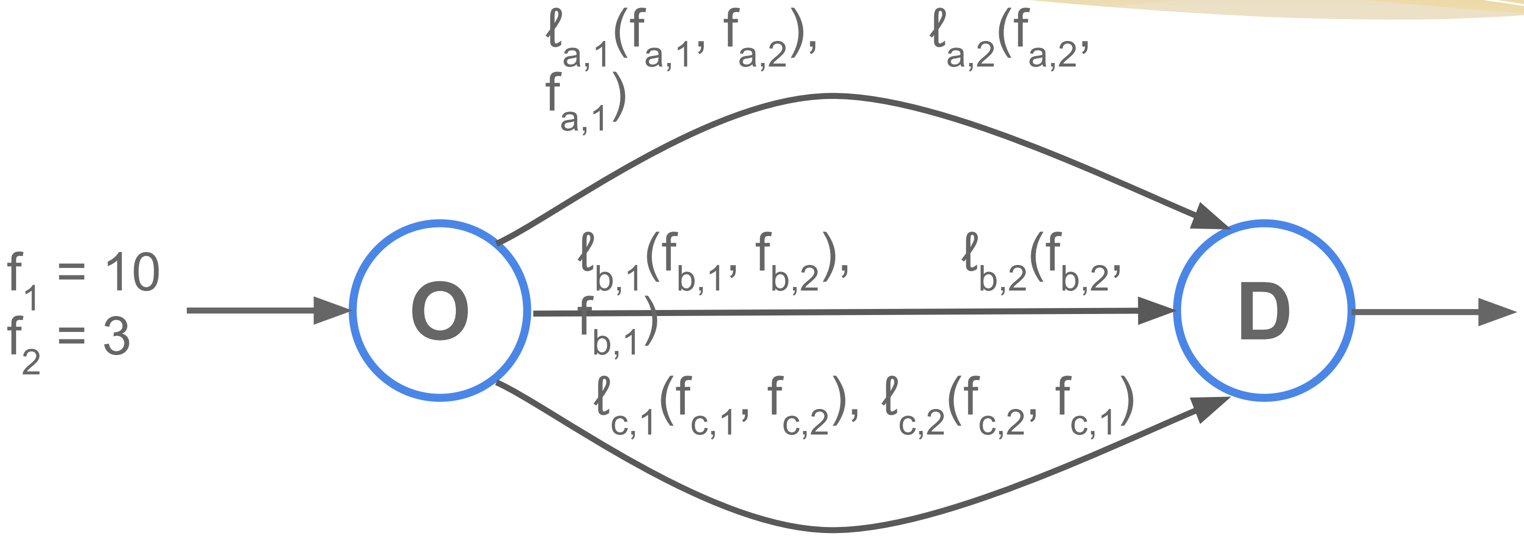
UE with two types



Distribute each type among the 3 routes:

$f_{a,1} + f_{b,1} + f_{c,1} = 10, \quad f_{a,2} + f_{b,2} + f_{c,2} = 3$

UE definition with two types

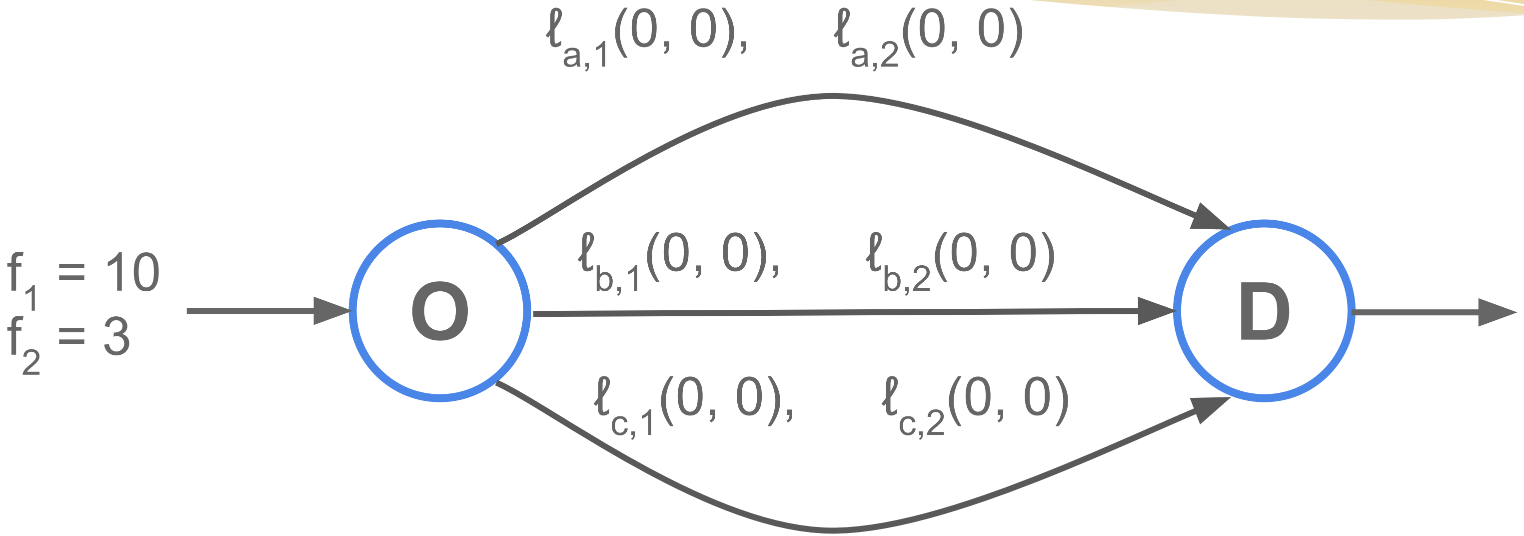


If $(f_{a,1}, f_{b,1}, f_{c,1}) = (6,4,0)$, $(f_{a,2}, f_{b,2}, f_{c,2}) = (0,1,2)$ is NE then

$\ell_{a,1}(f_{a,1}, f_{a,2}) = \ell_{b,1}(f_{b,1}, f_{b,2}) \leq \ell_{c,1}(f_{c,1}, f_{c,2})$

$\ell_{b,2}(f_{b,2}, f_{b,1}) = \ell_{c,2}(f_{c,2}, f_{c,1}) \leq \ell_{a,2}(f_{a,2}, f_{a,1})$

UE computation

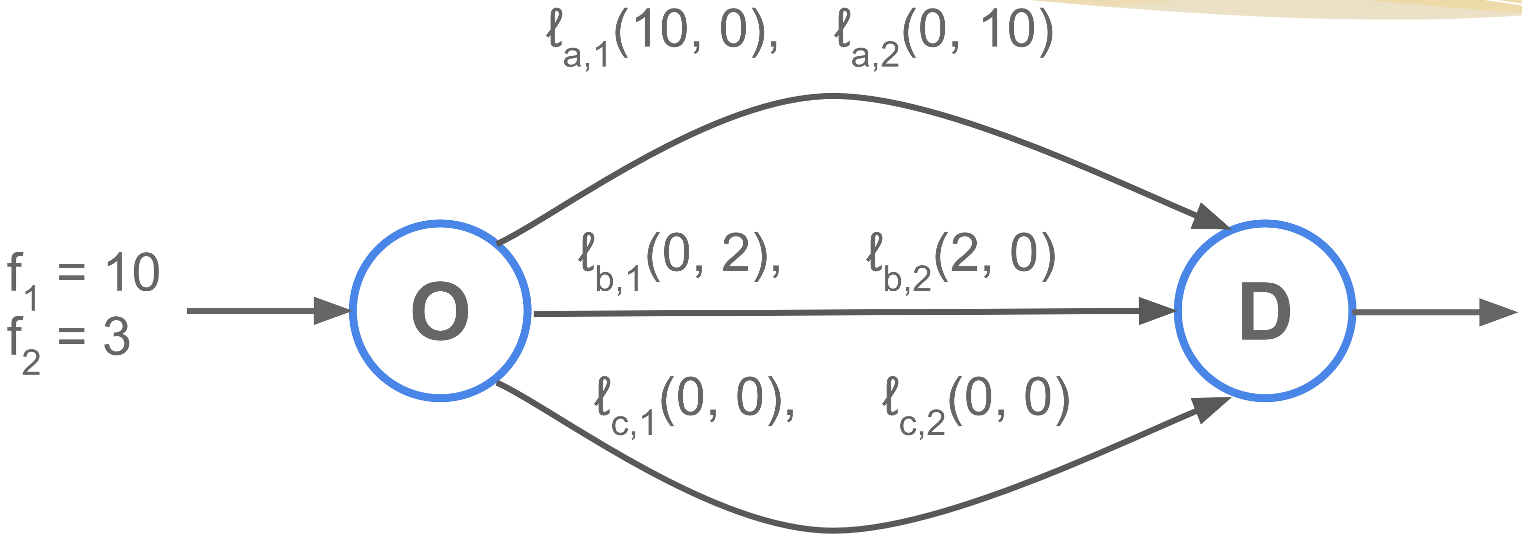


Suppose $\ell_{a,1}(0,0) < \ell_{b,1}(0,0) < \ell_{c,1}(0,0)$ and $\ell_{b,2}(0,0) < \ell_{c,2}(0,0) < \ell_{a,2}(0,0)$

route 10 units of type 1 on a

route 3 units to type 2 on b.

UE computation



Now $\ell_{b,1}(0,2) < \ell_{c,1}(0,0) < \ell_{a,1}(10,0)$ and $\ell_{c,2}(0,0) < \ell_{b,2}(2,0) < \ell_{a,2}(0,10)$

re-allocate part of type 1 units on b and part of type 2 units on c.

Convergence is guaranteed for smooth increasing costs.

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