FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

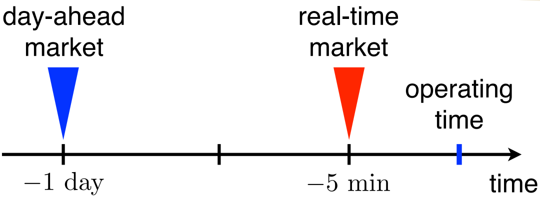
Risk-Limiting Dynamic Contracts for Direct Load Control

Insoon Yang, Duncan Callaway, Claire Tomlin

Department of Electrical Engineering and Computer Sciences
Energy and Resources Group
UC Berkeley, CA, USA

Berkeley University of California
Massachusetts Institute of Technology
NSF
Michigan
Vanderbilt University

Time Line of Electricity Market Operation and Financial Risk

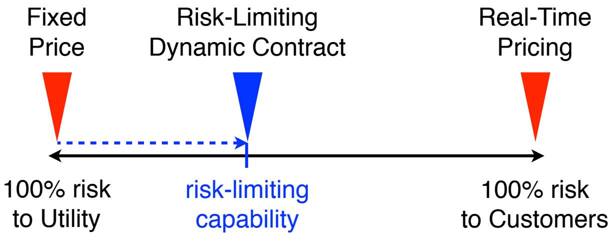


- Day-ahead market: market-clearing prices & unit commitments
  - Supply = Forecasted demand

- Real-time market (RM): balancing instantaneous demand
  - higher penetration of customers' solar & wind
    $\Longrightarrow$ higher imbalance fee

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Risk-Limiting Dynamic Contracts: Towards Financial Risk-Sharing on Demand Side



- Key Idea: Direct load control + Contract

Contributions and Features of This Work

- Contributions:
  - Financial risk management solutions for electricity markets using direct load control
  - Dynamic contracts with risk-limiting capability
  - Solution method for mean-variance constrained-stochastic optimal control via dynamic programming
- Features:
  - Risk-limiting capabilty
  - Scalability: decoupled optimal contract design
  - Decentralized control + central monitoring

Risk-Limiting Dynamic Contracts

- Contract: $(C^i, \{u^i_t\}_{0 \le t \le T})$ (Note: they are schemes!)
- For customer i (Payoff: $J^A_i[C^i, u^i]$)
  - Participation payoff condition:
    $$\mathbb{E}[J^A_i[C^i, u^i]] \ge b_i$$
  - Risk-limiting condition (risk measure - variance):
    $$\text{Var}[J^A_i[C^i, u^i]] \le S_i$$
- Mean and Variance can be independently adjusted!

Risk-Limiting Dynamic Contracts (continued)

- For utility (Payoff: $J^P[C, u]$)
  - Risk-sensitive control

$$\max_{C,u} \quad -\frac{1}{\theta}\log \mathbb{E}\left[\exp(-\theta J^P[C,u])\right]$$

$$\text{subject to} \quad dx_t^i = f_i(x_t^i, u_t^i)dt \qquad \text{– load dynamics}$$

$$\mathbb{E}[J_i^A[C^i, u^i]] \geq b_i$$

$$\text{Var}[J_i^A[C^i, u^i]] \leq S_i, \quad i = 1, \cdots, n$$

- Penalization of risk ($\theta > 0$: risk-averse decision making)

$$-\frac{1}{\theta}\log \mathbb{E}\left[\exp(-\theta J^P[C,u])\right] = \mathbb{E}[J^P[C,u]] - \frac{\theta}{2}\text{Var}[J^P[C,u]] + O(\theta^2)$$

High-Level Description of Proposed Solution Method

- The risk-limiting condition
  $=$ Conditions on the compensation and a new control variable γ_t^i

- Reformulation of the participation payoff condition:
  Introducing a new state y_t^i
  (customer's future expected payoff with a modified volatility)

- Reformulation of the risk-limiting condition:
  Introducing a new state z_t^i
  (remaining amount of risk that customer i can bear)

- Dynamic programming
  $\Longrightarrow$ n decoupled three dimensional Hamilton-Jacobi-Bellman equations

Risk-Limiting Compensation

Theorem (Construction of compensation)

Fix $u^i \in \mathbb{U}^i$ *and* $\gamma^i \in \Gamma^i$ *such that*

$$\mathbb{E}\left[\int_0^T (\gamma_t^i)^2 dt\right] \leq S_i.$$

The risk-limiting condition holds if and only if the end-time compensation, $C^i \in \mathbb{C}^i$*, satisfies*

$$C^i = \mathbb{E}[J_i^A[C^i, u^i]] - \int_0^T r_i^A(u_t^i, x_t^i)dt + \int_0^T \gamma_t^i dW_t^i.$$

Risk-Limiting Dynamic Contract Design

- Reformulation:

$$
\begin{aligned}
\max_{u,\gamma,\zeta} \quad & -\frac{1}{\theta}\log \mathbb{E}\left[\exp(-\theta \bar{J}^{P}[u,\gamma,\zeta])\right] \\
\text{subject to} \quad & dx_t^i = f_i(x_t^i, u_t^i)dt \\
& dy_t^i = -r_i^A(u_t^i, x_t^i)dt + (\gamma_t^i - \sigma_i^A(t) - \sigma_i(t))dW_t^i \\
& y_0^i = b_i \\
& dz_t^i = -(\gamma_t^i)^2 dt + \zeta_t^i dW_t^i \\
& z_0^i = S_i, \quad i = 1, \cdots, n
\end{aligned}
$$

- y_t^i: customer's future expected payoff with a modified volatility
- z_t^i: remaining amount of risk that agent can bear

Risk-Limiting Dynamic Contract Design (continued)

Theorem (Optimality)

Let (u^*, γ^*, ζ^*) be the solution to the reformulated problem. Define

$$C^{*i} := y_T^{*i} + \int_0^T \sigma_i(t) dW_t^i,$$

where W^i is the Brownian motion in the agent i's energy consumption model. If for $i = 1, \cdots, n$

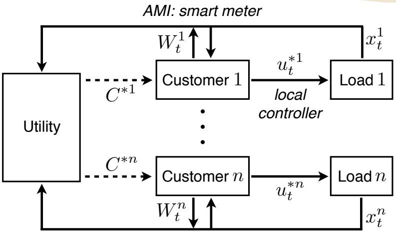
$$z_T^{*i} \geq 0,$$

then (C^*, u^*) is an optimal risk-limiting dynamic contact.

Remark:

- The problem can be decoupled for each agent: Scalability
- Solution method: dynamic programming

Practical Implementation: Decentralized Control + Central Monitoring

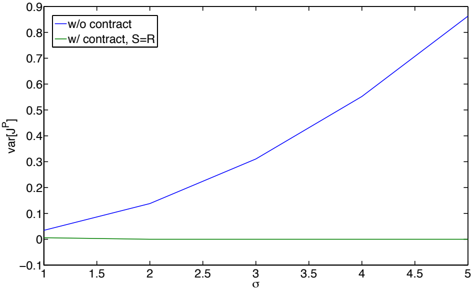


Minimum infrastructure required for TCL case:

- Smart meter ($120)
- Thermostat (installed in TCL or $25)
- Low-latency one-way data connection (Internet)
- Local controller in which the optimal control scheme in the contract is programmed

Effect of the uncertainty (forecast inaccuracy)

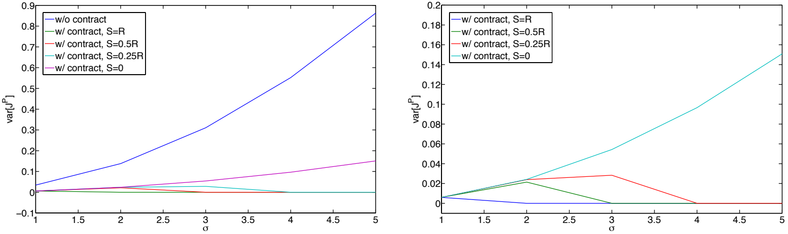
- Set the participation payoff as customer's optimal value in the case without a contract.
- R: customer's nominal risk (no contract case)
- Variance of utility's payoff vs volatility σ



FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Effect of the uncertainty (forecast inaccuracy)

- R: customer's nominal risk (no contract case)
- Variance of utility's payoff vs volatility σ



- $S \uparrow \Longrightarrow$ Risk management effectiveness $\uparrow$

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Ongoing & Future Research Directions

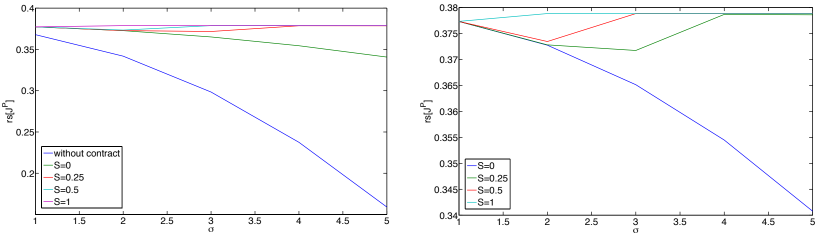
- Risk management solutions for electricity markets
  - Risk-limiting dynamic contracts for indirect load control
  - Risk-limiting dispatch + Risk-limiting dynamic contracts
    $\Longrightarrow$ Ultimate risk management solution for electric grid

- Scalable combinatorial optimization for control of interacting loads (with Sam Burden, Ram Rajagopal, Shankar Sastry, Claire Tomlin)
  - Guaranteed suboptimality bound
  - Noncooperative aggregators

- Scalability of implicit sampling in stochastic optimal control (with Matthias Morzfeld, Claire Tomlin, Alexandre Chorin)

Effect of the uncertainty (forecast inaccuracy)

- Risk-sensitive function of utility's payoff vs volatility σ



- $S \uparrow \Longrightarrow$ Risk sensitive function of utility's payoff $\uparrow$

- Very effective under high penetration of renewables

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS