



Resilient Fault Localization in Water Networks Using Multi-level Sensing

W. Abbas¹, L. Perelman², X. Koutsoukos¹, S. Amin²

¹Vanderbilt University

²MIT



Resilient Localization of Leakages in Water-Distribution Networks

Motivation:

- * Leakages in urban water networks can cause huge economic losses, and health risks.
- * Sensors for network monitoring are vulnerable to cyber attacks

Objective:

Sensor placement for an efficient and **resilient localization** of pipe failures in the presence of cyber-attacks.

Challenges:

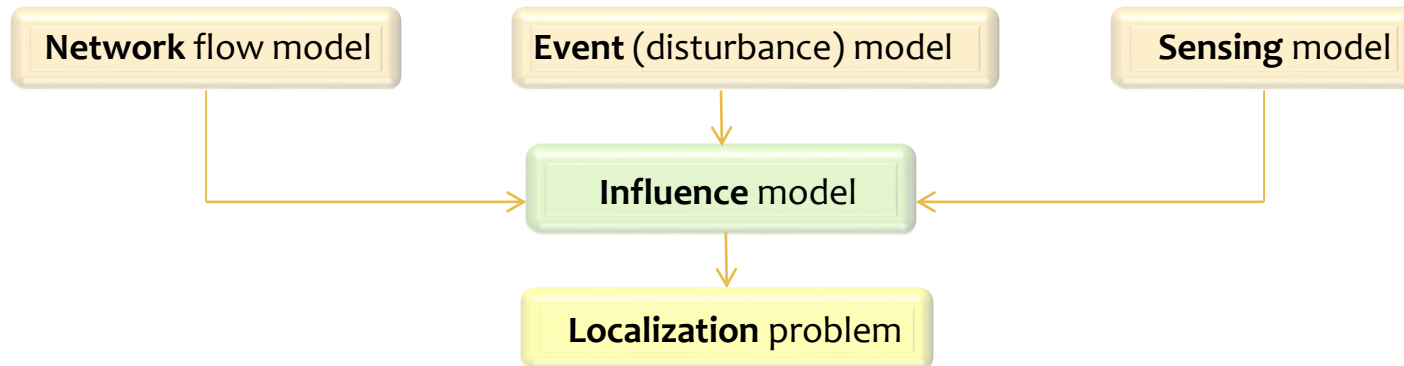
Pipe failure uncertainty, budget constraints, sensor errors etc.

Contributions:

- * Using combinatorial optimization, an efficient sensor placement algorithm for the localization of failure events in water networks. (*Automatica' 2016*)
- * Improved localization through multi-level sensing (*ACM BuildSys 2015*)
- * Localization in the presence of sensor attacks (errors)



Localization of Failure Events



Localization Problem:

Find the minimum number of sensors and their locations so that the maximum number of link failures can be uniquely detected and can be distinguished from one another.



To solve the minimum test cover problem, we proposed an approximate algorithm that is faster as compared to the other known solutions. ([Automatica 2016](#))

Localization Using Multi-level Sensing

In **single-level sensing (1-bit)**, output of the sensor is

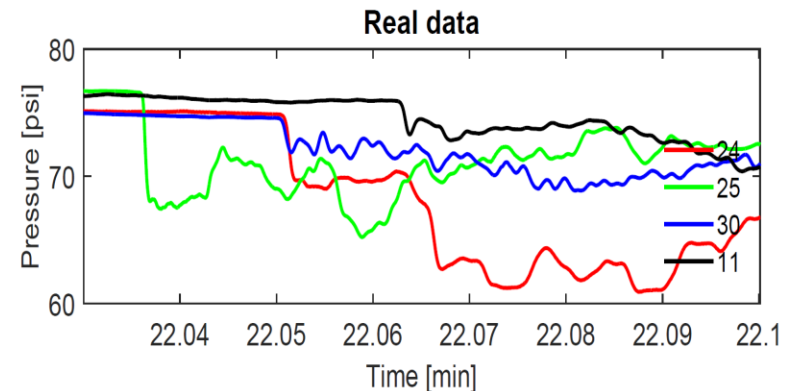
$$\begin{cases} 1 & \text{if failure event is detected, or} \\ 0 & \text{otherwise.} \end{cases}$$

In **multi-level sensing (σ -bit)**, a sensor in case of detection, captures some *extra information* about the failure event, such as time taken to detect the event etc.

Output of sensor consists of *multiple bits*.

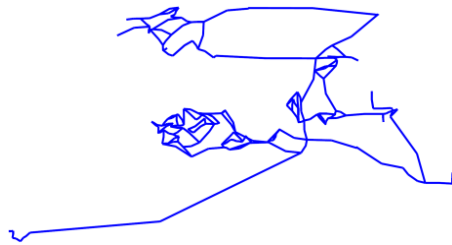
Case: Bi-level sensing

$$\begin{cases} 0 & 0 & \text{failure event is not detected,} \\ 1 & 0 & \text{event is detected early, i.e., in } [0 \quad t_1) \\ 0 & 1 & \text{event is detected later, i.e., in } [t_1 \quad T] \end{cases}$$

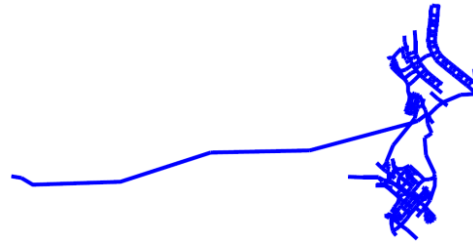


Simulations: Bi-Level Sensing

The **number of pair-wise link failures** that can be detected by σ -bit sensors is greater than in the case of 1-bit sensors.



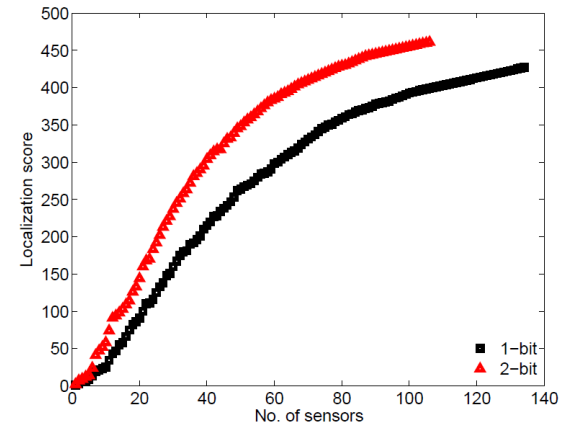
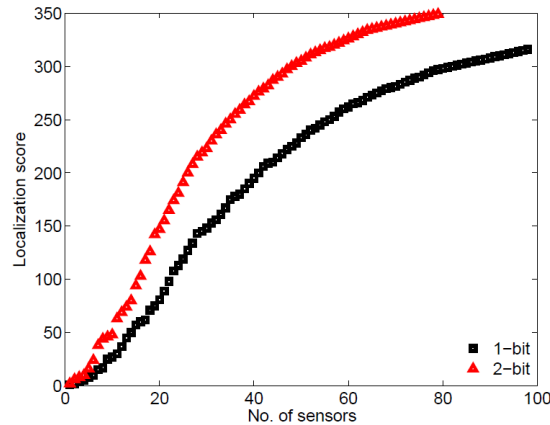
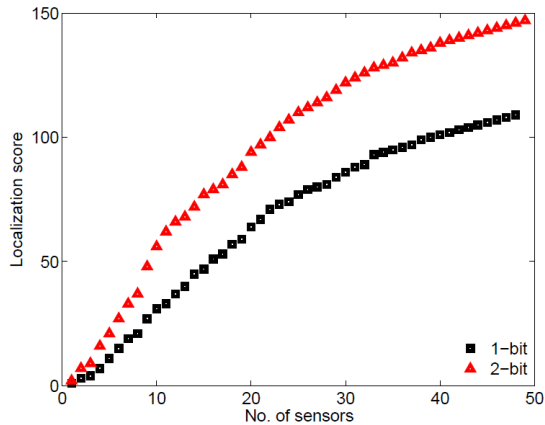
Net 1



Net 2



Net 3



Attacks Resulting in Sensor Errors

Single sensor attack



Error (attack): One or more of the output bits are flipped.

Example:

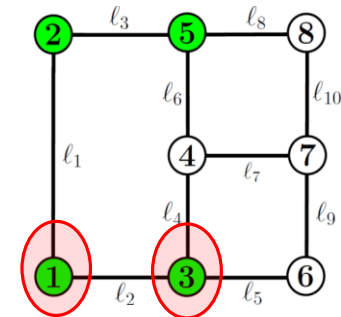
Correct 2-bit output: 0 1
 2-bit output with error can be any of the

0 0
 1 0
 1 1

Multiple sensors attack

Given a set of m sensors, at most e of them can give incorrect outputs for an event.

Example:



Sensors	S_1	S_2	S_3	S_5
Correct o/p	0 1	1 0	0 0	0 0
Possible o/p with 2 errors	1 0	1 0	1 1	0 0

Localization with Sensor Errors

Objective:

Design a resilient placement algorithm for a set of m sensors with multiple output bits to maximize the localization of n events given that at most e of the m sensors are attacked and can give incorrect outputs.

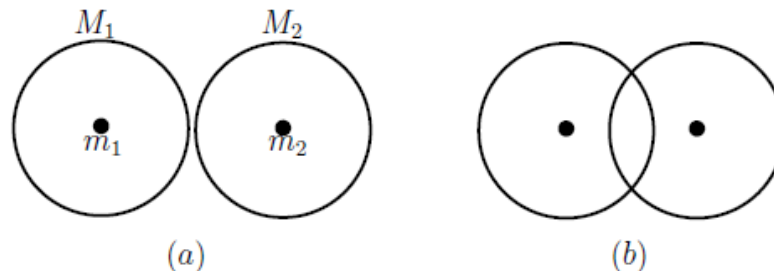
Two events can be distinguished in the presence of at most **e sensor errors (attacks)** as long as the hamming distance between their signatures is at least **$(2e + 1)$** .

Example:

$$m_1 = 0001$$

$$m_2 = 1100$$

$$e = 1$$



Select m sensors such that the number of event pairs that have a hamming distance of at least $(2e+1)$ between their signatures is maximized.

Set Multi-Cover for Localization with Errors

Select m sensors such that the number of event pairs that have a hamming distance of at least $(2e+1)$ between their signatures is maximized.



Set Multi-Cover Problem

Given a set of elements L , and a collection C of subsets of L . Select the minimum number of subsets in C such that each element in L is contained in at least k of selected subsets. For our problem, $k = 2e+1$.

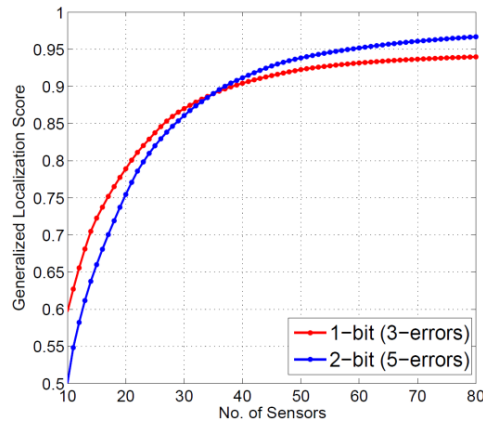
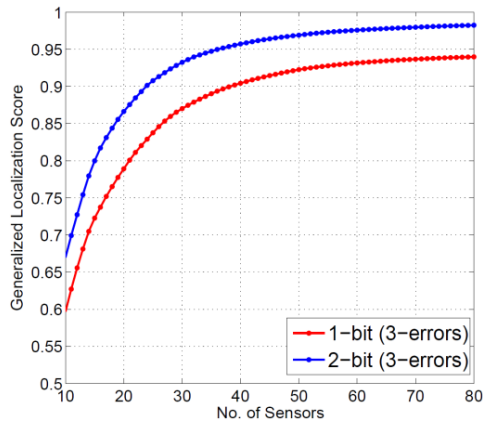
- * Simple greedy gives $(1+\ln d)$ -approximation algorithm *(Vazirani 2001)*
- * A randomized algorithm with an approximation ratio *(Berman et al. 2007)*
 $(1 + o(1)) \ln(d/k)$ if $(d/k) \geq 7.39$
 $1 + 2(d/k)^{1/2}$ if $(d/k) < 7.39$

High versus Low Level Sensing

Let $\sigma' > \sigma$

Fixing **number of sensors** and **e**,
Localization score in σ' -bit (high level) sensing is at least as good as in σ -bit (low level) sensing

Fixing **number of sensors** and **localization score**,
 σ' -bit (high level) sensing can handle at least as many **sensor errors (attacks)** as σ -bit (low level) sensing



How many more errors (attacks) σ' -bit sensing can handle as compared to σ -bit sensing?

Thank You

Localization Using Multi-level Sensing

The **maximum number of pair-wise link failures** that can be detected by σ -bit sensors is greater than in the case of 1-bit sensors.

1-bit

k : No. of **link failures** detected

\mathcal{P}_1 : No. of **pair-wise link failures** detected

$$\mathcal{P}_1 = k(n - k)$$

σ -bit ($\sigma > 1$)

k_i : No. of link **failures detected** by the i^{th} bit such that $\sum_{i=1}^{\sigma} k_i = k$

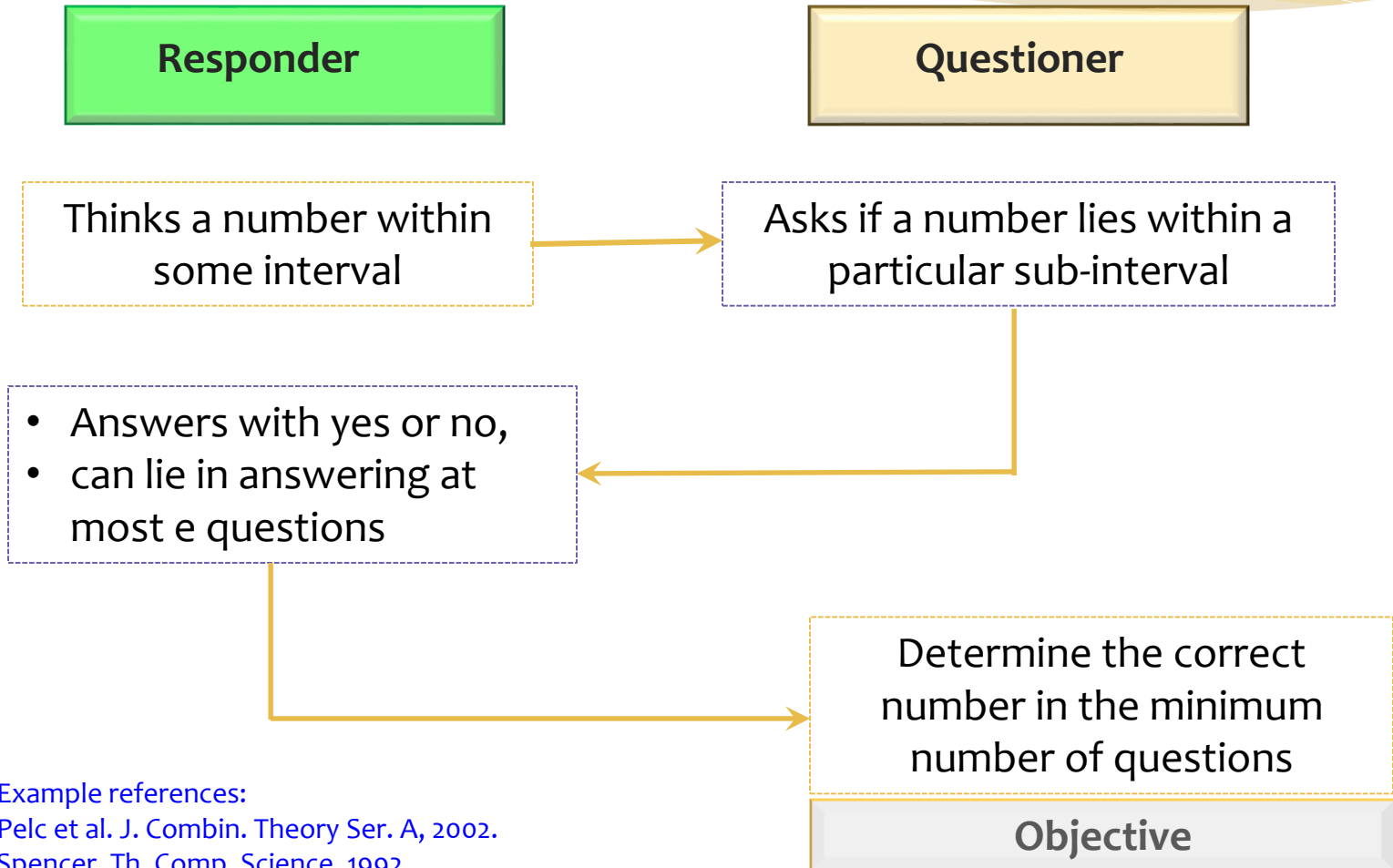
\mathcal{P}_σ : No. of **pair-wise link failures** detected

$$\mathcal{P}_\sigma = \mathcal{P}_1 + \left(\sum_{x=1}^{\sigma-1} \sum_{y>x}^{\sigma} k_x k_y \right)$$

e.g, for 2-bit: $\mathcal{P}_2 = \mathcal{P}_1 + k_1 k_2$

$$\mathcal{P}_1 + (\sigma - 1) \left(k - \frac{\sigma}{2} \right) \leq \mathcal{P}_\sigma \leq \mathcal{P}_1 + \left(\frac{k^2(\sigma-1)}{2\sigma} \right)$$

Ulam Games



Example references:

Pelc et al. J. Combin. Theory Ser. A, 2002.

Spencer, Th. Comp. Science, 1992.

Localization with Sensor Errors

	S_1	S_2	...	$S_{m'}$
event 1	1	1	...	0
event 2	0	1	...	1
⋮				
event n				

Influence matrix

Pair-wise event (1,2)
 Pair-wise event (1,3)
 ⋮
 Pair-wise event (n-1,n)

	S_1	S_2	...	$S_{m'}$
	1	0	...	0

Pair-wise influence matrix

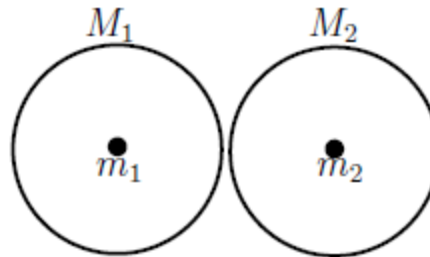
Two events can be distinguished in the presence of at most **e sensor errors** as long as the hamming distance between their signatures is at least **$(2e + 1)$** .

Example:

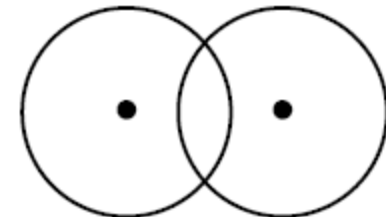
$$m_1 = 0001$$

$$m_2 = 1100$$

$$e = 1$$

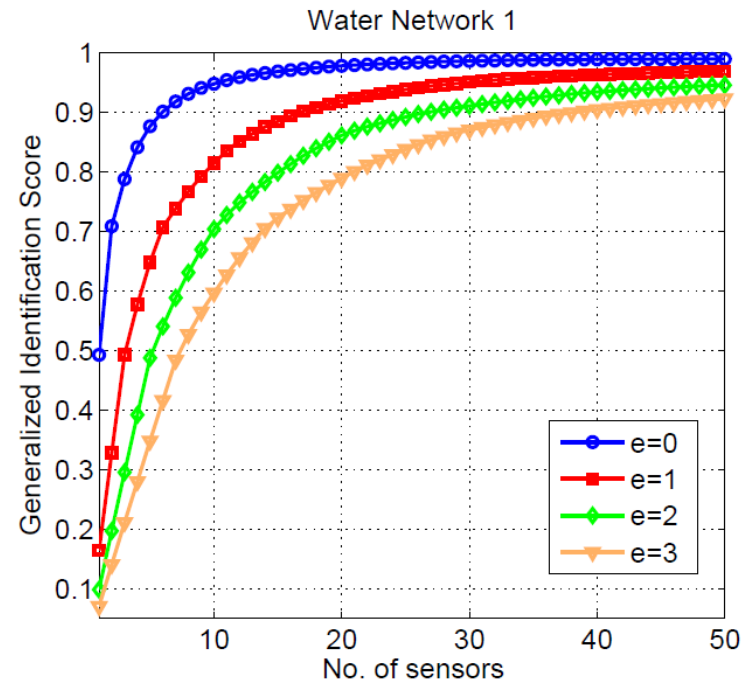
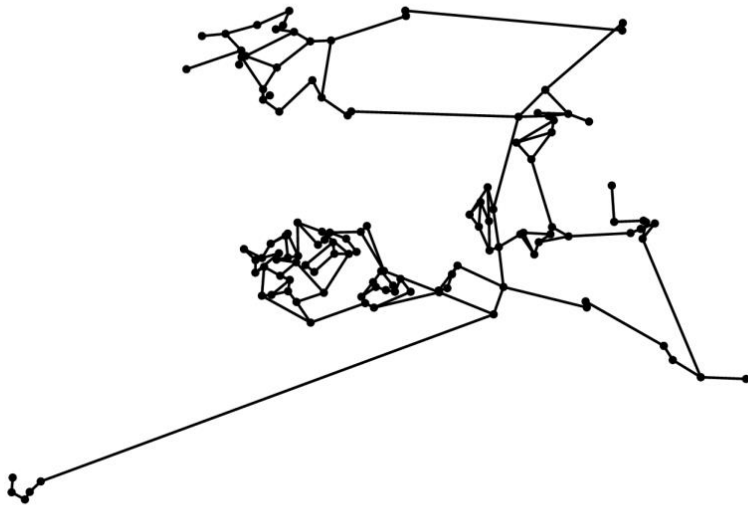


(a)



(b)

Simulations



Errors in High versus Low Level Sensing

Pair-wise influence matrix in σ -bit sensing

select m columns that maximally **K-cover** its rows

Special structure

Pair-wise influence matrix in σ' -bit sensing

select m columns that maximally **K'-cover** its rows

What is the maximum K' such that K' -coverage achieved with m sensors in σ' -bit pair-wise influence matrix is at least as good as K -coverage achieved in σ -bit pair-wise influence matrix ?

$$T_{\sigma} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2-coverage yields 0.833
localization score

$$T_{\sigma'} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

3-coverage yields 0.833
localization score