## Information in networked world

## Economic theory for CPS researchers

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Lecture 1－2：Hurwicz，Maskin，Myerson（2008） Asymmetric Information and Implementation


SFORCES

## Arrow impossibility theorem \& its progenies



We will make no distinction between transaction costs \& Contract costs
Our focus: column III

- modeling asymmetric info and costly contracts

Asymmetric info and contractual costs: any relationship?
■ risk \& decision making under uncertainty

## Travel through Time I

From prehistoric era: Nobel'71
■ Paul Samuelson Equilibrium theory (partial \& general eq.)

To ancient times: Nobel'72, '91

- Arrow Impossibility Theorem

■ Coase Transaction costs, property rights, institutions

To modern times: Nobel'94 Nash, Harsanyi, Selten

- Nash Equilibria, Bargaining Theory, Game Theory
- Harsanyi Bayesian games [tools for richer info structures]
- Selten Equilibrium refinements, multi-stage games


## Time Travel II: our coming journey

Past spring (2015)
We discussed Nobel'72,91,94 and started asymmetric information

To reaching XXI century: Nobel'2001 - our coming travel
■ Akerl of Information Imperfections [Lemons]

- Missing Markets

■ Stiglitz Markets with Asymmetric Info

- Grossman-Stiglitz impossibility result
- Spence Signaling


## Time Travel III: we will make excursions to

To Nobel'2007 - Today's lecture
■ Maskin Implementation
■ Myerson Mechanism Design
■ Hurwicz Enforcement

Nobel'2014 Analyzing Institutions, Markets and Information

- Tirole A footprint on everything [no unifying theory yet]
- Regulatory economics: market power and public goods
- Incomplete contracts
- Platforms (networked environments)


## Time Travel IV: Economic Institutions (applications)

Valuation of Derivatives: [attn Pricing Risks]
■ Scholes'97 Valuation of derivatives

- from Ito calculus to Black-Scholes formula [Samuelson was close ...]

■ Merton'97 Applications of Black-Scholes

From Coase to Institutions to Governance

- North'93 Theory of economic institutions
- Williamson'2009 The firm and its boundaries
- Ostrom'2009 Organization of commons
- public infrastructures \& property rights


## Time Travel and Hot Topics

Nobel'72, 91, 94, then Nobel'97, 2001, 93 \& 2009, 2007 \& 2014

- Merton, Scholes'97 Value of Derivatives [ = Pricing Risk]
- Akerlof, Stiglitz, Spence'2001 Asymmetric information
- North'93, Williamson \& Ostrom'2009 Institutions: theory, design and governance
- Maskin, Myerson, Hurwicz'2007 Implementation, Mechanism Design, Enforcement
- Tirole'2014 Regulations, market power, networks, contracts

Hot topics today \& tomorrow: Information and risks in networked world
■ Practical Issues in implementation, robust implementation

- Limits of Mechanism design

■ Information in Games, global games
■ Tomorrow? Risks in networks, institutions [why?]

## Time Travel: Glancing into the future

On route to Nobel?: Information and risks in networked world
■ Contracts, Enforcement, Institutions Costly contracts revisited
■ Institutional design from theory ['96,2009,2014] to practice

- Infrastructures (CPS): design in networked world
- Cyber security (CPS): pricing information in networked world
- Resilience to risks (CPS): Liability for information goods

■ Cyber-risks from valuing the derivatives ('97) \& information ('2001) to
managing cyber-risks

- cyber-risks (CPS): evaluating and pricing risks in networks
- cyber-risks (CPS): management \& liability assignment
- cyber-risks (CPS): management via cyber insurance


## Hurwicz quest

Designing resource allocation mechanisms
Hurwicz: Let us characterize environments for which informationally decentralized, incentive compatible, \& Pareto satisfactory (i.e., efficient) mechanisms can be designed.

## Classical environment

Desirable eq. features

- well behaved utility functions

■ no externalities (interdependencies)

- no indivisibilities
- pure exchange economy
- Pareto efficient [PE]
- Individually rational [IR]
- Incentive compatible [IC]

Standard imposition: Anonymity (to be defined)

## Hurwicz'72 Impossibility Theorem

Theorem (Hurwicz'72)
For any classical private goods environment $E$ with $n>1$ individuals, and I > 1 goods, there exists no (i) incentive-compatible direct revelation mechanism $\mathcal{M}$ whose dominant strategy equilibrium outcomes are (ii) Pareto efficient and (iii) individually rational.
where
$e^{i}=\left(u^{i}, \omega^{i}\right), e \in E-$ a space of all possible environments
$u^{i}-$ utility function, $\omega^{i}$ - endowment of individual $i$
$\mathcal{M}=(E, \delta, h), \delta$ - message correspondence, $h$ - outcome function $\delta: E \rightarrow \rightarrow M, h: M \rightarrow \rightarrow A$

Feasible set $A=\left\{x \in \mathbb{R}_{+}^{n /}: \sum_{i=1}^{n} x^{i} \leq \sum_{i=1}^{n} \omega^{i}\right\}$.

## Preliminaries: Definitions and Notation

Notation

| $n$ | \# of players | $n \geq 2$ |
| :--- | :--- | :--- |
| $I$ | a finite set of players, player $i, i \in I$ | $l=\{1, \ldots, i, \ldots, n\}$ |
| $l$ | private goods | $l \geq 2$ |
| $\mathbb{R}_{+}^{l}$ | commodity space |  |
| $\omega^{i}$ | endowment of $i$ | $\omega^{i} \in \mathbb{R}_{+}^{l}$ |
| $u^{i}$ | utility function of $i$ | $u^{i} \in \mathbb{R}_{+}^{l}$ |
| $e^{i}$ | characteristic of $i$ | $e^{i}=\left(u^{i}, \omega^{i}\right)$ |
| $E^{i}$ | a set of admissible characteristics for $i$ | $e^{i} \in E^{i}$ |
| $e \in E$ | a pure exchange economy | $e=\left(e^{i}, \ldots, e^{n}\right)$ |
| $E$ | a space of all possible economies | $E=\Pi_{i \in I} E^{i}$ |

$A$ the set of feasible allocations

$$
A=\left\{x \in \mathbb{R}_{+}^{n l}: \sum_{i=1}^{n} x^{i} \leq \sum_{i=1}^{n} \omega^{i}\right\}
$$

## Preliminaries: Definitions and Notation (cont.)

| $\varphi$ | PEIR correspondence (social goal) | $\varphi: E \rightarrow \rightarrow A$ |
| :--- | :--- | :--- |
| $\mathcal{M}$ | mechanism (a triple) | $(M, \delta, h)$ |
| $M$ | message space (Cartesian product) | of $\left\{M^{1}, \ldots M^{n}\right\}$ |
| $M^{i}$ | agent $i$ individual message space |  |
| $\delta$ | eq. message correspondence | $\delta: E \rightarrow \rightarrow M$ |
| $h$ | outcome function | $h: M \rightarrow \rightarrow A$ |
| $m$ | message vector $m=\left(m_{1}, \ldots, m_{n}\right)$ <br> $\hat{m}$ | $m \in M$ |
| if $\hat{m}^{i}=\hat{m}^{j}$ | equilibrium message vector$\Longrightarrow$ identical eq. allocation for $i \neq j$ | $=$ anonymity |

## Definition (PEIR social goal)

A correspondence $\varphi: E \rightarrow \rightarrow A$ is PEIR social goal if for each $e \in E$ the allocations in $A$ are $P E$ and $I R$.

Definition (Anonymous mechanism)
Mechanism $\mathcal{M}$ is anonymous if for identical eq. messages $\hat{m}^{i}=\hat{m}^{j}, i \neq j$, equilibrium allocations for individuals $i$ and $j$ are identical.

## Implementing social goals via mechanisms

Definition (PEIR implementation via mechanism $\mathcal{M}$ )
If for any $e \in E$ every dominant strategy equilibrium of $\mathcal{M}=(M, \delta, h)$ results in PE and IR allocations, then $\mathcal{M}$ weakly implements the PEIR social goal $\varphi$.
[i.e, for every $e \in E$ and every $m \in \delta(e)$, we have $h(m) \in \varphi(e)$ ]
Definition (Direct revelation mechanism)
Direct revelation mechanism $\mathcal{M}=(E, \delta, h)$, i.e, $M^{i}=E^{i} \forall i \in I$.
Definition (Incentive compatibility (IC))
For any given environment $E$ and direct revelation mechanism $\mathcal{M}=(E, \delta, h)$, the mechanism is IC if truth-telling is a dominant strategy equilibrium of $\mathcal{M}$, i.e., if all $e \in E, e \in \delta(e)$.

## Proof (by contradiction)

Two person two good economy, with 2 possible utilities, Cobb-Douglas and Linear:

$$
u_{C}^{i}\left(x_{1}^{i}, x_{2}^{i}\right)=x_{1}^{i} * x_{2}^{i} \text { and } u_{L}^{i}\left(x_{1}^{i}, x_{2}^{i}\right)=2 x_{1}^{i}+x_{2}^{i} .
$$

Let

$$
\omega_{1}=(1 / 2,0) \text { and } \omega_{1}=(1 / 2,1) .
$$

Let player 1and 2 utilities be $C$ and $L$, resp. Assume the truth telling eq. with eq. allocation $\left(x_{C}, x_{L}\right)$. From anonymity, if both agents lie: $\left(x_{L}, x_{C}\right)$.

$$
\text { Player } 2
$$

IC constraints


$$
\begin{aligned}
& u_{C}^{1}\left(x_{C}\right) \geq u_{C}^{1}\left(x_{L}\right) \\
& u_{L}^{2}\left(x_{L}\right) \geq u_{L}^{2}\left(x_{C}\right)
\end{aligned}
$$

## Proof (cont.)

IR constraints are:

$$
u_{C}^{1}\left(x_{C}\right) \geq 0 \text { and } u_{L}^{2}\left(x_{L}\right) \geq 2
$$

Truth-telling inducing IC constraints are:

$$
u_{C}^{1}\left(x_{C}\right) \geq u_{C}^{1}\left(x_{L}\right) \text { and } u_{L}^{2}\left(x_{L}\right) \geq u_{L}^{2}\left(x_{C}\right)
$$

or

$$
x_{2}^{1} \geq 1-x_{1}^{1} \quad \text { and } \quad x_{2}^{1} \leq 3 / 2-2 x_{1}^{1} \quad[I C] .
$$

We used total endowment ( $x_{1}=1, x_{2}=1$ ) to have $x_{1}^{1}=1-x_{1}^{2}$ and $x_{2}^{1}=1-x_{2}^{2}$
Note: For the original Hurwicz'72 proof see Jackson [2001]

## Proof of Hurwicz impossibility result: an illustration

Edgeworth box


PEIR and IC segments: no common points
The PE line segments ( $\left[O_{1}, T\right]$ and $\left[T, Q_{2}\right]$ ) and the IR (interval $\left[O_{1}, Q\right]$ ) do not intersect with IC segment (interval [RS]), which is a contradiction. $\square$

- example is general
- easy to construct similar ones
- strong and robust result


## Discussion: What drives Hurwicz impossibility result?


"Psst! If you have any stock tips to pass on, I can probably lighten your sentence for insider trading."

Hurwicz impossibility theorem is driven by INCENTIVES [to hide info]

## Discussion: Explaining Hurwicz impossibility result

Would you reveal your true valuation
■ when haggle with a car dealer?

- when haggle at Eastern bazaars?


Sometimes truth telling is not an eq.


For Travelers


## Discussion: What drives Hurwicz impossibility result?

With hidden info truth telling is infrequent.
■ IR and IC constraints differ (in general)

- for efficiency, both constraints should be binding

Intuition (informal): why to give up a valuable good (information) for free?

- exchange $\approx$ surplus sharing $\approx$ bargaining [bargaining, auctions, \& pricing are tools to share surplus]
■ hidden info could improve bargaining position
- truth telling $\approx$ giving up one's hidden info info is a good. when it is valuable, it could have a non-zero price.

Hurwicz impossibility is driven by INCENTIVES

"Psst! If you have any stock tips to pass on, I can probably lighten your sentence for insider trading."

## Revelation Principle: <br> from Mechanisms to Direct Revelation Mechanism

Theorem (Revelation principle)
If there exists some mechanism $\mathcal{M}=(M, \delta, h)$, which weakly implements any social goal $\varphi: E \rightarrow \rightarrow A$ for any $E$, then there is a direct revelation mechanism $\tilde{\mathcal{M}}=(E, \tilde{\delta}, \tilde{h})$ s.t. truth-telling is a dominant strategy for $\tilde{\mathcal{M}}$.
Revelation principle is as powerful. It allows to limit the search by truth telling mechanisms. Dasgupta, Hammond \& Maskin [1979], Myerson [1979].

Intuition of the proof (formal proof: F\&T [p. 255-256])
Assume there exists a (hypothetical) trusted mechanism coordinator (mediator) that has everyone's confidential info. First, the mediator would compute the dishonest eq. behavior (dishonest reports and disobedient actions). Then, the mediator will prescribe each player the plan with behavior that would have been chosen by that player in a dishonest equilibrium. The prescriptions will be followed. Such implementation is incentive compatible and truthful.

## Revelation Principle



Fig.1, Myerson [2008]
For any given eq. mechanism with dishonest reporting and disobedient actions mediator can implement the same eq. via equivalent incentive-compatible plan.

## Amplifying Hurwicz theorem by Revelation Principle

Theorem (Revelation Principle)
Given any $E$ and any social goal $\varphi$, if there exists a mechanism $\mathcal{M}=(M, \delta, h)$ that weakly implements $\varphi$ in dominant strategies, then there exists a direct revelation mechanism $\tilde{\mathcal{M}}=(E, \tilde{\delta}, \tilde{h})$ and truth-telling is a dominant strategy for $\tilde{\mathcal{M}}$.

Theorem (Hurwicz'72)
For any classical private goods environment $E$ with $n>1$ individuals, and $I>1$ goods, there exists no IC direct revelation mechanism $\mathcal{M}(E, \delta, h)$, whose dominant strategy equilibrium outcomes are $P E$ and $I R$.

## Corollary (Hurwicz theorem + revelation principle)

For any classical private goods environment with $n>1$ players, there is no mechanism $\mathcal{M}=(M, \delta, h)$ which weakly implements PEIR social goal in dominant strategies.

## Information, power, control

- Information: the negative reciprocal value of probability. Claude Shannon
- Information is power. But like all power, there are those who want to keep it for themselves. Aaron Swartz
- As a general rule, the most successful man in life is the man who has the best information. Benjamin Disraeli
- Information is power. Disinformation is abuse of power. Newton Lee
- Information, knowledge, is power. If you can control information, you can control people. Tom Clancy
- To live effectively is to live with adequate information. Norbert Wiener

Hurwicz: From formal proof of no truth-telling with hidden info to Focus on enforcement

## No truth-telling: a profound shift in thinking

rules of the (legal) game $=$
game-form = strategies-outcomes
= mechanism
$=$ implementation
true game strategies $=$
feasible strategies $=$
legal strategies + illegal strategies
true game $\neq$
legal game

Hurwicz:
If it is possible to violate legal rules, must enforce them


## Successful Implementation and Enforcement A profound shift in thinking: focus on institutions

Hurwicz'2008:

- Truth is not Nash equal. Nash equal. is not self-enforcing

■ People can use "illegal strategies" (not prescribed by the mechanism)
■ Hence, focus on institutions / enforcement (of legal rules)
Implementation $=$
mechanism design $=$
rules of the game $=$
institutional choices $=$ the rules and their enforcement

Institutional choices $=$ market, non-market (central planning, regulations)
Successful implementation requires enforcement
Enforcement $=$ oversight to ensure compliance with the mechanism

## Impossibility Theorems: from Arrow'51 to Hurwitz'72

Theorem (Arrow'51)
Any constitution (aka social choice rule) that respects transitivity, independence of irrelevant alternatives and unanimity is a dictatorship.

Theorem (Hurwicz'72)
In any classical private goods environment with two or more players, Pareto efficiency is not achievable via incentive-compatible, individually rational direct revelation mechanism with dominant strategy eq. outcomes.

Hurwicz on mechanisms: if IC and IR then no efficiency
If Arrow Theorem is a disappointment, Hurwicz Theorem is a misfortune.
Positive spin on Hurwicz'72
Design of constrained-efficient optima remains an active research area.
(Pareto) efficiency goes South, but research papers proliferation goes on.

## Usefulness of Hurwitz'72

Reviewing papers: When authors claim to reach efficiency. If setup
■ has hidden info $=$ the results are too good to be true.
■ has no hidden info $=$ the results are not robust if hidden info added

- How to find errors: where to look?
hint: check the constraints. in many cases, IC and IR do not bind simultaneously

For research with strategic agents (with IC and IR constraints)

- Search for constrained efficiency only
- When truth telling fails: Design other tools
alter info structure (at a cost); mandatory info revelation (with enforcement)
■ Are there more impossibility results? - yes. many more Akerlof'70, Grossman \& Stiglitz'80 $\longleftarrow$ to be covered, Hammond'79, Jordan'82


## Mechanism design: incentive compatibility perspective

Maskin's quest for incentive compatibility
A When designing incentive compatible mechanisms is possible?
B If mechanisms exist, what form might they take?
C When the existence of such mechanisms is ruled out theoretically?

Maskin's quest formalized
A' Under what conditions can a social choice rule be implemented?
B' What form does a mechanism take?
C' Which social choice rules cannot be implemented?

## Example 1: Social choice rule and its implementation

World has 2 states. Alice \& Bob know the state; but Authority does not.

$$
\begin{gathered}
\text { Alice }\left\{\begin{array}{l}
\text { gas } \succ \text { oil } \succ \text { coal } \succ \text { nuclear if st. } 1 \\
\text { nuclear } \succ \text { gas } \succ \text { coal } \succ \text { oil } \text { if st. } 2
\end{array}\right. \\
\text { Bob }\left\{\begin{array}{l}
\text { nuc } \succ \text { oil } \succ \text { coal } \succ \text { gas if st. } 1 \\
\text { oil } \succ \text { gas } \succ \text { coal } \succ \text { nun } \quad \text { if st. } 2
\end{array}\right.
\end{gathered}
$$

- Social choice rule: $f(1)=$ oil, and $f(2)=$ gas
- state $1 \&$ state 2 , with resp. low \& high discounting in st. 1 - care for present, in st. 2 - for future
- Alice - pref. for convenience now, anticipates tech advances Bob - safe \& clean now, afraid of nuclear waste in the future Maskin: reasonably happy, for ex. getting one's 1st or 2nd choice


## Example 1: Implementation

Naive mechanism is not incentive compatible.
Authority asks consumers to report state. Alice always will report state 2, and Bob - state 1. $\rightarrow$ No truthful revelation of the state.

| Table 1 |  |  |  |
| :--- | :--- | :--- | :--- |
| state 1 |  | state 2 |  |
| Alice | Bob | Alice | Bob |
| gas | nuc | nuc | oil |
| oil | oil | gas | gas |
| coal | coal | coal | coal |
| nuc | gas | oil | nuc |
| $f(1)=$ oil |  | $f(2)=$ gas |  |

Incentive compatible mechanism
Energy authority (mechanism designer) requests the consumers to participate in the mechanism:

| Alice | T | Bob |  |
| :---: | :---: | :---: | :---: |
|  |  | L | R |
|  |  | oil | coal |
|  |  | nuc | gas |


|  |  | Bob |  |
| :---: | :---: | :---: | :---: |
|  |  | state 1 |  |
| Alice | state 2 |  |  |
|  | st. 1 | oil, oil | coal, coal |
|  | st. 2 | nuc, nuc | gas, gas |
|  |  |  |  |

Nash eq. of the game coincides with social choice. i.e., the mechanism implements social choice rule in Nash equilibrium.

## Social choice rule \& implementation: formal definitions

## Definition

Social choice rule $f$ : $\Theta$ is a correspondence from each state $\theta$ into the set of optimal outcomes $\mathbf{a}$, where $\mathbf{a} \subset A$.

Designing an incentive compatible mechanism
Authority should suggest a protocol ( a game), such that consumers reveal the true state and social choice rule is implemented as a Nash equilibrium.

## Definition

Let $S_{i}$ denote player $i$ strategy space. A mechanism for a society with $n$ individuals is a mapping $g: S_{1} \times \ldots \times S_{n} \rightarrow A$, where $g\left(s_{1}, \ldots, s_{n}\right)$ is the outcome prescribed by the mechanism when player strategies are $\left(s_{1}, \ldots, s_{n}\right)$. Mechanism $g$ implements social choice rule $f$ in Nash equilibrium if for all $\theta$ we have: $f(\theta)=N E_{g}(\theta)$, where $N E_{g}(\theta)$ is the set of Nash eq. outcomes of $g$ in state $\theta$.

## Implementation and monotonicity

## Definition

Let $f(\theta)=$ a. If a does not fall in anyone's ranking relative to other alternatives when $\theta$ goes to $\theta^{\prime}$, then monotonicity requires $f\left(\theta^{\prime}\right)=a$.

If for someone, the ranking of $a$ falls (relative to some $b$ ), monotonicity imposes not restriction.
Theorem (Maskin'77)
If a social choice rule implementable, it much be monotonic.

## Definition

Let all players (but one) agree that outcome a is the best. Then, if social choice rule satisfies no veto power, a must be socially optimal.

Theorem (Maskin'77)
When there are at least 3 individuals, and social choice rule satisfies monotonicity and no veto power, it is implementable.
With 3 or more individuals, monotonicity guarantees implementation.

## Example 2: Social choice rule with no implementation

World has 2 states. Alice \& Bob know the state; but Authority does not.

Alice $\left\{\begin{array}{cl}\text { gas } \succ \text { oil } \succ \text { coal } \succ \text { nuclear } & \text { if } s t .1 \\ \text { gas } \succ \text { oil } \succ \text { nuc } \succ \text { coal } & \text { if st. } 2\end{array}\right.$

$$
\text { Alice } \begin{cases}\text { gas } \succ \text { oil } \succ \text { coal } \succ \text { nuc } & \text { if st. } 1 \\ \text { nuc } \succ \text { gas } \succ \text { coal } \succ \text { oil } & \text { if st. } 2\end{cases}
$$

Bob $\left\{\begin{array}{l}\text { nuc } \succ \text { oil } \succ \text { coal } \succ \text { gas } \\ \text { if } s t .1 \\ n u c ~ \\ \text { oil } \\ \succ \text { coal } \succ \text { gas }\end{array}\right.$ if st. 2
Bob $\begin{cases}\text { nuc } \succ \text { oil } \succ \text { coal } \succ \text { gas } & \text { if st. } 1 \\ \text { oil } \succ \text { gas } \succ \text { coal } \succ \text { nun } & \text { if st. } 2\end{cases}$

■ Social choice rule: $f(1)=o$, and $f(2)=n$
■ state 1 is identical in both examples, state 2 - differs

- example 2 :
- Bob's prefs. are identical in both states
- Alice's prefs. differ by the order of coal and nuc only


## Example 2: social choice rule is not implementable

| Table 1 (example 1) |  |  |  |
| :--- | :--- | :--- | :--- |
| state 1 |  | state 2 |  |
| Alice | Bob | Alice | Bob |
| g | n | n | $\circ$ |
| o | o | g | g |
| c | c | c | c |
| n | g | o | n |
| $f(1)=o$ |  |  |  |

Table 4 (example 2)

| st. 1 |  | st. 2 |  |  |  |
| :--- | :--- | :--- | :--- | :---: | :---: |
| Alice | Bob | Alice | Bob |  |  |
| g | n | g | n |  |  |
| o | o | o | o |  |  |
| c | c | n | c |  |  |
| n | g | c | g |  |  |
| $f(1)=o$ |  |  | $f(2)=n$ |  |  |

Proof that no implementation exists
Assume there exists an implementation and let strategies $\left(s_{A}, s_{B}\right)$ be Nash eq. in state 1 inducing oil. Claim: then, $\left(s_{A}, s_{B}\right)$ must be Nash eq. in state 2. Bob will not deviate, 'cause his prefs. are identical in states $1 \& 2$. Alice would only deviate to induce gas, but then, she would have deviated in state 1 as well. But if $\left(s_{A}, s_{B}\right)$ is Nash eq. in state 2 , it will induce oil, but social optimum requires nuclear.

Example 2 violates (Maskin's) monotonicity
Proof: From monotonicity, oil must remain optimal in state 2, but social choice rule prescribes $f(2)=n$.

## Example 1 re-interpreted: FBI \& Apple encryption dispute

World has 2 states. Apple \& FBI know the state; but CA Judge does not. How should the Judge rule in FBI and Apple encryption dispute?

| gas | g | Good to go (Apple wins) | G |
| :--- | :--- | :--- | :--- |
| oil | o | Order to Apple (unlocking is cheap \& fast) | O |
| coal | C | Order to Apple (has a key; Can unlock) | C |
| nuclear | n | Order to Apple (has No key; unlocking is pricey \& slow) | N |


| Table 1 |  |  |  |
| :---: | :---: | :---: | :---: |
| state 1 |  | state 2 |  |
| Alice | Bob | Alice | Bob |
| g | n | n | o |
| $\bigcirc$ | $\bigcirc$ | g | g |
| c | c | c | c |
| n | g | - | n |
| $f(1)=0$ |  | $f(2)$ |  |

Alice \& Bob lower-case letters

Apple \& FBI upper-case letters

Table 1 Apple \& FBI

| st. 1 insecure |  |  | st. 2 secure |  |
| :--- | :--- | :--- | :--- | :---: |
| Apple | FBI | Apple | FBI |  |
| G | N | N | O |  |
| O | O | G | G |  |
| C | C | C | C |  |
| N | G | O | N |  |
| $f(1)=O$ |  |  |  |  |

## FBI \& Apple encryption dispute

| FBI vs Apple dispute is identical to example 1 |  |  |
| :---: | :---: | :---: |
| g | Good to go (Apple wins) | G |
| o | Order to Apple (unlocking is cheap \& fast) | O |
| C | Order to Apple (has a key; Can unlock) | C |
| n | Order to Apple (No key; unlocking is pricey \& slow) | N |
| Social choice rule: $f(1)=O$, and $f(2)=G$ |  |  |

- st. 1 - care for present (insecure), st. 2 - for future (secure)

| Table 1 |  |  |  |
| :--- | :--- | :--- | :--- |
| st. 1 |  | st. 2 |  |
| Apple | FBI | Apple | FBI |
| G | N | N | O |
| O | O | G | G |
| C | C | C | C |
| N | G | O | N |
| $f(1)=O$ |  |  |  |

- Apple: in st. 1 (insecure) prefers no order, but is ok to give a key cheaply; anticipates tech advances making unlocking slow in st. 2.
- FBI needs a key, and especially in state 2 (secure)



## Example 1: Implementation for Apple and FBI?

Naive mechanism is not incentive compatible.
The Judge asks the public to report state. Apple always will report state 2 , and FBI - state $1 . \rightarrow$ No truthful revelation of the state.
Note: FBI announced obtaining the key from own source: Cellebrite (Sun's subsidiary). But FBI eq. action is to report having a key in all states.

| Table 1 |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| st. 1 |  | st. 2 |  |  |
| Apple | FBI | Apple | FBI |  |
| G | N | N | O |  |
| O | O | G | G |  |
| C | C | C | C |  |
| N | G | O | N |  |
| $f(1)=0$ | $f(2)=G$ |  |  |  |



Incentive compatible mechanism
Could the Judge (mech. designer) request Apple \& FBI to participate in the mechanism?


The mechanism implements social choice rule in Nash equilibrium.

## Example 2: as Apple vs FBI

Then, social choice rule is not implementable

Not implementable: monotonicity violated

| emple 1) |  |  |  |
| :---: | :---: | :---: | :---: |
| state 1 |  | state 2 |  |
|  |  |  |  |
| G | N | N | O |
| 0 | 0 | G | G |
| c | c | c | c |
|  | 6 |  | N |
| $f(1)=0 \quad f(2)=6$ |  |  |  |
| Table 4 (example 2) |  |  |  |
| st. 1 |  | st. 2 |  |
| Apple | FBI | Apple |  |
| G | N | G | N |
| 0 | 0 | 0 | 0 |
| C | C | N | C |
| N | G | C | G |
| $f(1)$ | $=0$ | $f(2)$ | $=N$ |

Let FBI vs Apple dispute be identical to example 2

| FBI vs Apple dispute as example 2 |  |  |
| :---: | :---: | :---: |
| g | Good to go (Apple wins) | G |
| o | Order to Apple (unlocking is cheap \& fast) | O |
| C | Order to Apple (has a key; Can unlock) | C |
| n | Order to Apple (No key; unlocking is pricey \& slow) | N |
| Social choice rule: $f(1)=O$, and $f(2)=N$ |  |  |

## Maskin on Implementation: when is it possible

When market is the best [aka when Libertarian doctrine is correct]
I.e., when implementation via market is possible?

Hammond [1979], Jordan [1982] Maskin [2008] formulates the conditions
If these conditions are violated
$\Leftrightarrow$
Mechanisms improving market allocation are generally possible.
FORCES task $=$ design the mechanisms to improve market allocation.
Q: Is Coase Theorem (mostly) correct after all !?
Answer: more or less, if certain (strict) conditions imposed.
We will cover the conditions later

## Afterthoughts and plans

The weakest link in the problems of robust mechanism design?

- Humans

Standard human preferences revisited interesting $\succ$ informed $\succ$ reasonable $\succ$ lucid s. Illing, Feb. 1 2016. .... calculated con job.

As far as an individual human is concerned

- Societal efficiency $\neq$ individual objective
- So called market failures are frequent

The challenge
Design robust institutions (mechanisms) with objective to min

- conflicts between the society and individuals?
- inefficiencies?
- occurrences of market failures?


## Myerson [2008] Game 1 Moral Hazard

Extensive form game with hidden information: $P \& A$

| $K$ | capital |
| :--- | :--- |
| $R$ | revenue |
| $p_{G}$ | success prob. if $A_{G}$ |
| $p_{B}$ | success prob. if $A_{B}$ |
| $B$ | $A^{\prime}$ s hidden benefit |
| $w$ | $A^{\prime}$ s wage if success |
| $A$ | $A^{\prime}$ 's collateral |

Numerical example
K 100
R 240
$p_{G} \quad 1 / 2$
$p_{B} \quad 1 / 4$
B 30


$$
\begin{aligned}
\Pi^{P} & =p_{G}(R-w)+\left(1-p_{G}\right) A-K \\
\Pi^{A} & =\left\{\begin{array}{cc}
p_{G} w-\left(1-p_{G}\right) A & \text { if } A_{G} \\
B+p_{B} w-\left(1-p_{B}\right) A & \text { if } A_{B}
\end{array}\right.
\end{aligned}
$$

$$
p_{B} R+B<K<p_{G} R, \quad A<K \text { [makes Game interesting] }
$$

## Myerson Game 1 MH: Solution

A's IC and IR constraints are:

$$
\begin{gathered}
p_{G} w-\left(1-p_{G}\right) A \geq p_{B} w-\left(1-p_{B}\right) A+B \quad[I C] \\
p_{G} w-\left(1-p_{G}\right) A \geqslant 0 \quad[I R]
\end{gathered}
$$

then, in eq.

$$
w^{*}=w_{I C}^{\min }=\frac{B}{\left[p_{G}-p_{B}\right]}-A>B \frac{\left[1-p_{G}\right]}{\left[p_{G}-p_{B}\right]}=w_{I R}^{\min },
$$

A expects positive rent $\Pi^{A}>0$ if

$$
\begin{gathered}
\Pi^{A}=p_{G} w^{*}-\left(1-p_{G}\right) A=p_{G}\left\{\frac{B}{\left[p_{G}-p_{B}\right]}-A\right\}-\left(1-p_{G}\right) A \\
A<p_{G} \frac{B}{\left[p_{G}-p_{B}\right]} \Leftrightarrow \Pi^{A}>0 .
\end{gathered}
$$

## Myerson Game 1 MH: Discussion

| Numerical example |  |
| :---: | :--- |
| $K$ | 100 |
| $R$ | 240 |
| $p_{G}$ | $1 / 2$ |
| $p_{B}$ | $1 / 4$ |
| $B$ | 30 |

$$
A<p_{G} \frac{B}{\left[p_{G}-p_{B}\right]} \Leftrightarrow \Pi^{A}>0
$$

If $A^{\prime}$ 's assets are below 60 , his has rents: $\Pi^{A}>0$
Q: Will A be truthful if $A=65$ ? Or some $A \in[60,100]$ ?
A: No, if A could, he will pretend that $A<60$.
$P$ contracts with A only if expects non-negative profit $\Pi^{P}$

$$
\begin{gathered}
\Pi^{P}=p_{G} R-\frac{p_{G} B}{\left[p_{G}-p_{B}\right]}+A-K=\left\{p_{G} R-K\right\}-\left\{\frac{B p_{G}}{\left[p_{G}-p_{B}\right]}-A\right\} \\
\Pi^{P}>0 \Leftrightarrow A>\frac{B p_{G}}{\left[p_{G}-p_{B}\right]}-\left\{p_{G} R-K\right\}
\end{gathered}
$$

## Myerson Game 1 MH : implementation

In our numerical example:

$$
\Pi^{P} \leq 0 \Leftrightarrow A \leq \frac{B p_{G}}{\left[p_{G}-p_{B}\right]}-\left\{p_{G} R-K\right\} \Leftrightarrow A \leq 40
$$

With collateral below 40, mechanism fails: project is non-viable. To sum:

$$
\begin{gathered}
\Pi^{P}=0 \quad \text { and } \quad \Pi^{A}=0, \quad \text { if } \quad A \in[0,40) \\
\Pi^{P}=A-40 \quad \text { and } \quad \Pi^{A}=60-A, \quad \text { if } \quad A \in[40,60] \\
\Pi^{P}=20 \quad \text { and } \quad \Pi^{A}=0, \quad \text { if } A \in[60,100)
\end{gathered}
$$

Next - another instrument: punishing A for failures.

## Myerson Game 2

## MH with collateral and punishment

$K$ capital
$R$ revenue
$p_{G}$ success prob. if $A_{G}$
$p_{B}$ success prob. if $A_{B}$
$B \quad A$ 's hidden benefit
w A's wage if success
A A's collateral
z A's punishment

$$
\begin{gathered}
\Pi^{P}=p_{G}(R-w)+\left(1-p_{G}\right) A-K \\
\Pi^{A}=\left\{\begin{array}{cc}
p_{G} w-\left(1-p_{G}\right)(A+z) & \text { if } A_{G} \\
B+p_{B} w-\left(1-p_{B}\right)(A+z) & \text { if } A_{B}
\end{array}\right. \\
p_{B} R+B<K<p_{G} R, \quad A<K \text { [makes Game interesting] }
\end{gathered}
$$

## Myerson Game 2 MH: Solution

$P$ have to chooses $w$ and $z$ to maximize his objective $\Pi^{P}$

$$
\Pi^{P}=p_{G}(R-w)+\left(1-p_{G}\right) A-K, \text { s.t. } w \geq A \text { and } z \geq 0
$$

constrained by [IC] and [IR]:

$$
\begin{gathered}
p_{G} w-\left(1-p_{G}\right)(A+z) \geq p_{B} w-\left(1-p_{B}\right)(A+z)+B \quad[I C] \\
p_{G} w-\left(1-p_{G}\right)(A+z) \geqslant 0 \quad[I R]
\end{gathered}
$$

Similar to Game 1:

$$
w^{*}=w_{I C}^{\min }=\frac{B}{\left[p_{G}-p_{B}\right]}-(A+z) \geq w_{I R}=\frac{\left(1-p_{G}\right)}{p_{G}}(A+z)
$$

## Myerson Game 2 MH: Results

$$
\begin{gathered}
\Pi^{A}=p_{G} w-\left(1-p_{G}\right)(A+z)=p_{G}\left\{\frac{B}{\left[p_{G}-p_{B}\right]}-(A+z)\right\}-\left(1-p_{G}\right)(A+z) \\
\Pi^{A} \geq 0 \Longleftrightarrow z+A<\frac{p_{G} B}{\left(p_{G}-p_{B}\right)}=60,
\end{gathered}
$$

To keep $\Pi^{A}=0, P$ will set

$$
z=\frac{p_{G} B}{\left(p_{G}-p_{B}\right)}-A=60-A
$$

which gives eq. wage $w^{*}$

$$
w^{*}=w_{I C}^{\min }=\frac{\left(1-p_{G}\right) B}{\left(p_{G}-p_{B}\right)}
$$

## Myerson Game 2 MH: Discussion

In Game 2, $P$ profit $\Pi^{P}$ becomes non-positive if

$$
\begin{gathered}
\Pi^{P}=p_{G}\left\{R-\frac{\left(1-p_{G}\right) B}{\left(p_{G}-p_{B}\right)}\right\}+\left(1-p_{G}\right) A-K<0 \Leftrightarrow \\
A<\frac{p_{G} B}{\left(p_{G}-p_{B}\right)}-\frac{\left\{p_{G} R-K\right\}}{\left(1-p_{G}\right)}=60-40=20 .
\end{gathered}
$$

When A's collateral falls below 20, $P$ 's profit becomes negative.
Next, let $A=0$ (egalitarian society). Then, could implement only if IR is violated. Then, eq. is driven by the threat of punishment only:

$$
w^{*}=0 \text { and } z^{*}=120
$$

## Myerson Game 3 Adverse Selection with production risk

## $K$ capital

$R$ revenue
$p_{G} \quad$ success prob. if type $A_{G}$
$p_{B}$ success prob. if type $A_{B}$
$g_{G}$ project prob. if $A_{G}$ reported
$g_{B}$ project prob. if $A_{B}$ reported
w A's wage if success

$\alpha$ prob. of good type $A_{G}$
$\Pi^{P}=\alpha q_{G}\left[p_{G}\left(R-w_{G}\right)+\left(1-p_{G}\right) A-K\right]+(1-\alpha) q_{B}\left[p_{B}\left(R-w_{B}\right)+\left(1-p_{B}\right) A-K\right]$

$$
\Pi^{A_{G}}= \begin{cases}p_{G} W_{G}-\left(1-p_{G}\right) A & \text { if } A_{G} \text { reports } A_{G} \\ p_{G} W_{B}-\left(1-p_{G}\right) A & \text { if } A_{G} \text { reports } A_{B}\end{cases}
$$

$$
\Pi^{A_{B}}= \begin{cases}p_{B} w_{G}-\left(1-p_{B}\right) A & \text { if } A_{B} \text { reports } A_{G} \\ p_{B} w_{B}-\left(1-p_{B}\right) A & \text { if } A_{B} \text { reports } A_{B}\end{cases}
$$

$p_{B} R+B<K<p_{G} R, \quad A<K$ [makes Game interesting]

## Myerson Game 3 AS: Solution

$P$ choice variables are $\left(q_{G}, w_{G}, q_{B}, w_{B}\right)$ to max his payoff $\Pi^{P}$
$\alpha q_{G}\left[p_{G}\left(R-w_{G}\right)+\left(1-p_{G}\right) A-K\right]+(1-\alpha) q_{B}\left[p_{B}\left(R-w_{B}\right)+\left(1-p_{B}\right) A-K\right]$
Resource constraints

$$
w_{G} \geq-A \text { and } w_{B} \geq-A \text { and } q_{G}, q_{B} \in[0,1] .
$$

Participation constraints:

$$
p_{G} w_{G}-\left(1-p_{G}\right) A \geq 0 \text { and } p_{B} w_{B}-\left(1-p_{B}\right) A \geq 0
$$

Information incentive constraints (truth telling)

$$
\begin{aligned}
q_{G}\left[p_{G} w_{G}-\left(1-p_{G}\right) A\right] \geq q_{B}\left[p_{G} w_{G}-\left(1-p_{G}\right) A\right] \\
q_{B}\left[p_{B} w_{B}-\left(1-p_{B}\right) A\right] \geq q_{G}\left[p_{B} w_{B}-\left(1-p_{B}\right) A\right]
\end{aligned}
$$

In optimum [even with no collateral $(A=0)$ ]

$$
q_{G}=1, q_{B}=0 \text { and } w_{G}=w_{B}=0 .
$$

## Myerson's interpretation of results

Societal property rights (Collectivism / Public goods properties)

- Help with adverse selection facilitate honest communication
- Hurt with moral hazard*** (CPS)

Individual property rights

- Help with moral hazard [for some parameter ranges]

■ Hurt with adverse selection

Comments: complex trade-offs
No perfect remedy for info asymmetry. Instruments:

- info rents and punishments [sticks and carrots]
- risk reallocation


## Implementation: yesterday, today and tomorrow

## Summary

root Arrow = impossibility theorem
■ Maskin = yes, we can implement (when?) [in search for possibilities: necessary \& sufficient conditions?]
■ Hurwicz = If IC and IR then no Efficiency have to incentivize to share info \& follow social (choice) rule
■ Myerson = designing the mechanisms [to be continued next lecture]

Literature $\infty$ unbounded and increasing

- The interest in mechanism design and implementation continues


## Implementation literature: yesterday and today

## Literature $\infty$

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## Mechanism design literature: yesterday and today

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## Afterthoughts and plans

Standard human preferences revisited
Humans are the weakest link in the problems of robust mechanism design. interesting $\succ$ informed $\succ$ reasonable $\succ$ lucid s. Illing, Feb. 1 2016. .... calculated con job.

## Next Lecture

Volkswagen emissions scandal from agency theory perspective.
We will demonstrate how P \& A framework applies to complex engineering problems.

## WV scandal from P\&A perspective

The gap between ex ante (required) and ex post (real)

## the gap between RULES AND REALITY

NOx emissions have been dropping in Europe. But the difference between the legal limits (dark shading) - which auto companies comply with in their lab tests - and the actual on-road emissions (light shading) has persisted.


Nested P\&A settings

- $\mathrm{P} 1=\mathrm{R}(E P A, C A R B)$
- $\mathrm{A} 1=\mathrm{VW}$ (hidden info)
- P2 = VW (top) management
- As = VW engineers (from separate divisions):
- engine electronics

■ diesel motor development

- motor testing


