Understanding the Impact of Parking on Urban Mobility via Routing Games on Queue–Flow Networks

Daniel Calderone, Eric Mazumdar, Lillian J. Ratliff, S. Shankar Sastry

Abstract— We derive a new routing game model for urban centers that takes into account parking-related traffic along with all other traffic. In particular, we combine a queuing game model for on-street parking with a classical routing game framework and consider two types of populations: parking and throughput. While the throughput traffic plays the standard routing game by selecting a route from their origin to their destination, the parking traffic selects a parking zone (block-face) in addition to their route. We show that the routing game on a queue–flow network of this type is a potential game. We construct practical examples by using subsets of the Seattle downtown area to illustrate the usefulness of this novel modeling paradigm. We verify that parking–related traffic can have a large impact on the routing choices of the throughput drivers as well as the overall congestion and social cost. By varying the cost of parking in different parking zones, we demonstrate that parking–related traffic can be adjusted to satisfy a particular objective.

I. INTRODUCTION

Transportation systems are the backbone of cities since they support a large number of crucial interactions be they economic transaction, resource distribution, or emergency response in nature. Due to the *great urban sprawl* [1]–[3], transportation infrastructure in cities is being taxed to its limits. As a result, cities incur large economic costs from transport-related inefficiencies [3], [4]. There is an urgent need for urban municipalities to find new and economical ways of improving urban mobility that do not require a complete overhaul of the existing system.

Congestion is a particularly pressing transportation-related inefficiency that urban areas face. Across the U.S., traffic congestion is responsible for nearly 4 billion gallons of wasted fuel a year and nearly 7 billion extra hours of travel time [5]. Beyond economic costs, congestion has adverse effects on public health, the environment, and general quality of life in cities [6], [7].

The most common way urban municipalities address this problem is through the use of congestion charges. These have been implemented in many of the states and cities within the U.S. as well as in international cities such as London, Singapore, and Stockholm among others with varying levels

D. Calderone, E. Mazumdar, and S. S. Sastry are in the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, Berkeley, CA, 94707, USA {danjc,mazumdar,sastry}@eecs.berkeley.edu
L. Ratliff is in the Department of Electrical Engineering at the University of Washington, Seattle, WA, USA and the Department of Electrical Engineering and Computer Sciences at the University of California, Berkeley, Berkeley, CA, USA {ratliffl}@uw.edu

of success [8]–[10]. Congestion charges have long been touted by economists to be a successful, if not *the* successful, mechanism for decreasing congestion, and yet the theory has confronted stiff opposition from the public, with criticism that they disproportionately target the poor, push traffic towards more residential neighborhoods, and have a negative impact on local economies by incentivising people to stay away from urban areas [11], [12]. The idea behind congestion charges is to make each driver pay the marginal social cost of their trip. However, this is not easily computable and thus, a significant obstacle to thir implementation is determining the right cost to charge drivers.

A second, and perhaps less intuitively obvious, approach to decreasing congestion in urban centers is through parking pricing. A significant amount—up to 40% in U.S. cities— of all arterial traffic in urban areas stems from drivers looking for parking [13]. This approach has the added benefit that it minimizes a significant source of congestion and can be implemented using largely pre-existing infrastructure. Many pilot programs to test pricing schemes have been implemented (see, e.g., [14]–[16]).

We aim to develop a novel theoretical framework for understanding the effect of parking-related driving behaviors such as *circling while looking for parking* on overall congestion and route choice. In particular, we combine the classical routing game [17]–[20] on a network with a queueing model of parking. Given a road network topology, we allow for multiple populations of drivers, some populations being designated as potential parkers, to select a route from their origin to their destination by maximizing their own utility. Regular throughput traffic populations travel from their origin to their destination while potential parker populations try to find on-street parking near the *attraction* that is their destination. Each attraction has several *parking areas*—collections of blockfaces—associated to it.

We extend the routing game framework go beyond the standard routing choices and account for the additional decision that the parking population must make with regard to selecting the blockface along which they park. We show that this modified routing game is a potential game [21]. We demonstrate through several examples—including realistic urban transportation network topologies—the impact parking-related congestion and route choice has on the over all congestion. We analyze the user-selected equilibrium for the game and present some insights into how parking populations respond to the cost of parking and influence congestion on the links in the queue–flow network. In addition, we compare the user-selected equilibrium (*Wardrop*) induced

welfare to the socially optimal welfare.

The paper is organized as follows. In Section II, we present the queuing model for on-street parking. The queue model is used to inform the potential function of the routing game on queue–flow networks which is introduced in Section III. In Section IV we present simulation results for using real-world networks taken from the downtown Seattle area. We conclude with a discussion of future work in Section V.

II. OBSERVABLE QUEUING GAME

We aim to combine a queuing model of on-street parking, which is inherently discrete, with the routing game so that we can assess the effect of parking-related congestion due to potential parkers circling looking for an available spot on the overall congestion which includes congestion due to throughput traffic.

To motivate the routing game model on queue–flow networks, we consider a simple observable queue game in which arriving customers observe the queue length and choose to join by maximizing their utility which is a function of the reward for having parked, the cost of additional wait time due to circling, the cost of parking itself. We use this queue game to inform the additional cost we will add to the routing game to account for populations of potential parkers.

Abstractly, the queue length represents the amount of parking related congestion on a collection of roadways that make up a parking area which is a collection of blockfaces. The *nominal expected utility* of an arriving customer to the system with queue length k is given by

$$\alpha_k = R - \frac{C_w(k+1)}{\mu c}. \tag{1}$$

The *total expected utility* for parking is

$$\beta_k = R - \frac{C_w(k+1)}{\mu c} - \frac{C_p}{\mu}. \tag{2}$$

where the cost of parking is C_p per unit time. On the other hand, if the customer were to balk, the total expected utility is zero.

The optimal strategy for a customer arriving to a queue with length k and then deciding whether or not to join by maximizing their expected utility is to join the queue if and only if $\beta_k \geq 0$. In this case, if the decision to join the queue depends on the customer optimizing their individual utility, then the system will be a M/M/c/n_b queue where

$$n_b = \left\lfloor \frac{(R\mu - C_p)c}{C_w} \right\rfloor \tag{3}$$

is the *balking level*, i.e. it is the maximum queue length after which arriving customers decide not to drive to a parking area and instead, select some outside option such as taking the bus. The balking level is determined by solving $\alpha_{n_b-1} \geq 0 > \alpha_{n_b}$ (respectively, $\beta_{n_b-1} \geq 0 > \beta_{n_b}$).

III. FLOW NETWORK MODEL

In this section, we present the routing game where some users select their parking destination in addition to their route. To combine the queuing model of the previous section with this framework, we relax the discrete nature of the queue

length and consider the incremental expected utility a mass of population experiences if they choose park, i.e. join the queue. In particular, we define the marginal cost of one driver entering a parking area and thus, one of the queues. Since the routing game is inherently static, we consider the case when the balking level is less than the maximum congestion reached in the queue–flow network system (meaning, we exclude the outside option for the time being).

A graph is a natural abstraction of an urban area where aterials are modeled as edges and intersections, origins, and destinations are modeled as nodes. Drivers, each having an origin and destination, select a path among the edges that leads them from their origin to their destination and they do so by finding the path that has the least cost to traverse. The *routing game* is one method of determining the equilibrium given this game framework and is formulated as follows.

To formulate the routing game we consider a directed graph given by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of verticies or nodes corresponding to intersections, origins, or destinations and $\mathcal{E}$ is the set of directed edges corresponding to roads joining the nodes. Edges $e \in \mathcal{E}$ take the form $(i, j) \in \mathcal{V} \times \mathcal{V}$. Along with the sets of nodes and edges, we define a set of *attractions* $\mathcal{A}$ that drivers are traveling to as well as a set of *parking areas* $\mathcal{P}$.

A parking area $p \in \mathcal{P}$ consists of a set of nodes denoted $\mathcal{N}^p \subset \mathcal{V}$ and the edges that connect them denoted $\mathcal{E}^p \subset \mathcal{E}$. We will define indicator vectors for each of these sets respectively $\mathbf{N}^p \in \mathbb{R}^{|\mathcal{V}|}$ and $\mathbf{E}^p \in \mathbb{R}^{|\mathcal{E}|}$:

$$(\mathbf{N}^p)_i = \begin{cases} 1, & \text{if node } i \text{ is in parking area } p \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$(\mathbf{E}^p)_e = \begin{cases} 1, & \text{if edge } e \text{ is in parking area } p \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

An attraction $a \in \mathcal{A}$ consists of a set of parking areas $\mathcal{P}^a$ that drivers traveling to that attraction can choose from. An individual population starts at a single origin node $o \in \mathcal{V}$ and travels to a specific attraction $a \in \mathcal{A}$. We will denote the population associated with this *origin-attraction* pair as s_o^a. The size of these populations are given *a priori*. Furthermore, traffic that parks in area p will travel through the network to one of the nodes in $\mathcal{N}^p$ and will then become *circling traffic* which is added to the edges in $\mathcal{E}^p$. We note that several attractions may share parking areas.

There are two types of populations. The first type are *throughput drivers* and these populations are associated to an attraction that has only one parking area associated to it and this parking area consists of a single node. Hence, they only choose their route in the routing game. The second type are *parkers* and, while their attraction is fixed, in addition to selecting their route, they choose their parking area and the destination node within the parking area.

We use $d \in \mathcal{N}^p$ to denote a node (destination) drivers in a parking–type population travel to in order to enter that parking area. Thus, s_o^{ap} will be the portion of population traveling from origin o to attraction a and parking in area

Fig. 1. Illustration of $\mathcal{A}$ and $\mathcal{P}$ and parking area nodes.
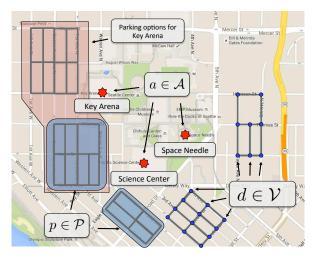
p and s_{od}^{ap} will be the subset of this population that enters parking area p through node d. These subpopulations satisfy the following relationships:

$$\left.\begin{array}{rl} s_o^a & = \sum_{p\in\mathcal{P}} s_o^{ap} \\ s_o^{ap} & = \sum_{d\in\mathcal{N}^p} s_{od}^{ap} \\ s^{ap} & = \sum_{o\in\mathcal{V}} \sum_{d\in\mathcal{N}^p} s_{od}^{ap} \end{array}\right\} \quad (6)$$

As an example, we illustrate what the sets $\mathcal{A}$ and $\mathcal{P}$ along with the entry nodes into each parking represent in Figure 1. We note that we can simply include all the nodes in a parking area as possible destination nodes. Since the cost of traveling to an internal node before beginning to circle will always be higher than simply traveling to a node on the edge of the parking area since no mass will travel to these nodes at equilibrium.

The total amount of traffic that reaches a given parking area coming from any origin and traveling to any attraction (if multiple populations are using that parking area) is

$$s^p = \sum_o \sum_a s_o^{ap} = \sum_o \sum_a \sum_d s_{od}^{ap}. \quad (7)$$

We use the notation $\mathbf{s} \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{A}|\times|\mathcal{P}|\times|\mathcal{V}|}$ as a short hand for the vector of all populations.

The contribution to traffic flow on edge e from each individual population s_{od}^{ap} is given by $(\mathbf{x}_{od}^{ap})_e$. We denote the vector of all these edge flows as $\mathbf{x}_{od}^{ap} \in \mathbb{R}^{|\mathcal{E}|}$. The total congestion vector is given by

$$\mathbf{x} = \sum_p \left[\sum_a \sum_o \sum_d \mathbf{x}_{od}^{ap}\right] + \alpha^p \mathbf{E}^p s^p \quad (8)$$

$$= \sum_p \sum_a \sum_o \sum_d \left[\mathbf{x}_{od}^{ap} + \alpha^p \mathbf{E}^p s_{od}^{ap}\right] \quad (9)$$

with congestion on an individual edge e denoted $\mathbf{x}_e$. The first portion of the above sum is the contribution from traffic flow in transit from origin to destination. The second portion is the parking traffic that circles in a given parking area after it arrives while waiting to park. The term α_p defines the fraction of traffic that is circling on a given link in parking

area p at any time. We assume that circling traffic will spread out uniformly over the parking area and thus we let

$$\alpha_p = \frac{1}{|\mathcal{E}^p|}. \quad (10)$$

We define the *node–edge incidence* matrix $A = (a_{ib}) \in \{-1, 0, 1\}^{|\mathcal{V}|\times|\mathcal{E}|}$ by

$$a_{ib} = \begin{cases} 1, & \text{if } i \text{ is the origin of edge } b \\ -1, & \text{if } i \text{ is the destination of } b \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

for all $b \in \mathcal{E}$ and for all $i \in \mathcal{V}$. The the flow vector for each origin-attraction pair that chooses a specific parking area and entry node satisfies

$$A\mathbf{x}_{od}^{ap} = \mathbf{s}_{od}^{ap}, \qquad \mathbf{x}_{od}^{ap} \geq 0 \quad \forall o, d, a, p \quad (12)$$

where we define the demand vector $\mathbf{s}_{od}^{ap}$ as

$$(\mathbf{s}_{od}^{ap})_i = \begin{cases} s_{od}^{ap}, & \text{if } i = o \\ -s_{od}^{ap}, & \text{if } i = d \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

We define the potential function

$$P(\mathbf{x}, \mathbf{s}) = \sum_e \int_0^{\mathbf{x}_e} \tau l_e(u) \, du + \sum_p \int_0^{s^p} C^p(u) \, du + \\ \sum_{a,p} \int_0^{s^{ap}} -R^{ap} \, du \quad (14)$$

where l_e is a standard edge latency function that is assumed to be strictly increasing, τ (in units of money/time) is a parameter that represents the population's time money tradeoff, R^{ap} is the reward for parking in area p for drivers traveling to attraction a, and $C^p(u)$ is the cost of parking derived from the queuing model defined by

$$C^p(u) = \frac{C_{\mathrm{p}}^p}{\mu^p} + \frac{C_w^p}{\mu^p c^p} u \quad (15)$$

with u being a variable representing the mass a small amount of population contributes to the congestion in parking area p. Conceptually, we can think of u as adding a driver to the queue in the discrete formulation of the queuing game and (15) as the marginal utility. Moreover, having separate rewards for parking-attraction pairs allows us to model parking areas shared between several attractions to be convenient for some and inconvenient for others. Note that since $C_{\mathrm{p}}^p/(\mu^p c^p)$ is positive, $C^p(u)$ is strictly increasing. As a result, the potential function $P(\mathbf{x}, \mathbf{s})$ is strictly convex.

The optimization problem that is used to find the equilibrium for the parking-routing game is given as follows:

$$\min_{\mathbf{x}, \mathbf{s}} \quad P(\mathbf{x}, \mathbf{s}) \quad (16a)$$

$$\text{s.t.} \quad A\mathbf{x}_{od}^{ap} = \mathbf{s}_{od}^{ap}, \quad \forall \, o, d, a, p \quad (16b)$$

$$s_o^a = \sum_p \sum_d s_{od}^{ap}, \quad \forall \, o, a \quad (16c)$$

$$\mathbf{x}_{od}^{ap} \geq 0, \quad \forall \, o, d, a, p \quad (16d)$$

$$s_{od}^{ap} \geq 0, \quad \forall \, o, d, a, p \quad (16e)$$

$$\mathbf{x} = \sum_p \sum_a \sum_o \sum_d \left[\mathbf{x}_{od}^{ap} + \alpha_p \mathbf{E}^p s_{od}^{ap}\right] \quad (16f)$$

$$s^p = \sum_o \sum_a \sum_d s_{od}^{ap}, \quad \forall \, p \quad (16g)$$

Note that s_o^a is given *a priori*. Since $P(\mathbf{x}, \mathbf{s})$ is strictly convex, (16) has a unique global minimizer.

Let $\mathcal{R}_{od}$ be the set of all routes from an origin node o to destination node d. Let $r \in \mathcal{R}_{od}$ be a specific route and let the set of edges that compose r be denoted $(o \to d)_r \subset \mathcal{E}$.

For each population associated with an origin-attraction pair (o, a), we define a set of strategies

$$U_o^a = \{(p, d, r) \mid p \in \mathcal{P}^a, \ d \in \mathcal{N}^p, \ r \in \mathcal{R}_o^d\}. \quad (17)$$

We define the cost associated with a particular strategy $(u_o^a)_i = (p_i, d_i, r_i) \in U_o^a$ to be

$$\ell_o^a\left((u_o^a)_i\right) = \underbrace{\sum_{e \in (o \to d_i)_{r_i}} \tau l_e(\mathbf{x}_e)}_{\text{travel latency}} + \underbrace{\alpha_{p_i} \sum_{e \in \mathcal{E}^{p_i}} \tau l_e(\mathbf{x}_e)}_{\text{circling latency}}$$
$$\underbrace{- (R^{ap_i} - C^{p_i}(s^{p_i}))}_{\text{parking cost}} \quad (18)$$

and we use $(s_{od_i}^{ap_i})^{r_i}$ to denote the population that chooses this strategy $(u_o^a)_i$.

We say $(\mathbf{x}, \mathbf{s})$ is feasible if it satisfies the constraints of (16). We say $(\mathbf{x}, \mathbf{s})$ is a *Wardrop Equilibrium* of the parking-routing game if and only if it is feasible and for any (o, a) pair, any two strategies $(u_o^a)_i, (u_o^a)_j \in U_o^a$ satisfy

$$\ell_o^a((u_o^a)_i) \leq \ell_o^a((u_o^a)_j) \quad (19)$$

if $(s_{od_i}^{ap_i})^{r_i} > 0$ and $(s_{od_j}^{ap_j})^{r_j} = 0$ and

$$\ell_o^a((u_o^a)_i) = \ell_o^a((u_o^a)_j) \quad (20)$$

if both $(s_{od_i}^{ap_i})^{r_i} > 0$ and $(s_{od_j}^{ap_j})^{r_j} > 0$.

Intuitively this says that for every origin-attraction pair, no infinitesimal mass of drivers can improve their cost by switching to another parking area, another destination node, or another route.

Theorem 1: The optimizer of (16) satisfies the Wardrop Equilibrium Condition for the parking-routing game.

Proof: In order to write the Lagrangian of (16) we introduce the following Lagrange multipliers:

$$\pi_{od}^{ap} \in \mathbb{R}^{|\mathcal{V}|}, \qquad \forall\, o, d, a, p \qquad \text{for Equation (16b)} \quad (21)$$
$$\kappa_o^a \in \mathbb{R}, \qquad\qquad \forall\, o, a \qquad \text{for Equation (16c)} \quad (22)$$
$$\xi_{od}^{ap} \in \mathbb{R}_+^{|\mathcal{E}|}, \qquad \forall\, o, d, a, p \qquad \text{for Equation (16d)} \quad (23)$$
$$\nu_{od}^{ap} \in \mathbb{R}_+, \qquad \forall\, o, d, a, p \qquad \text{for Equation (16e)} \quad (24)$$

We use π, κ, ξ, and ν as short hands for each of the sets of Lagrange multipliers. The Lagrangian is given by

$$\mathcal{L}(\mathbf{x}, \mathbf{s}, \pi, \kappa, \xi, \nu) = P(\mathbf{x}, \mathbf{s}) + \sum_{o,d,a,p} \left[(\pi_{od}^{ap})^T \right.$$
$$\left. \cdot (A\mathbf{x}_{od}^{ap} - \mathbf{s}_{od}^{ap}) + \kappa_o^a(s_{od}^{ap} - s_o^a) - (\xi_{od}^{ap})^T \mathbf{x}_{od}^{ap} - \nu_{od}^{ap} s_{od}^{ap} \right]$$
$$\quad (25)$$

The optimality condition $\frac{\partial \mathcal{L}}{\partial (\mathbf{x}_{od}^{ap})_e} = 0$ where $e = (i, j)$ yields

$$l_e(\mathbf{x}_e) + (\pi_{od}^{ap})_i - (\pi_{od}^{ap})_j = (\xi_{od}^{ap})_e \quad (26)$$

Note that $(\xi_{od}^{ap})_e \geq 0$ with equality achieved only when $(\mathbf{x}_{od}^{ap})_e > 0$ by complimentary slackness.

Similarly, the optimality condition $\frac{\partial \mathcal{L}}{s_{od}^{ap}} = 0$ yields

$$\alpha_p \sum_{e \in \mathcal{E}^p} \tau l_e(\mathbf{x}_e) - R^{ap} + C^p(s^p)$$
$$- (\pi_{od}^{ap})_o + (\pi_{od}^{ap})_d + \kappa_o^a = \nu_{od}^{ap} \quad (27)$$

where $\nu_{od}^{ap} \geq 0$ with equality achieved when $s_{od}^{ap} > 0$ by complimentary slackness.

For any (o, a) pair, consider any strategy $(u_o^a)_m \in U_o^a$. Summing Equation (26) along path r_m yields

$$\sum_{e \in (o \to d_m)_{r_m}} l_e(\mathbf{x}_e) + (\pi_{od_m}^{ap_m})_o - (\pi_{od_m}^{ap_m})_{d_m}$$
$$= \sum_{e \in (o \to d_m)_{r_m}} (\xi_{od_m}^{ap_m})_e \quad (28)$$

Summing Equations (28) and (27) yields

$$\sum_{e \in (o \to d_m)_{r_m}} l_e(\mathbf{x}_e)$$
$$+ \alpha_{p_m} \sum_{e \in \mathcal{E}^{p_m}} \tau l_e(\mathbf{x}_e) - R^{ap_m} + C^{p_m}(s^{p_m}) + \kappa_o^a$$
$$= \nu_{od_m}^{ap_m} + \sum_{e \in (o \to d_m)_{r_m}} (\xi_{od_m}^{ap_m})_e \quad (29)$$

which is equivalent to

$$\ell_o^a\left((u_o^a)_m\right) + \kappa_o^a = \nu_{od_m}^{ap_m} + \sum_{e \in (o \to d_m)_{r_m}} (\xi_{od_m}^{ap_m})_e \quad (30)$$

For any two strategies $(u_o^a)_m, (u_o^a)_n \in U_o^a$ with positive mass we have

$$\nu_{od_m}^{ap_m} = \nu_{od_n}^{ap_n} = 0 \quad (31)$$

since $s_{od_m}^{ap_m} > 0$ and $s_{od_n}^{ap_n} > 0$. Also

$$\sum_{e \in (o \to d_m)_{r_m}} (\xi_{od_m}^{ap_m})_e = \sum_{e \in (o \to d_n)_{r_n}} (\xi_{od_n}^{ap_n})_e = 0 \quad (32)$$

since $(\mathbf{x}_{od_m}^{ap_m})_e > 0$ for all $e \in (o \to d_m)_{r_m}$ and $(\mathbf{x}_{od_n}^{ap_n})_e > 0$ for all $e \in (o \to d_n)_{r_n}$. It follows that

$$\ell_o^a\left((u_o^a)_m\right) = -\kappa_o^a = \ell_o^a\left((u_o^a)_n\right) \quad (33)$$

satisfying Condition (20). If a strategy $(u_o^a)_n \in U_o^a$ has mass 0, either no mass in population (o,a) chose parking area p_n and destination d_n, i.e. $s_{od_n}^{ap_n} = 0$ implying $\nu_{od_n}^{ap_n} \geq 0$; or no mass chose route r_n implying that there exists $e \in (o \to d_n)_{r_n}$ such that $(\mathbf{x}_{od_n}^{ap_n})_e = 0$ and thus $\xi_{od_n}^{ap_n})_e \geq 0$. Either way we have

$$\ell_o^a\left((u_o^a)_m\right) = -\kappa_o^a$$
$$= \ell_o^a\left((u_o^a)_n\right) - \underbrace{\nu_{od_n}^{ap_n} + \sum_{e \in (o \to d_n)_{r_n}} (\xi_{od_n}^{ap_n})_e}_{\geq 0} \quad (34)$$

which yields

$$\ell_o^a\left((u_o^a)_m\right) \leq \ell_o^a\left((u_o^a)_n\right) \quad (35)$$

satisfying Condition (19).

∎

We remark that we choose to formulate the routing game using the *edge formulation* as opposed to the *path formulation* because the path formulation requires that all paths be enumerated in the computation step and for large graphs and complex networks like those arising from urban arterials this may be time consuming. We remark that the
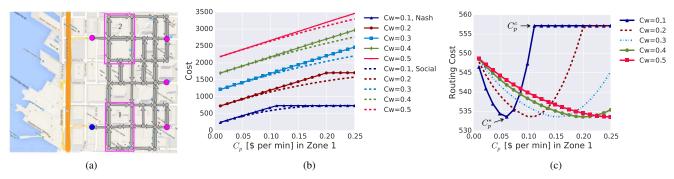
Fig. 2. Setup and results for Example 1 (SR-99). (a) Queue–flow network: Magenta nodes are parking population sources and the magenta boxes are the parking areas. Through traffic begins at every node and flows to the destination node shown in blue. (b) The social cost evaluated at the Nash equilibrium and the socially optimal solution as a function of the cost of parking C_p^1 in area 1 (with $C_\mathrm{p}^2 = 0.01$ fixed) for $C_w = C_w^1 = C_w^2 \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. (c) The portion of the social cost due to routing under the Nash equilibrium as a function of the cost of parking C_p^1 in parking area 1 (with $C_\mathrm{p}^2 = 0.01$ fixed) for $C_w \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. We point out for $C_w = 1$ the critical point C_p^c at which the routing demand is no longer flexible to increases in price and the price C_p^* that minimizes the routing costs (i.e. the latency experienced by all the populations).
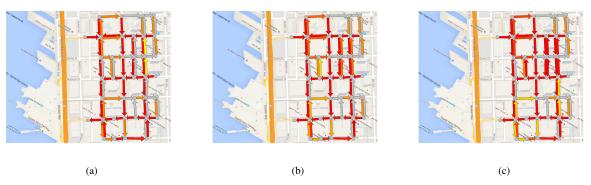

Fig. 3. All flow for Example 1 (SR-99) with $C_\mathrm{p}^2 = 0.01$ in area 2 and (a) $C_\mathrm{p}^1 = 0.01$, (b) $C_\mathrm{p}^1 = 0.124$, and (c) $C_\mathrm{p}^1 = 0.2$ in area 1. From left to right ((a) to (c)), we see that the traffic in the two parking zones shift from equally distributed between the two areas to largely being in area 2. On the spectrum from yellow to red, we show the flow intensity with red indicated a large amount of flow and yellow indicating less flow.

edge formulation is *not equivalent* to the path formulation in that the set of feasible flows for the edge formulation is a superset of the set of feasible flows for the path formulation. In particular, the set of feasible edge flows contains the commodity cycle flows which are not in the set of feasible path flows [22, Theorem 3.5]. However, these cycle flows will not be selected by the users since the cost along links is positive and therefore accumulates.

IV. RESULTS

We now present several rich examples that effectively illustrate the usefulness of the queue–flow modeling paradigm both for analysis of congestion in multi-use transportation networks as well as for design of system paramters.

First, we take care of some notational preliminaries. In the examples of this section, we measure flows on each edge ($\mathbf{x}_e$) in cars per unit time. We use linear latencies that were derived from the Bureau of Public Roads (BPR) *link performance function* which is given by

$$l_e^{\mathrm{BPR}}(\mathbf{x}_e) = t_e \left(1 + 0.15 \left(\tfrac{\mathbf{x}_e}{c_e}\right)^4\right) \quad (36)$$

where t_e is the free-flow travel time on link e (length/speed limit) and c_e is the capacity of link per unit time [23]. We

heuristically take c_e to be

$$c_e = \tfrac{50 \text{ cars}}{\mathrm{mi}} \times \binom{\text{speed}}{\text{limit}} \quad (37)$$

assuming cars are travelling in *free-flow* with approximately 50 cars per mile (approximately 100 feet per car). We chose the linear latency that agrees with this function at $\mathbf{x}_e = 0$ and $\mathbf{x}_e = 3c_e$ at free-flow and when traffic is moving 4 times slower:

$$l_e(\mathbf{x}_e) = t_e \left(1 + 4\tfrac{\mathbf{x}_e}{c_e}\right). \quad (38)$$

To demonstrate the usefulness of the routing game for queue–flow networks, we explore two examples using different regions in the Seattle downtown area and its arterials as the basis for the network topologies.

A. Example 1: SR-99

In the first example, we construct a queue–flow network using a portion of the Seattle downtown area near state route 99 (SR-99), a heavily traversed road. The example consists of two parking zones with parking traffic coming from a small number of nodes and one destination node for all throughput traffic—with origins at every node in the network—exiting onto SR-99. This, for instance, would be
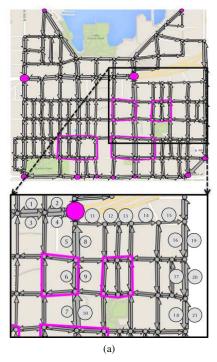
(a)

Fig. 4. Structure of Amazon area simulation. Magenta dots are traffic sources (with radius indicating size relative to other sources.) Parking areas are polyhedra. The lower figure shows location of the edges from Figure5.

consistent with constituents exiting the downtown area after work while others seek to find parking close to a nearby restaurant in the two parking areas.

In Figure 2a, we show the network with the exit node for all throughput traffic depicted in blue and the origin nodes for the parking populations and parking zones depicted in magenta. In Figures 2b and 2c we show the social cost under the user-selected (Wardrop) equilibrium and socially optimal strategy and the routing portion of the Wardrop-induced social cost, respectively. The Wardrop-induced cost is always higher than the socially optimal cost as expected. Both costs increase with $C_w = C_w^1 = C_w^2$ and with the cost of parking C_p^1. Interestingly, the Wardrop-induced cost plateaus when the price of parking reaches a value after which the selfish routing no longer changes. This indicates a critical point exists after which there is no flexibility in the demand for values of C_p^1 larger than the critical point. This critical point can be seen in Figure 2c for each value of C_w (we highlight the critical value C_p^c for $C_w = 0.1$); indeed, it is the cost of parking at which the routing cost becomes fixed for all larger values of C_p^1.

In addition, in Figure 2c, we see that for each value of C_w, the routing cost obtains a minimum for some value of C_p^1 (indicated for $C_w = 0.1$ by C_p^*). These points represent the optimal price of parking that a municipal service provider should charge if its objective is to minimize the latency experienced by the total population—note this objective might not align with minimizing social cost. Moreover, as the cost of waiting C_w increases, the price of parking that minimizes the routing cost becomes larger. We remark that

in Seattle, like many municipalities, there are regulatory constraints on the maximum value that can be charged per hour for on-street parking. This value tends to be around $7; hence, if the cost of waiting is too large, then it may not be possible to optimally design the price of parking to minimize the latency experienced by the total population. This suggests that understanding preferences over waiting (time spent circling or in traffic) should be better understood and perhaps, incorporated into regulatory policies that cap parking prices.

In Figure 3, we show the total traffic flow (parking plus throughput populations) for three different values of C_p^1. It can be seen that as C_p^1 increases, the flow shifts from being evenly distributed between the two parking areas to largely being in area 2 (the top area).

B. Example 2: Amazon Campus

The next example we explore is the effect of parking on congestion in the region around Amazon's headquarters. This area has seen increased congestion over the last half decade due to Amazon's presence. We hold the total throughput traffic fixed and increase the amount of parking traffic to simulate a potential rush hour scenario as Amazon employees are driving to work.

In Figure 4 we show the network graph along with a key section of the network which has interesting routing behaviors that emerge as the proportion of parking-related traffic increases relative to the volume of throughput traffic. All traffic enters through the magenta nodes. The through traffic travels (uniformly) to other magenta nodes and the parking traffic travels to a parking area indicated by the boxed magenta regions. The size of the magenta nodes indicates the relative magnitude of both the throughput and parking traffic coming from those nodes. For each parking region, we use the queuing parameters shown in the following table:

$R^p(\$)$	C_w^p ($/min)	μ^p (spots/min)	c^p (# spots)	C_p^p ($/min)	τ ($/hr)
100	0.1	1/120	50	0.01	30

We fix the total throughput traffic to be 75 cars/minute distributed among the source nodes according to their relative size and we vary the amount of parking traffic in the interval $[0, 300]$.

In Figure 5, we show the total flow, the throughput traffic flow, and the parking flow for key edges in the graph enumerated and depicted in Figure 4. We remark that the throughput traffic decreases as parking traffic decreases in the parking regions. Also, note how even links that are significantly removed from the parking regions are affected by parking traffic as the network adjusts to the demand. Moreover, it is interesting to see that in while the through traffic flow in Figure 5h increases along edges $13 - 15$ for the first several values of parking population, the parking flow remains zero. This indicates that through traffic is being rerouted to these edges to avoid parking traffic on some other edges. This is similarly true for edge 9 (see Figures 5e and 5f) and edge 1 and 2 (see Figures 5b and 5c).
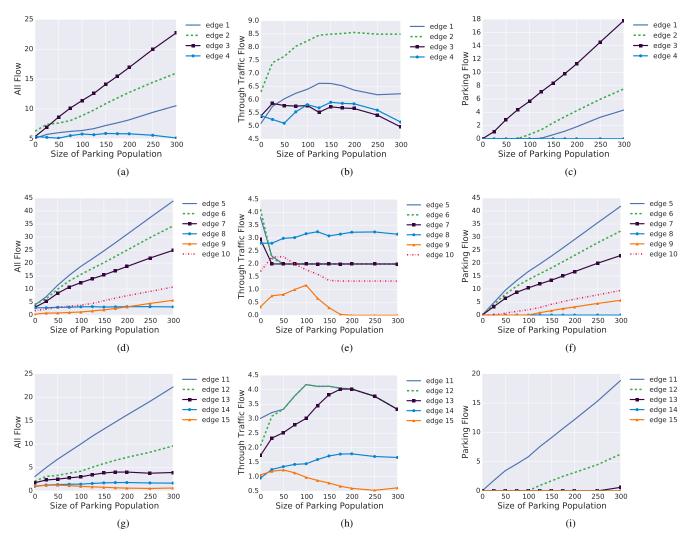
Fig. 5. Total, through, and parking traffic flow for Example 2 (Amazon) along edges: (a)–(c) 1–4; (d)–(f) 5–10; (g)–(i) 11–15. See Fig. 4 for the edge labels in the Amazon example.

In Figure 6 we show qualitatively how the network becomes more congested as the parking populations grow for a fixed through-traffic population size. Of note is the fact that increasing the parking population not only increases congestion in and around parking areas, but can also have adverse effects on arterial streets further removed the parking areas. This is due to the fact that through traffic is rerouted through previously un-congested streets.

V. DISCUSSION AND FUTURE WORK

We presented a novel routing game framework over queue–flow networks that allowed us to capture the interaction between populations of drivers seeking on-street parking and populations of drivers flowing through an urban area. Further, the combination of the routing game framework with the queuing game gave us a computational tool for analyzing the impact of circling traffic on congestion in large and realistic flow networks.

The work in the paper is the first steps towards the development of a modeling paradigm for urban mobility that accounts for drivers having different objectives and intended uses of transportation infrastructure. There is still a significant amount of work to create a comprehensive theoretical framework for queue–flow networks. In particular, the routing game framework presented is inherently *static* while the queuing model is inherently *dynamic*. We are investigating dynamic routing game frameworks that can more seemlessly integrate with the dynamic queueing model. We are also formulating the pricing problem as a bilevel optimization problem so that in this new dynamic framework, we will be able to design dynamic pricing schemes that more effectively take into account demand (which is naturally time varying). We aim to extend the queuing game framework to consider more interesting behaviors such as jockeying between queues after the initial parking area has been selected, balking to other modes of transit, and reneging to off-street parking. Finally, in the near term, we are working on incorporating more heterogeniety of different populations into our framework so that we can consider a range of user preference models that span very rich and
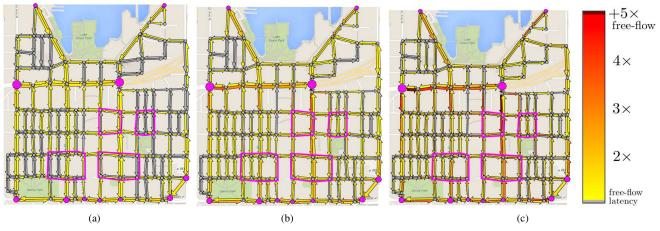
Fig. 6. Evolution of the congestion in the Amazon Campus area network as the parking populations increase. Through traffic population is fixed at 75 cars/min (a) No parking population (b) Total parking population of size 150 cars/min (c) Total parking population size of 300 cars/min.

diverse socioeconomic features that are representative of urban spaces.

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