Price of Anarchy of Mobility-as-a-Service Rivalry

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Outline

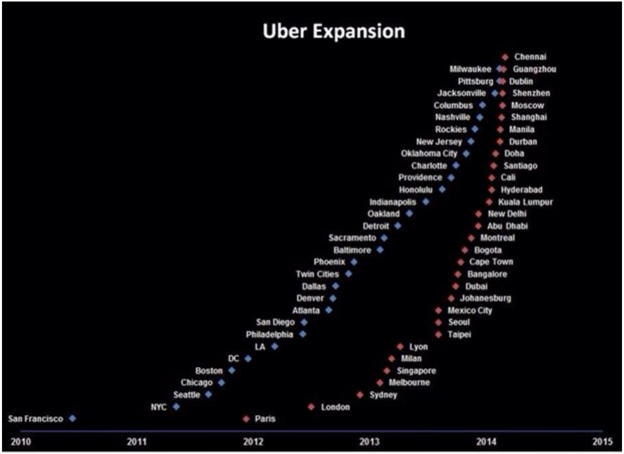
Motivation

Queueing-theoretical framework

Game-theoretical framework

Equilibrium: existence, uniqueness, computation

Price of anarchy



Lyft may expand to 100 cities globally in 2015

Josh Lipton | @CNBCJosh
Wednesday, 17 Sep 2014 | 3:01 PM ET
CNBC

Expansion of Mobility-as-a-Service (MaaS) systems.

Collaborative Consumption | Lyft | Uber

Uber Strikes Back, Claiming Lyft Drivers And Employees Canceled Nearly 13,000 Rides

Posted Aug 12, 2014 by Ryan Lawler (@ryanlawler), Contributor

CNN Money | Business | Markets | Tech | Media | Personal Finance | Small Biz | Luxury

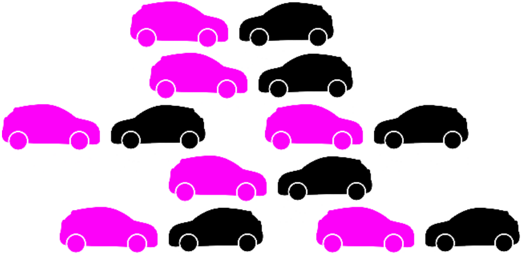
Innovation Nation

Uber's dirty tricks quantified: Rival counts 5,560 canceled rides

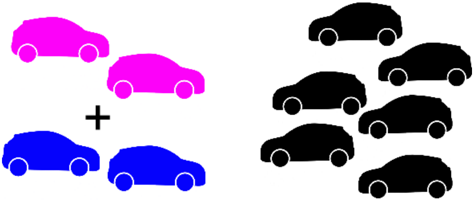
Uber, Lyft Battle It Out In San Francisco With Ultra-Low Prices On Carpool Rides

By Salvador Rodriguez @sal19 s.rodriguez@ibtimes.com on January 26 2015 6:33 PM EST

Fierce competition between MaaS systems.



Analyze Price of Anarchy (PoA): increased traffic, VMT, VHT.



Merging two companies to compete against a larger one.

TRANSPORT › CARS

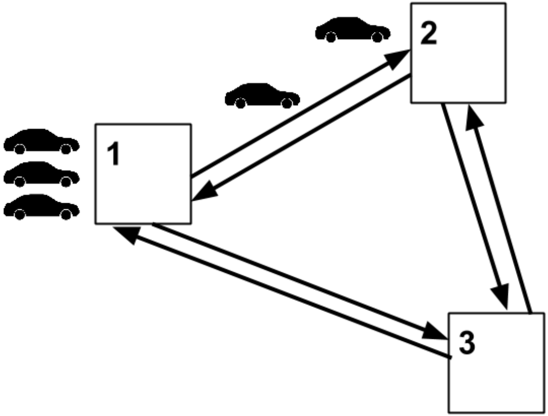
China's Ride-Sharing Apps Raising Big Money to Thwart Uber

Shai Oster, Bloomberg - Jun 15, 2015 3:00 pm

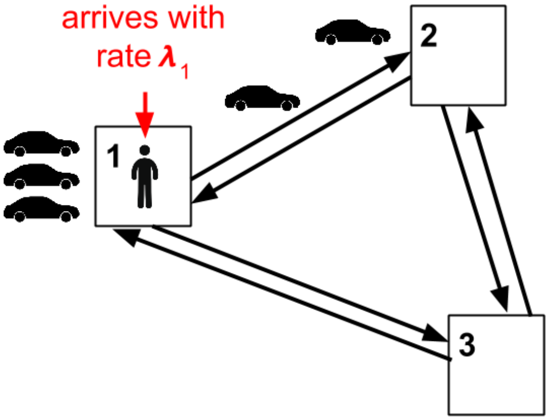
Objective

How to regulate the MaaS war.

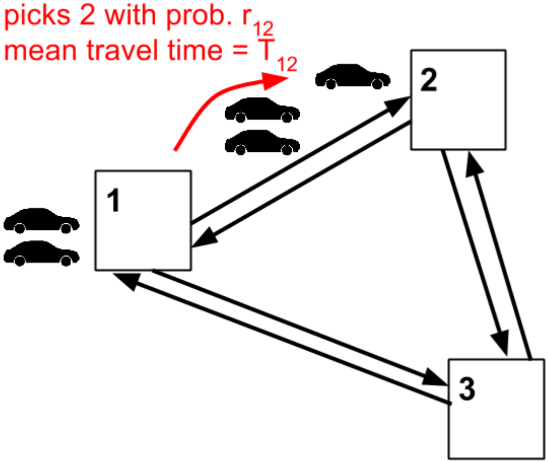
Outline



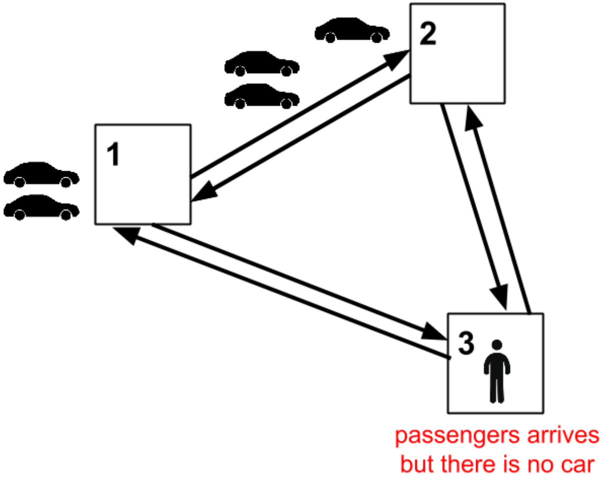
Example: network with three stations.



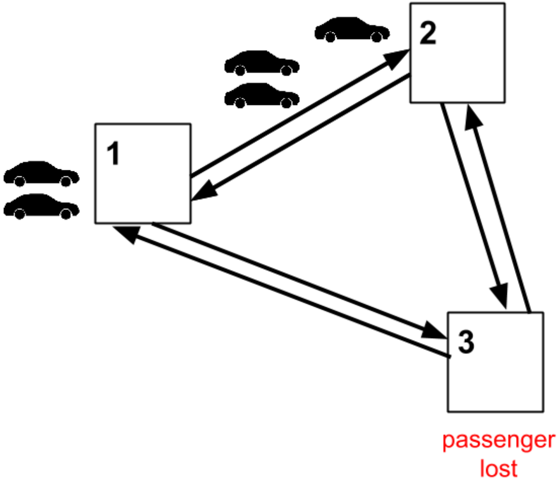
Customer arrives at station 1 with rate λ_1 and gets a car.



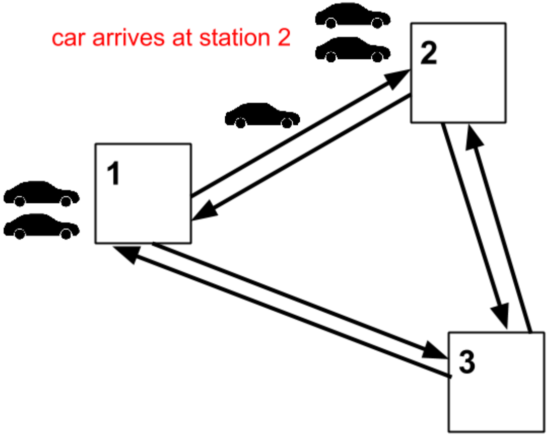
Picks up destination 2 (resp. 3) with probability r_{12} (resp. r_{13}).



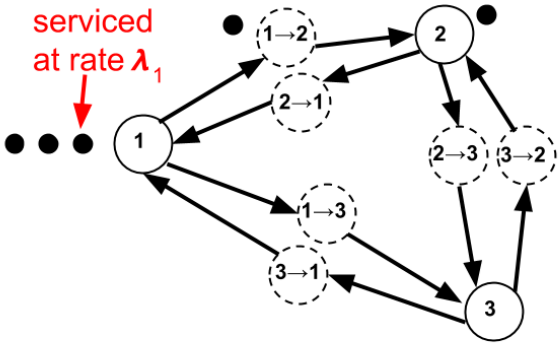
Customer arrives at station 3 with rate λ_3.



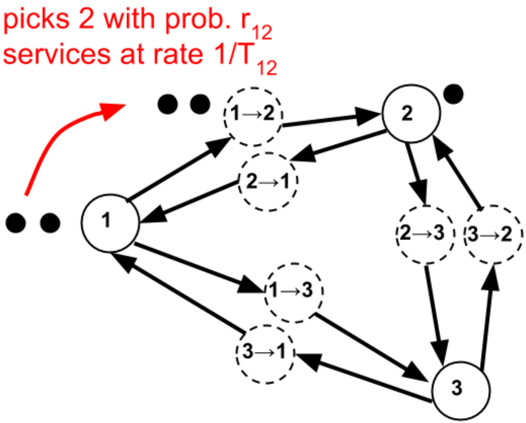
No car at station 3: passenger leaves the system.



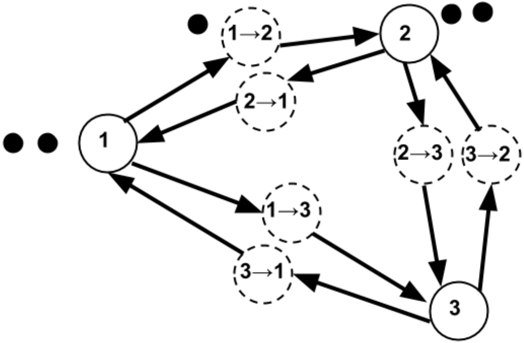
Car arrives at station 2.



Jackson network: station nodes + route nodes between pairs of stations.

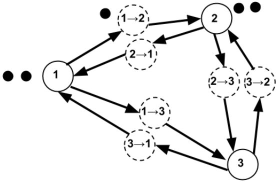


Car (packet) leaves station 1 to go to route node 1→2.



After spending T_{12} on route 1→2, arrives at station 2.

Casting into a Jackson network



- 1st car in line processed with rate ϕ_i (customer arrival rate at i)
- Routed to node $i \to j$ with probability α_{ij}
- Processed with rate $1/T_{ij}$ (T_{ij} = mean travel time from i to j)
- Routed to station j with probability 1
- Full specification

$$\text{Service rate:} \quad \mu_i = \phi_i \qquad \mu_{i\to j} = 1/T_{ij}$$

$$\text{Routing probabilities:} \quad p_{i,\,i\to j} = \alpha_{ij} \qquad p_{i\to j,\,j} = 1$$

Stationarity results

- In equilibrium, arrival rates π_i of cars at station i:

$$\pi_i = \sum_j p_{ji} \pi_j \quad \text{(balance equations)}$$

- $\gamma_i :=$ relative utilization $= \pi_i / \mu_i$ satisfies

$$\gamma_i = \sum_j \frac{p_{ji} \mu_j}{\mu_i} \gamma_j$$

- $X_i :=$ number of vehicles in queue at station i (random variable)
- Availability at station i:

$$\Pr[X_i \geq 1] \propto \gamma_i = \pi_i / \mu_i$$

Outline

Literature review

	arrival rate	routing	authors	contribution
	ϕ_i	α_{ij}	George & Xia	Framework
	ψ_i	β_{ij}	Zhang & Pavone	Balancing
	v_i	κ_{ij}	Thai, Yuan & Bayen	Cybersecurity
	ψ_i^1, ψ_i^2	$\beta_{ij}^1, \beta_{ij}^2$	Thai, Yuan & Bayen	Game theory

Stochastic control

- Company 1 (reps. 2) only sees arrival:

$$\lambda_i^1 = \phi_i + \psi_i^1 \qquad (\text{resp. } \lambda_i^2 = \phi_i + \psi_i^2)$$

- Routing probabilities for 1 (reps. 2)

$$p_{ij}^1 = \frac{\alpha_{ij}\phi_i + \beta_{ij}^1\psi_i^1}{\phi_i + \psi_i^1} \qquad \left(\text{resp. } p_{ij}^2 = \frac{\alpha_{ij}\phi_i + \beta_{ij}^2\psi_i^2}{\phi_i + \psi_i^2}\right)$$

- Maximum rate of dispatch for company $r \in \{1, 2\}$

$$\sum_j \psi_j^r \leq \tau \sum_j \phi_j$$

Balance equations

- Rates of arrivals for company $r \in \{1, 2\}$

$$\pi_i^r = \sum_j p_{ji}^r \pi_j^r$$

$$\pi_i^r = \sum_j \frac{\alpha_{ji}\phi_j + \beta_{ji}^r \psi_j^r}{\phi_j + \psi_j^r} \pi_j^r$$

- Fleet size limitation

$$\sum_i \pi_i^r = \pi_0 N^r$$

Re-balancing framework

- Recall availabilities for company r

$$\Pr[X_i^r] \propto \gamma_i^r = \frac{\pi_i^r}{\phi_i + \psi_i^r} \quad \forall\, i$$

- Fairness:

$$\gamma_i^r = \gamma_j^r \;\forall\, i, j \quad \iff \quad \phi_i + \psi_i^r = \sum_j \alpha_{ji}\phi_j + \beta_{ji}^r \psi_j^r$$

- Hence

$$\pi_i^r = \frac{\pi_0 N^r(\phi_i + \psi_i^r)}{\sum_j \phi_j + \psi_j^r}$$

Prisoner's dilemma

- Benefit available at each station i

$$w_i := \phi_i \, E[T \mid i] = \phi_i \sum_j \alpha_{ij} T_{ij}$$

- Share of w_i for each company r

$$\text{proportion of cars from } r \text{ station } i = \frac{\pi_i^r}{\sum_s \pi_i^s}$$

- Objective for company r, with $\pi_i^{-r} := \sum_{s \neq r} \pi_i^s$

$$\max \quad \sum_i w_i \frac{\pi_i^r}{\sum_s \pi_i^s} \quad \Longleftrightarrow \quad \min \quad \sum_i w_i \frac{\pi_i^{-r}}{\pi_i^r + \pi_i^{-r}}$$

- $\sum_j \psi_j^r = \tau \sum_j \phi_j$ is necessary for optimality, hence

$$\pi_i^r = \alpha N^r (\phi_i + \psi_i^r) \quad \text{with} \quad \alpha = \frac{\pi_0}{(1+\tau) \sum_j \phi_j}$$

Nash equilibrium problem

Putting everything together

$$\min \quad \sum_i \frac{w_i \pi_i^{-r}}{\pi_i^{-r} + \alpha N^r(\phi_i + \psi_i^r)}$$

subject to

$$\phi_i + \psi_i^r = \sum_j \alpha_{ji}\phi_j + \beta_{ji}^r \psi_j^r \quad \forall\, i \qquad \textit{fairness}$$

$$\sum_j \psi_j^r = \tau \sum_j \phi_j \qquad \textit{rate of dispatch}$$

$$\psi_j^r \geq 0,\ \beta_{ij}^r \geq 0,\ \sum_j \beta_{ij}^r = 1 \qquad \textit{balancing process}$$

Outline

Flow constraints

Define the rate of balancing $x_{ij}^r := \psi_i^r \beta_{ij}^r$

$$\min \quad f^r(x_{ij}^r, x_{ij}^{-r}) := \sum_i \frac{w_i \pi_i^{-r}}{\pi_i^{-r} + \alpha N^r(\phi_i + \sum_j x_{ij}^r)}$$

subject to

$$\sum_j x_{ji}^r - x_{ij}^r = s_i \quad \forall\, i \qquad \textit{flow conservation}$$

$$\sum_{ij} x_{ij}^r = b \qquad \textit{total flow}$$

$$x_{ij}^r \geq 0 \qquad \textit{positive flow}$$

with parameters

$$s_i = \phi_i - \sum_j \alpha_{ji}\phi_j, \quad b = \tau \sum_j \phi_j, \quad \alpha = \frac{\pi_0}{(1+\tau)\sum_j \phi_j}$$

Convex payoff

- Gradient of the payoff

$$\frac{\partial f^r}{\partial x_{ij}^r} = \frac{-w_i \pi_i^{-r} \alpha N^r}{(\pi_i^{-r} + \alpha N^r(\phi_i + \sum_j x_{ij}^r))^2} \tag{1}$$

- Hessian of the payoff

$$\frac{\partial^2 f^r}{\partial x_{ij}^r x_{kl}^r} = \frac{2 w_i \pi_i^{-r} (\alpha N^r)^2}{(\pi_i^{-r} + \alpha N^r(\phi_i + \sum_j x_{ij}^r))^3} \delta_{ik} \tag{2}$$

- Hessian is positive semi-definite
- Hence the objective is convex

Existence and uniqueness of an equilibrium

Existence

- Constraints are convex compact
- Objective is convex

Uniqueness

- Equilibrium non-unique a priori
- Unique with a regularization term $\frac{1}{2}\sum_{ij} {x_{ij}^{r}}^{2}$

Computation

- Update for each player in parallel (Jacobi scheme)
- Guarantee of convergence (with the regularization)

"Convex Optimization, Game Theory, and Variational Inequality Theory", Scutari et al.

Outline

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Game-theoretical framework

Equilibrium: existence, uniqueness, computation

Price of anarchy

Combining two players

- Total rate is the sum of the rates

$$\pi_i^m = \pi_i^1 + \pi_i^2$$

- Fleet size

$$\sum_i \pi_i^m = \pi_0(N^1 + N^2)$$

- With fairness assumption

$$\pi_i^m = \alpha(N^1 + N^2)(\phi_i + \psi_i^m)$$

- Study effect of merging 2 companies to compete against a larger one

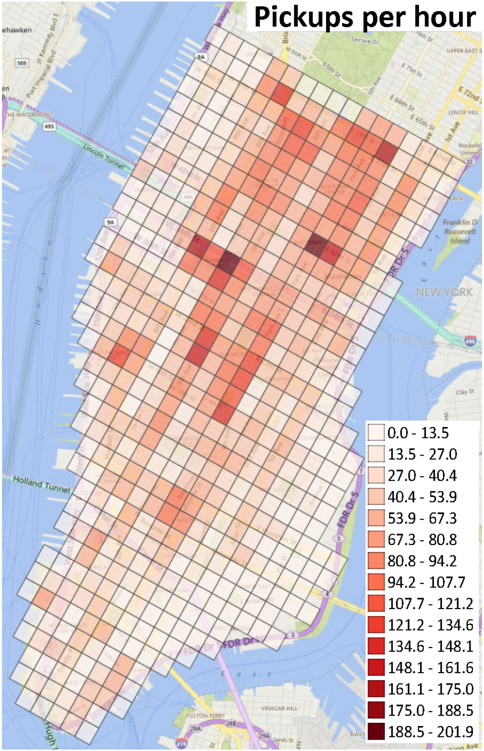
Price of Anarchy (PoA)

- Social optimum: minimize congestion with fleet size $N = \sum_r N^r$

$$
\begin{aligned}
\min \quad & \sum_{ij} T_{ij}\psi_i\beta_{ij} \\
\text{s.t.} \quad & \phi_i + \psi_i = \sum_j \alpha_{ji}\phi_j + \beta_{ji}\psi_j \quad \forall\, i && \textit{fairness} \\
& \sum_j \psi_j = \tau \sum_j \phi_j && \textit{rate of dispatch} \\
& \psi_j \geq 0,\ \beta_{ij} \geq 0,\ \sum_j \beta_{ij} = 1 && \textit{balancing process}
\end{aligned}
$$

- Price of Anarchy (PoA):

$$
\sum_{ij} T_{ij}\left(\frac{\sum_r \pi_i^r \psi_i^r \beta_{ij}^r}{\sum_s \pi_i^s}\right) - \sum_{ij} T_{ij}\psi_i^\star \beta_{ij}^\star
$$


Pickups per hour
0.0 - 13.5
13.5 - 27.0
27.0 - 40.4
40.4 - 53.9
53.9 - 67.3
67.3 - 80.8
80.8 - 94.2
94.2 - 107.7
107.7 - 121.2
121.2 - 134.6
134.6 - 148.1
148.1 - 161.6
161.1 - 175.0
175.0 - 188.5
188.5 - 201.9

Ongoing work

Proposed a rigorous framework for the analysis of the PoA

- Jackson network to model the taxi companies
- Game-theoretical framework for the selfish behavior
- Existence, uniqueness, and computation of the Nash equilibrium

Get numerical results

- Rough estimate of the PoA of the Uber-Lyft rivalry
- What is the effect of merging two companies? Effect of fleet sizes?
- Propose some regulations to get closer to the Social Optimum