



Game Theoretic Issues in Security of CPS

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Outline

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- Salient features of CPS' security

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- Key sources of difficulties in stochastic dynamic games with asymmetric information (SDGAS)

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- State-of-the-art in SDGAS

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- State-of-the-art in SDGAS
 - Our contribution

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- Salient features of CPS' security
- Key sources of difficulties in stochastic dynamic games with asymmetric information (SDGAS)
- Challenges/Consequences of Difficulties
- State-of-the-art in SDGAS
 - Our contribution
- Open research issues/problems in SDGAS for CPS security

Salient Features of CPS' Security

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 - Security status of CPS is affected by the DMS' actions.
- All features => **Stochastic Dynamic Games with Asymmetric Information (SDGAS)**

Illustrative Example

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Energy System

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- Two agents

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 - Attacker (A)

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 - (A) knows the components of the network (computers) it has under control

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 - (A) can compromise a sensor, inject false data in automatic generation controller

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 - (A) has incomplete information of above
 - (A) can compromise a sensor, inject false data in automatic generation controller
 - (D) does not perfectly know what part of network was attacked
 - (D) may see effects of attack but does not see the origin (fault or malicious)

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- Features => **SDGAS**

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 - P_t^i := Agent i's private information at t
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 - $g_t^i(C_t, P_t^i)$:= Agent i's strategy at t

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 - **Beliefs** about the status of the game
 - **Prediction** about other agent strategies
- Beliefs about status of game are, in general, different among agents (asymmetric information)
- Agents' actual strategies may be different from their prediction (agents' strategies are their own private information)

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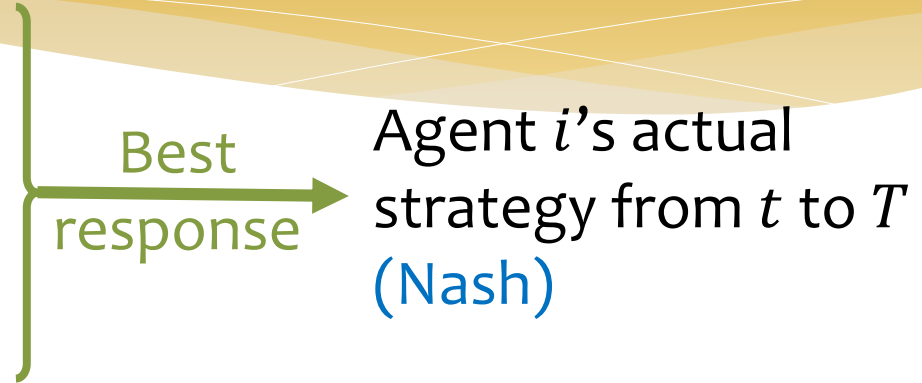
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- Belief and strategy predictions must be defined/specified:
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- Above specification is necessary because agents' strategies are their own private information

Challenges/Consequences of Difficulties

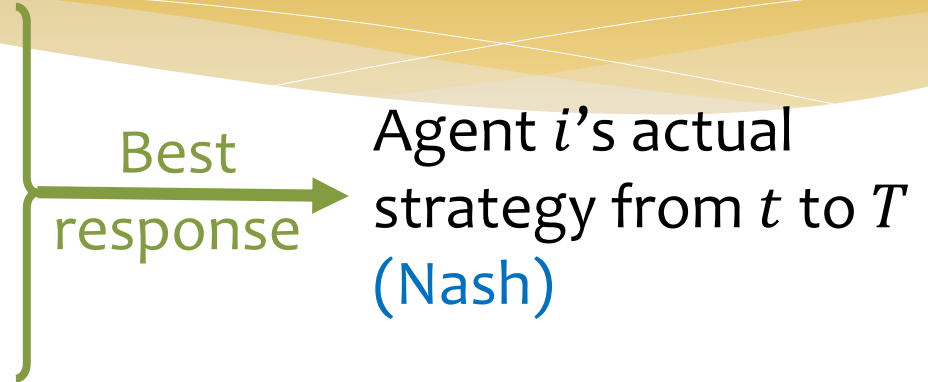
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- Agent i 's **belief** at t
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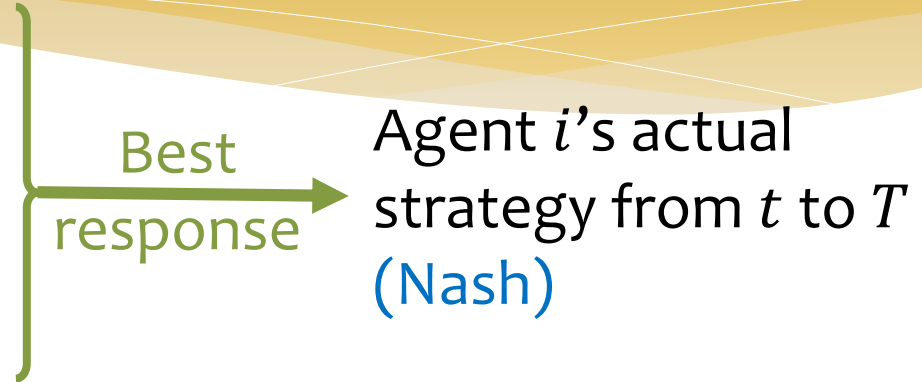
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 Agent i 's actual strategy (Nash)

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- **Agent i 's belief** at t



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- Agent i 's actual strategy (Nash)
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- ← **Best response** →
- Agent i 's actual strategy (Nash)
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- ← **Signaling** →
- Agent i 's actual strategy (Nash)
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Best response

Agent i 's actual strategy from t to T (Nash)

- Agent i 's strategy prediction $\hat{g}_{1:t}^i$

— Agent i 's actual strategy (Nash)

- **Agent i 's belief** at t

← Signaling Agents i 's prediction of $\hat{g}_{1:t-1}^{-i}$

- Strategies and beliefs are **inter-dependent** over time
- The domain of strategy $g_t^i(C_t, P_t^i)$ increases over time.
- What are appropriate equilibrium concepts?

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- } Assessment
 $(\hat{g}_{1:T}^i, \mu_{1:T}^i)$,
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$$\forall t, C_t, P_t^i, i=(A), (D)$$

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- Off-equilibrium**: μ_t^i must comply with $\hat{g}_{1:t-1}^{(A)}, \hat{g}_{1:t-1}^{(D)}$ via Bayes' rule whenever possible

Adopted Equilibrium Concept – Perfect Bayesian Equilibrium

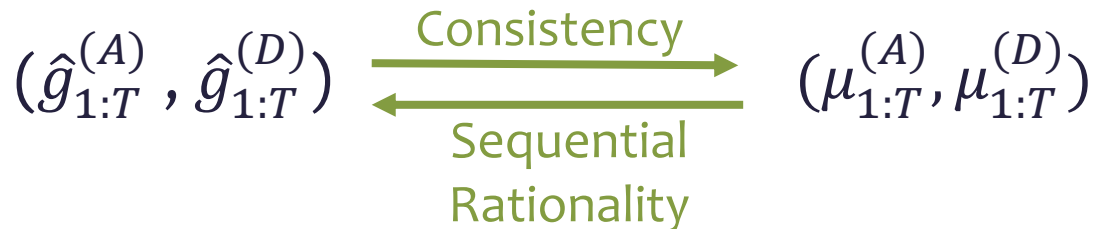
Adopted Equilibrium Concept – Perfect Bayesian Equilibrium

- **Definition.** An assessment $(\hat{g}_{1:T}^{(A)}, \hat{g}_{1:T}^{(D)}, \mu_{1:T}^{(A)}, \mu_{1:T}^{(D)})$ is a **Perfect Bayesian equilibrium (PBE)** if it is sequentially rational and consistent.

Adopted Equilibrium Concept – Perfect Bayesian Equilibrium

Key difficulties of determining a PBE

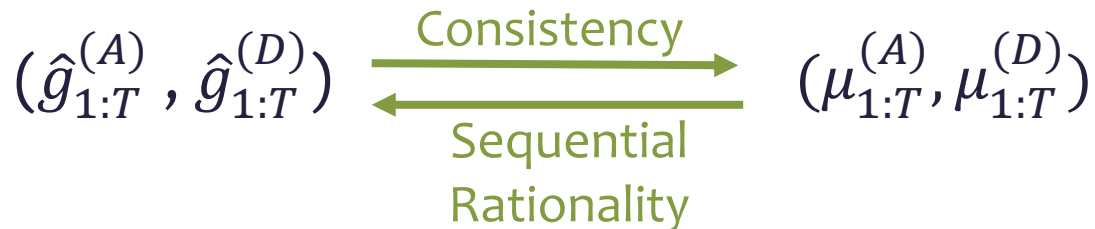
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- **Growing complexity** of Agent i 's strategy $g_t^i(C_t, P_t^i)$ with increasing t (and thus, its prediction $\hat{g}_t^i(C_t, P_t^i)$).

State-of-the-art in SDGAS

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- Common-information-based (CIB) approach to SDGAS

Our Contribution: The Common Information Approach

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 - Π_t is **common knowledge** among all agents

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Work on SDGAS Supported by FORCES Grant

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