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Modeling Fuel Flow Rate using Gaussian Processes

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Motivation and Aim

* Aircraft engine fuel flow rate
  * Engine performance
  * Pollutant emissions
  * Operating costs to airline

* Flight recorder data can model performance of a real engine in operation

* Aim: Model aircraft engine as a statistical system and apply machine learning techniques to flight data
  * Model aircraft engine fuel flow rate
  * Evaluate model for predictive performance on held-out test data

Gaussian Process Regression

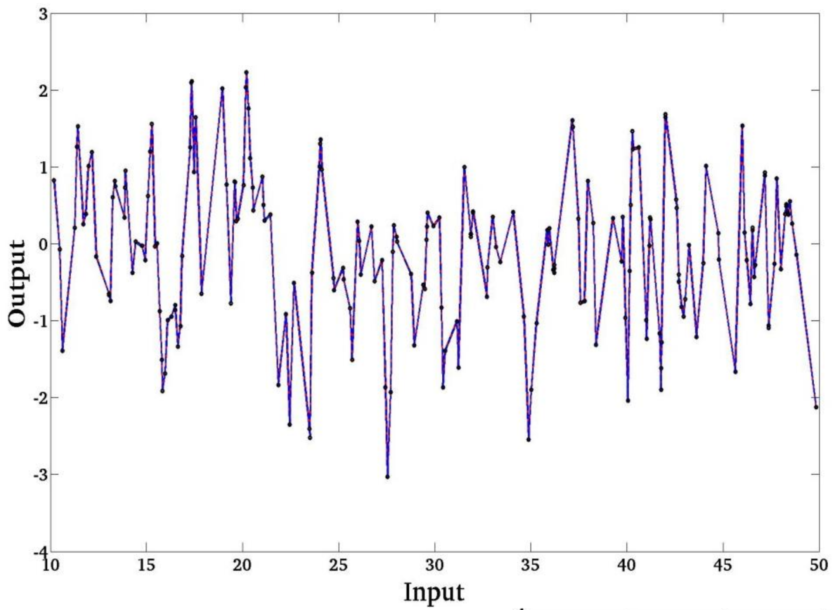
- Bayesian nonlinear and nonparametric regression technique
- Gaussian Process (GP) prior on underlying latent function:

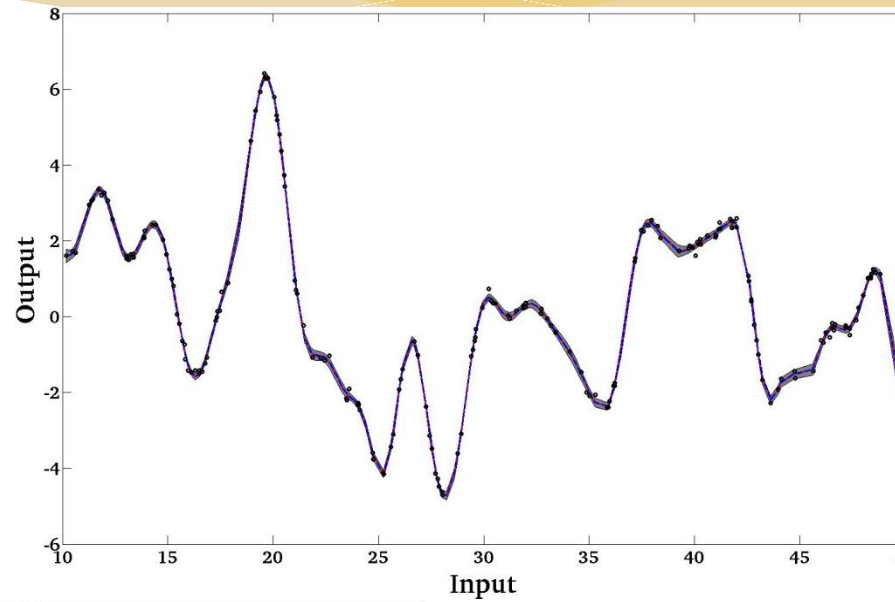
$$y_i = \mathrm{f}(x_i) + \varepsilon,\ \varepsilon \sim \mathrm{N}(0,\sigma^2)$$
$$\mathrm{f} \sim GP(0, k(x, x'))$$

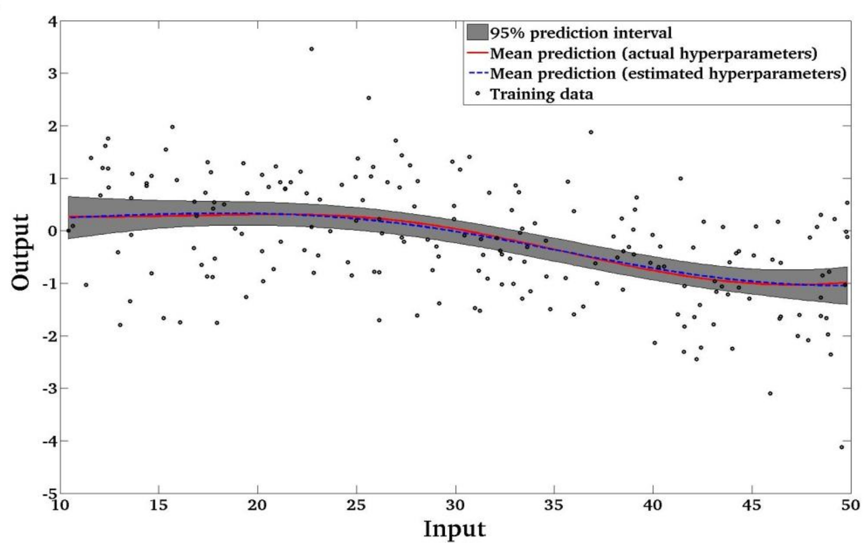
- Marginal likelihood and posterior predictive distribution:

$$p(\boldsymbol{y}|\boldsymbol{X},\boldsymbol{\theta}) = N\left(\boldsymbol{y}\middle|\mathbf{0}, \boldsymbol{K}_{\boldsymbol{n},\boldsymbol{n}} + \sigma^2\boldsymbol{I}\right)$$
$$p\left(\boldsymbol{y}_*\middle|\boldsymbol{X}_*, \mathbf{y}, \mathbf{X}, \widehat{\boldsymbol{\theta}}\right) = N(\boldsymbol{K}_{*,\boldsymbol{n}}\left(\boldsymbol{K}_{\boldsymbol{n},\boldsymbol{n}} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{y},\ \boldsymbol{K}_{*,*} - \boldsymbol{K}_{*,\boldsymbol{n}}\left(\boldsymbol{K}_{\boldsymbol{n},\boldsymbol{n}} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{K}_{\boldsymbol{n},*})$$

Examples of Gaussian Process Regression







Gaussian Process Regression on Real Aircraft Data

- Airbus A320-200 aircraft in ascent, cruise, descent
- Fuel flow rate regressed on aircraft altitude, ground speed, vertical speed, takeoff mass
  - Variables standardized before regression
- Priors on hyperparameters of kernels:
  - Inverse Gamma priors on variance and noise hyperparameters
  - Gamma priors on length scale (same/different for each predictor)

$$k(r)_{sq\_exp} = \sigma^2{}_f e^{-\frac{r^2}{2l^2}}$$

$$k(r)_{exp} = \sigma^2{}_f e^{-\frac{r}{l}}$$

$$k(r)_{Matern\ 3/2} = \sigma^2{}_f \left(1 + \frac{\sqrt{3}r}{l}\right) e^{-\frac{\sqrt{3}r}{l}}$$

$$k(r)_{Matern\ 5/2} = \sigma^2{}_f \left(1 + \frac{\sqrt{5}r}{l} + \frac{5r^2}{3l^2}\right) e^{-\frac{\sqrt{5}r}{l}}$$

$$r(x, x') = ||x - x'||_2$$

Model Evaluation on Held-Out Data

Phase	Percentage Mean Error	Coverage Per Unit Mean Length of Prediction Interval	Predictive Log Likelihood
Ascent	3.11	6.03	17421
Cruise	3.47	15.06	1516
Descent	23.63	1.52	8971

Ascent: 26658 training observations, 9553 held-out observations
Cruise: 1433 training observations, 551 held-out observations
Descent: 54178 training observations, 20488 held-out observations
Results shown for the best covariance function on de-standardized data

Comparison with Other Models on Held-Out Data*

Model	Percentage Mean Error	Coverage Per Unit Mean Length of Prediction Interval
Gaussian Process Regression **	4.79	21.87
Robust Least Squares (quadratic basis)	6.33	20.56
Parametric Bayesian Multiple Linear Regression (quadratic basis)	6.23	18.41
Classification and Regression Trees (CART)	6.39	11.60
Boosted Regression Trees	4.01	20.63

*: only for cruise

**: squared exponential kernel

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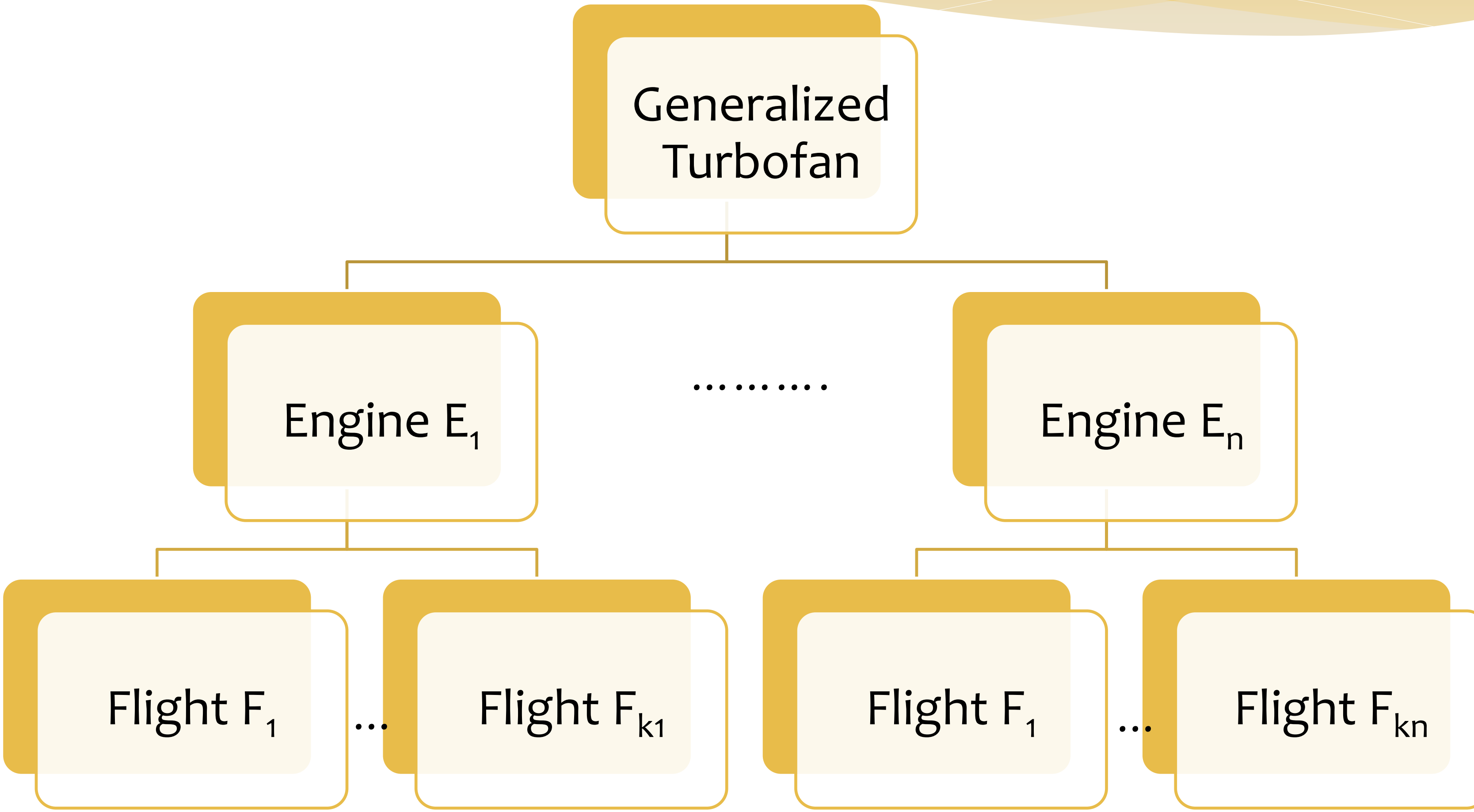
Summary

* Gaussian Process Regression can be used to achieve good prediction capability while modeling aircraft engine fuel flow rates – better than other parametric methods
* GP predicts the latent function directly – no need to worry about choice of basis functions
* Model flexibility by choice of different kernel functions/hyperparameters
* Gives the complete predictive distribution (no need of bootstrapping)
* Amenable to hierarchical regression

Hierarchical Model



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THANK YOU

BACKUP SLIDES

Model Evaluation on Held-Out Dataset

Model	Phase	Percentage Mean Error	Coverage Per Unit Mean Length of Prediction Interval
Gaussian Process Regression	Ascent	3.11	6.03
	Cruise	3.47	15.06
	Descent	23.63	1.52
Boosted Regression Trees	Ascent	1.60	20.34
	Cruise	3.71	21.06
	Descent	13.15	3.74

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