



Games of Strategy

For modeling conflict and cooperation

Schwartz/Teneketzis



CPS Infrastructures : Effect of Technological Advancements

- * Conflict and cooperation of CPS providers: an increase in
 - * Cooperation (driven by technological complementarities)
 - * Competition (driven by availability of numerous substitutes)

Complex parties' incentives \Leftrightarrow multiple externalities

- * The role of central authority: improving parties' incentives
 - * Higher competition and conflict \Leftrightarrow importance of resolving conflicts
 - * Threats of security failures \Leftrightarrow Importance of investing in security

Central authority's role: conflict mitigation, coordination, and regulation

CPS viewed as two-sided market : Models both Conflict and Cooperation

- * Increased role of Central Authority : necessity to harmonize interactions and improve CPS robustness
 - * Managers of infrastructure (platform)
 - * Producers (suppliers)
 - * Consumers (individual users)
 - * Central authority (coordinator/regulator)
- * Example: Electric grid
 - * Platform Managers: Transmission/Distribution operators
 - * Suppliers: Generation plants
 - * Users: Customers (large and small)
 - * Coordinator: ISO

Today's talk: Two game-theoretic models

- * Resource allocation in strategic multi-battlefield conflicts:
Our focus is to solve Blotto games with
 - * possibility to add extra fields and alliances
 - * possibility to form alliances (cooperation)
- * Network design with random faults and strategic attacks
 - * Arbitrary (fixed) network structure (a given network topology)
 - * Defender faces system failures, and cannot distinguish between
 - * Random faults (reliability failures)
 - * Strategic attacks (security failures)

An age-old quest in Game theory: How to allocate scarce resources?

- * Under given assumptions on
 - * information
 - * order of moves (simultaneous vs sequential)
 - * player actions: (discrete vs continuous)
 - * player objective: (plurality vs majority)
 - * players: symmetric (homogeneous) vs asymmetric (heterogeneous)
 - * game: one-shot vs repeated / dynamic
- * Original Colonel Blotto setup: deceptively trivial benchmark

Colonel Blotto setup

- scarce resource (time, money, effort) R
 - battlefields $n = 3$ (town, state, node, server, ...) $j = 1, 2, \dots, n$
 - the values of battlefields: identical $v_j = v$
 - contest (tournament) of two identical players, $i = A, B$ or $i = 1, 2$ or $i = \alpha, \beta$
 - order of moves: simultaneous
 - player actions: continuous
 - player objective (*plurality vs majority*)
 - players are symmetric (identical) $R_\alpha = R_\beta$

Applications of Blotto

- traditional warfare
 - 1 routing choices for supply support
 - 2 across distinct battlefields
- politics (elections under different rules)
 - optimization of fund raising strategy (time allocation)
 - optimization of resources during the contest
- Economics (IO) [Corporate strategy for multi-product markets]
 - research and development (R&D)
 - resource allocation for advertizement

Our research plan:

Blotto for CPS infrastructure protection & defense

Colonel Blotto: Identical players

- Two-players constant-sum complete information static game
- Identical (symmetric) resource case [formulated Borel 1921]
- Battlefields: identical value
- N battlefields ($N = 3$) [solved Borel & Ville (1938)]
- N battlefields ($N > 3$) [Gross & Wagner (1950)]

Colonel Blotto: Non-identical players

- Player α has resource α
- Player β has resource $\beta < \alpha$
- Battlefields: identical value
- with ($N = 2$) [Gross & Wagner (1950)]
- with ($N > 2$)[Roberson (2006), Kvasov (2007)]

Who wins the battlefield?

In a generic case, Colonel Blotto game admits no pure strategy equilibrium. CSF is a probability for player i ($i = \alpha, \beta$) to win each contest j ($j = 1, \dots, n$), and let $x_{i,j}$ and $x_{-i,j}$ player i resources (investments) for contest j , other player(s) resources for j

$$p(x_{i,j}, x_{-i,j}) = \frac{x_{i,j}}{x_{i,j} + x_{-i,j}}$$

If noise is present, and $m \geq 0$ some noise is present, with $m = 0$ – all is noise, investments are irrelevant, if a higher m noise decreases.

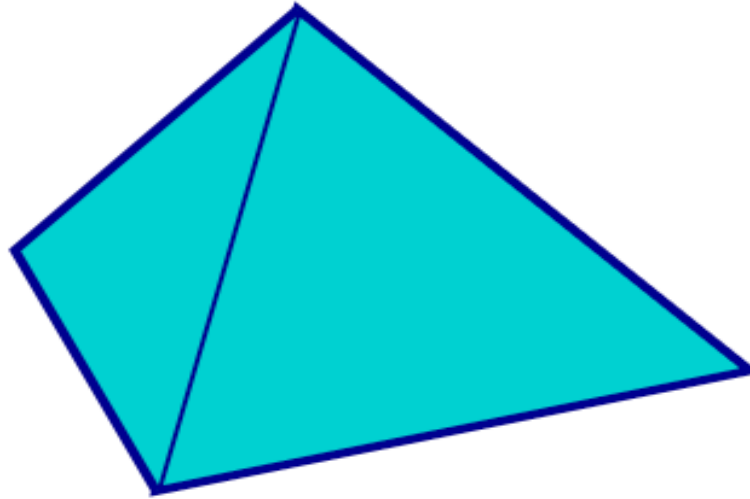
$$p(x_{i,j}, x_{-i,j}) = \frac{x_{i,j}^m}{x_{i,j}^m + x_{-i,j}^m}$$

Payoffs

- Player A decides on the allocation of resources across the battlefields $\mathbf{a} = \{\alpha_1, \dots, \alpha_N\}$
- Player B, decides on $\mathbf{b} = \{\beta_1, \dots, \beta_N\}$
- Constraint Regions: $\mathcal{A} := \{\mathbf{a} : \sum_{i=1}^N \alpha_i \leq \alpha\}$ and $\mathcal{B} := \{\mathbf{b} : \sum_{i=1}^N \beta_i \leq \beta\}$
- On each battlefield, player with maximum resource wins and receives a payoff $\frac{1}{N}$
- If equal resource on a battlefield, both player share the payoff, that is, each player receives a payoff $\frac{1}{2N}$

Overview of Results

- \mathcal{A} and \mathcal{B} looks like a solid simplex



- No Nash equilibrium in pure strategies
- Nash equilibrium exists in mixed strategies with unique player payoffs (Roberson 2006)
- Measures $\mu_i \in \wp(\mathbb{R}_+^N)$ such that $\text{supp}(\mu_A) \subset \mathcal{A}$ and $\text{supp}(\mu_B) \subset \mathcal{B}$

Results

- The payoff functions are expressed as

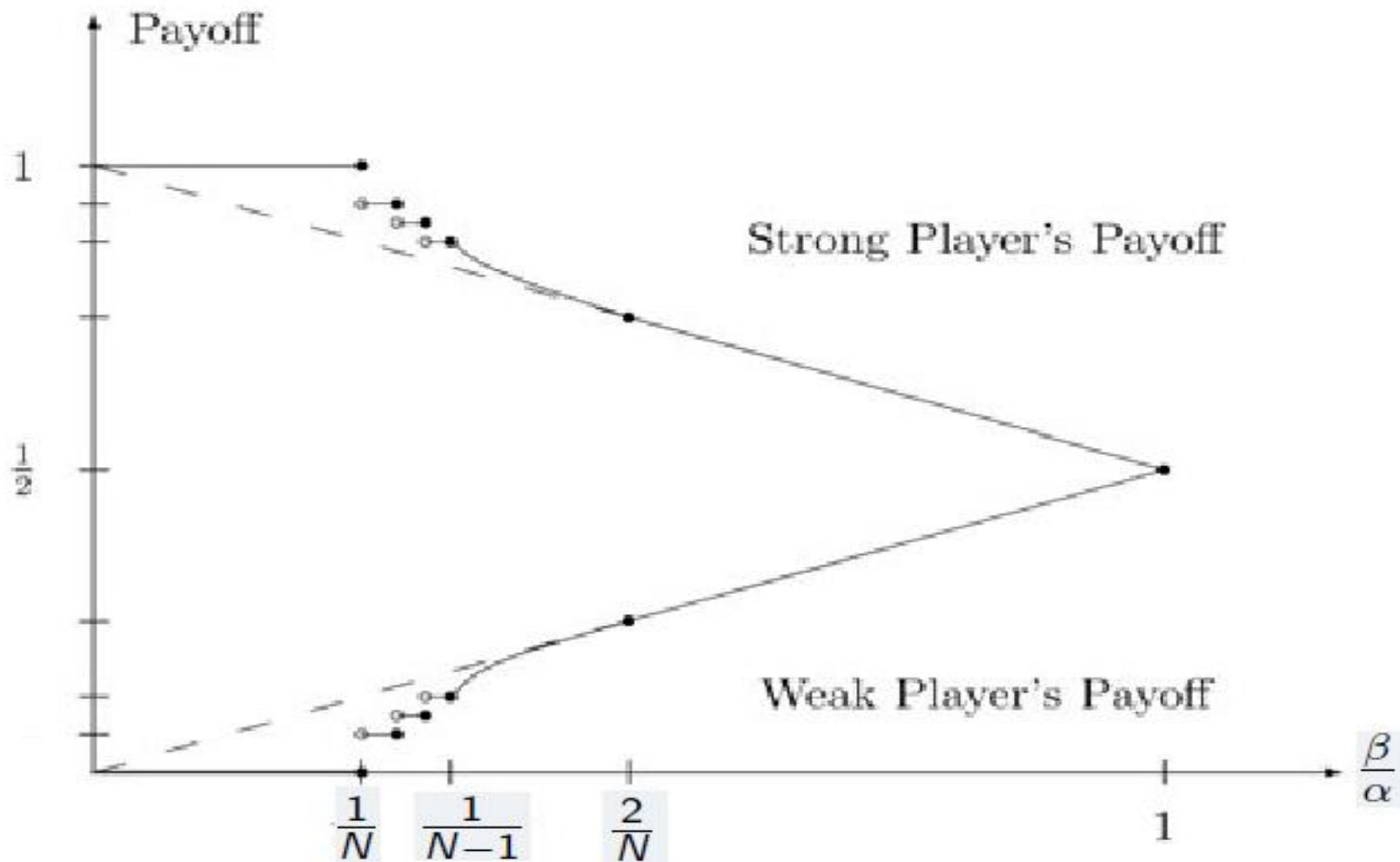
$$J_i(N, \mu_A, \mu_B) = \sum_{j=1}^N \frac{1}{N} \int_0^\infty \pi_{\#}^j \mu_{-i}([0, x)) \pi_{\#}^j \mu_i(dx)$$

- Payoffs dependent on the marginal distributions on the battlefields

Main Result for $\alpha \leq \beta$

Case	Player A's Payoff	Player B's Payoff
$\frac{2}{N} \leq \frac{\beta}{\alpha} \leq 1$	$1 - \frac{\beta}{2\alpha}$	$\frac{\beta}{2\alpha}$
$\frac{1}{N-1} \leq \frac{\beta}{\alpha} < \frac{2}{N}$	$1 - \frac{2}{N} + \frac{2\alpha}{N^2\beta}$	$\frac{2}{N} - \frac{2\alpha}{N^2\beta}$
$\frac{1}{N} \leq \frac{\beta}{\alpha} < \frac{1}{N-1}$	complicated expression	1 – comp. expr.
$\frac{\beta}{\alpha} < \frac{1}{N}$	1	0

Graphical Representation



Proof Outline

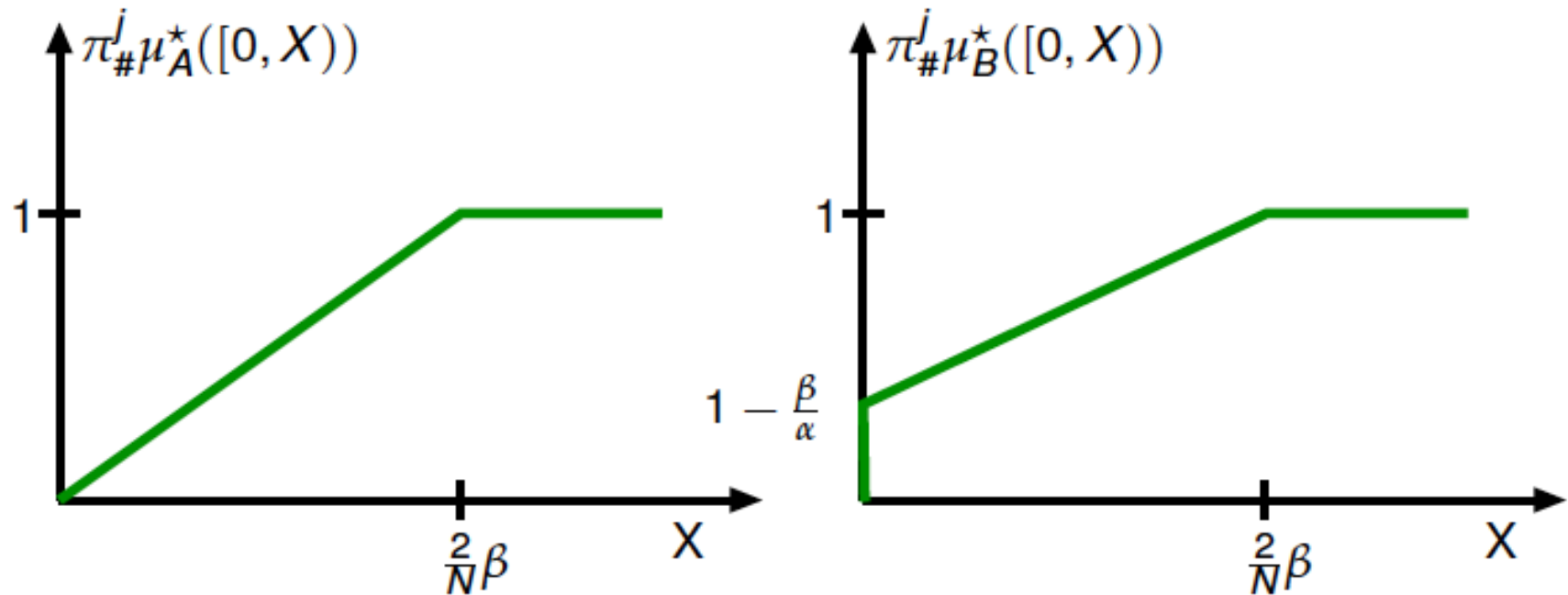
$$J_i(N, \mu_A, \mu_B) = \sum_{j=1}^N \frac{1}{N} \int_0^\infty \pi_{\#}^j \mu_{-i}([0, x)) \pi_{\#}^j \mu_i(dx)$$

such that $\text{supp}(\mu_A) \subset \mathcal{A}$, $\text{supp}(\mu_B) \subset \mathcal{B}$

- Three steps in the proof
- Step 1: Find the reaction curves of the players
 - For a fixed strategy of one player, marginal distribution on each battlefield?
- Step 2: Equilibrium marginal distributions on the battlefields?
- Step 3: Is there a joint distribution that satisfies the constraint on $\text{supp}(\mu_i)$

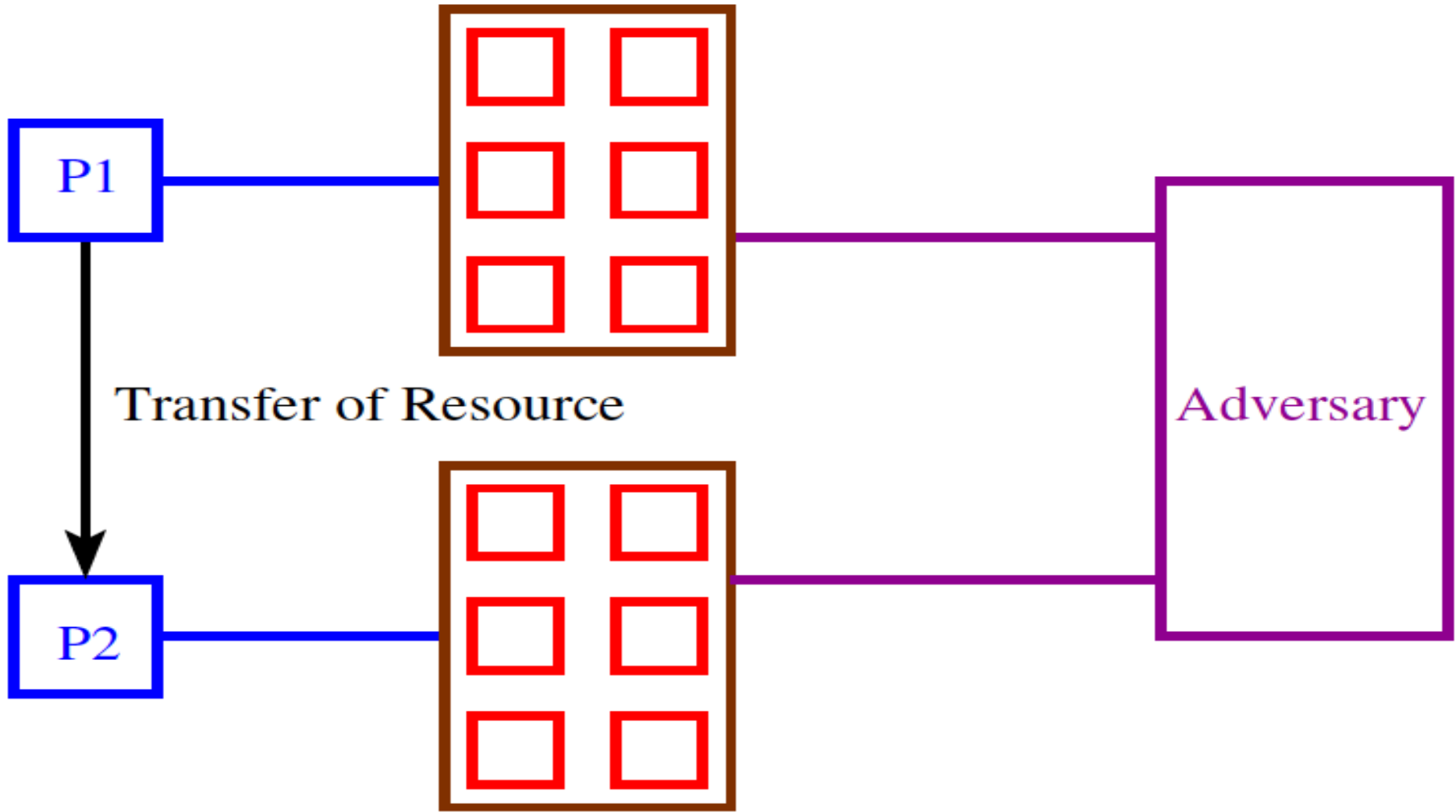
Marginal distributions at Nash Equilibrium

Assume $\frac{2}{N} \leq \frac{\beta}{\alpha} \leq 1$

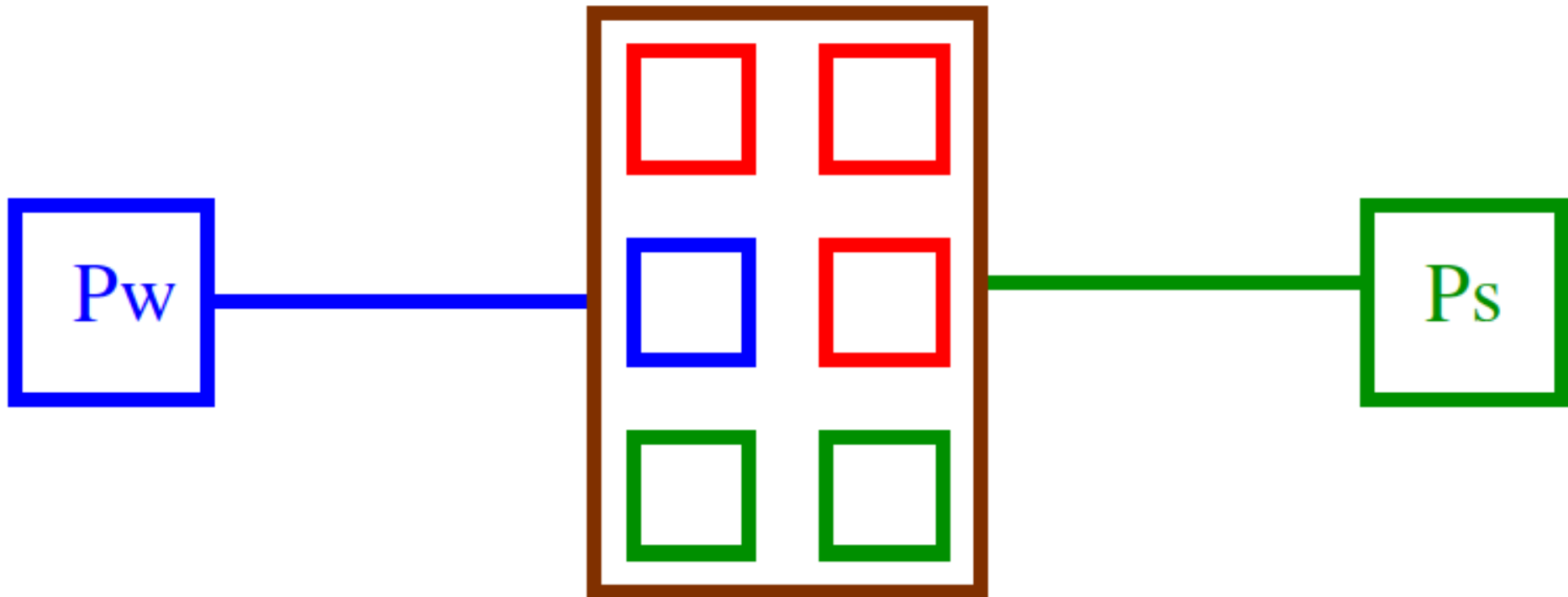


Joint distributions μ_1 and μ_2 are constructed using the marginal distributions in the paper

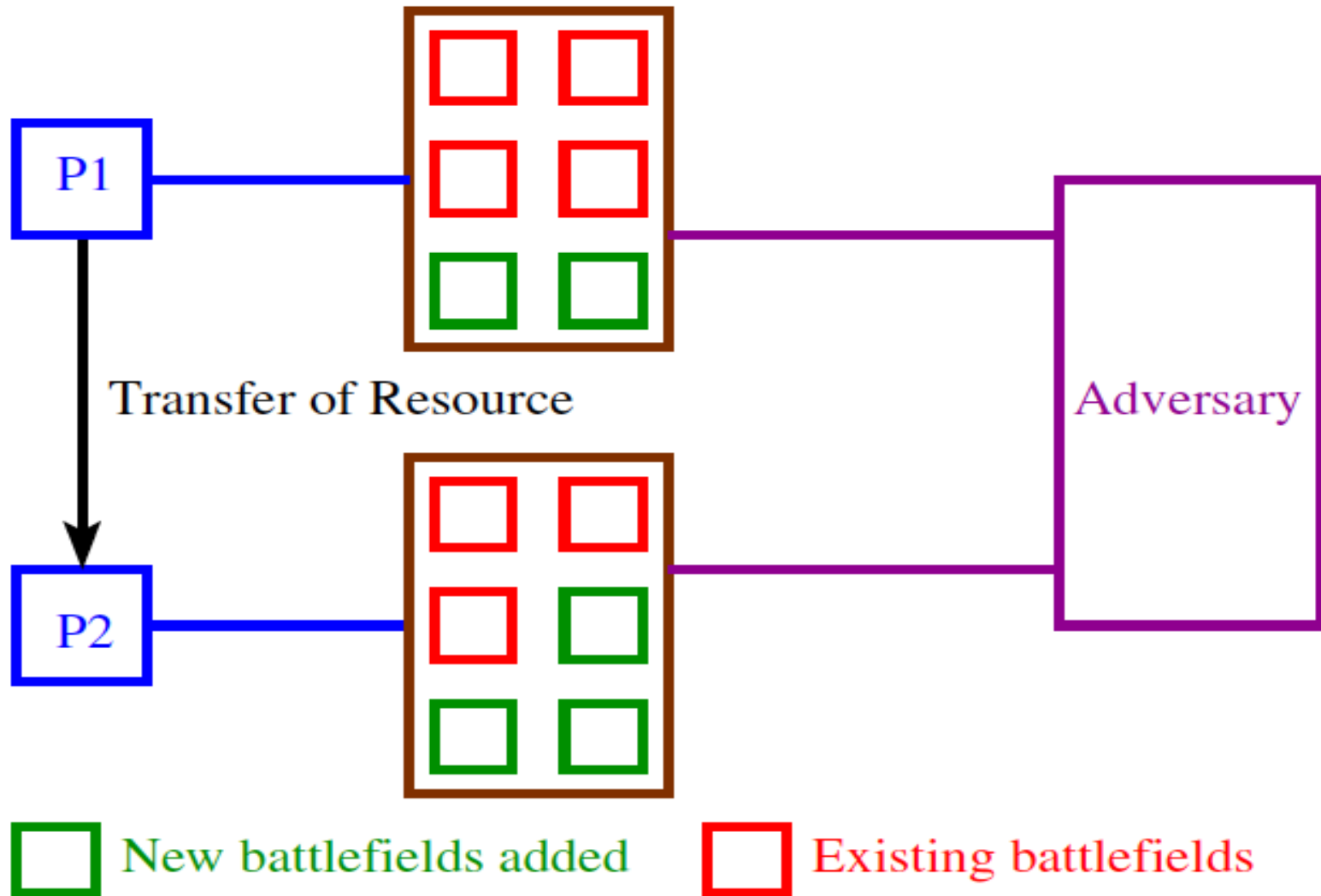
Extension I: Coalitional Colonel Blotto



Extension II: Colonel Blotto with Endogenous Battle fields



Our focus: Coalitional Colonel Blotto with Endogenous Battle Fields



Summary of Results on Blotto Games

- Blotto Games provide key insights into resource allocation
- Generically, Nash equilibrium only in mixed strategies
- Adding battlefields tend to be beneficial to players if it is “cheap enough”
- Voluntary coalitions may be beneficial to players due to effects on equilibrium behavior of the adversary
- Our work
 - We generalize two extensions
 - provides characterization of Nash equilibrium in certain parameter regions

Network Design Game for CPS

A problem of information deficiency

Due to prohibitive costs of determining the cause of a failure, reliability and security failures are frequently **indistinguishable**.

Game with reliability-security failures

- Network: Undirected weighted graph $G = (V, E, w)$
- Network manager: defender
- Failures:
 - Reliability failure R: Due to *Nature*/random fault (π)
 - Security failure S: Due to a strategic attack ($1 - \pi$)
- How should defender design his defenses?

[G. Schwartz, S. Amin, et al.]

Reliability and Security Failures

Game with reliability-security failures

- Network manager: defender
- Failures:
 - Reliability R: random faults/*Nature*
($P(R) = \pi$)
 - Security S: strategic attacker
 $P(S) = 1 - \pi$

- Failure probability of an edge $e \in E$:

$$P(f_e) = \pi\gamma_e + (1 - \pi)\beta_e, \quad \forall e \in E,$$

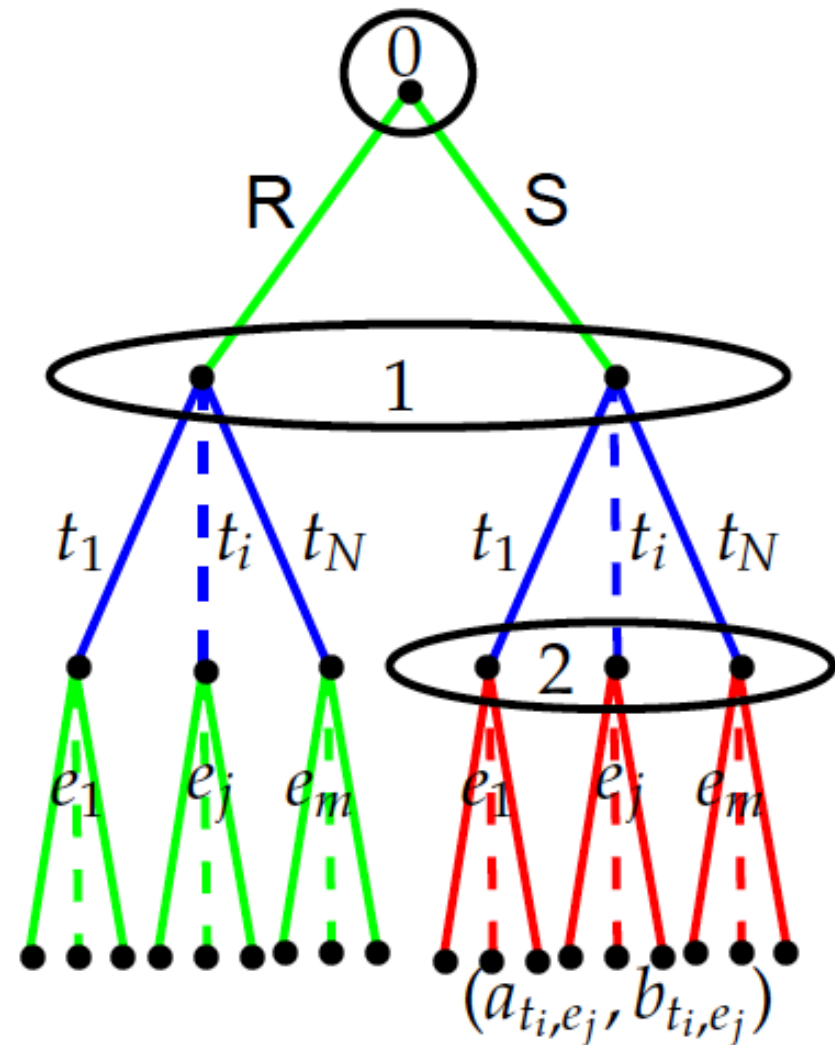
- $\gamma_e = P(f_e | R), \beta_e = P(f_e | S)$

- Fault probabilities:

$$\gamma = (\gamma_{e_1}, \dots, \gamma_{e_j}, \dots, \gamma_{e_m})^T$$

- Attack probabilities:

$$\beta = (\beta_{e_1}, \dots, \beta_{e_j}, \dots, \beta_{e_m})^T$$



Network Design Game

Attacker-Defender subgame

- Defender: chooses a spanning tree $t \in \mathcal{T}$
- Attacker: chooses an edge $e \in E$

Nature-Defender subgame

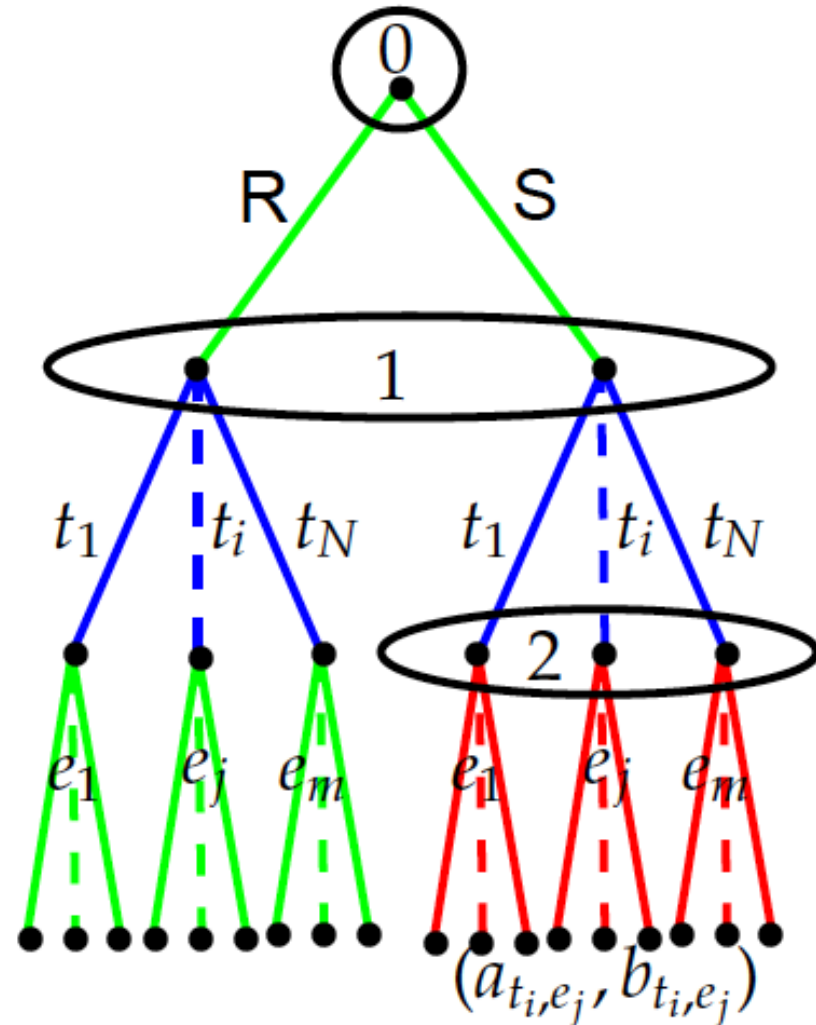
- Defender: chooses a spanning tree $t \in \mathcal{T}$
- Nature: given failure prob. γ_e over edges

Attacker-Nature-Defender game

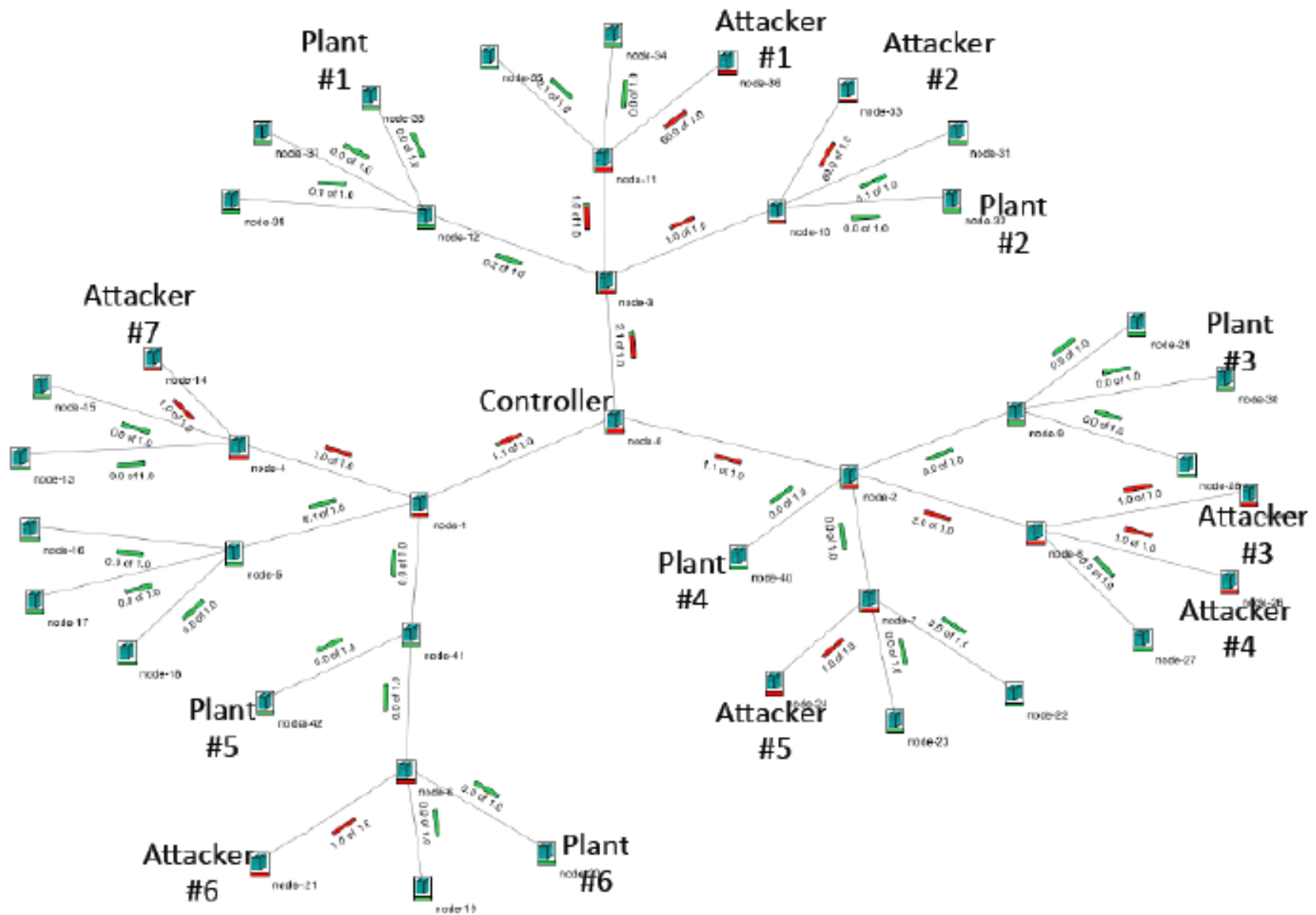
- Imperfect information: defender faces aggregate failure probabilities:

$$P(f_e) = \underbrace{\pi\gamma_e}_{\text{reliability}} + \underbrace{(1-\pi)\beta_e}_{\text{security}}$$

- Common knowledge: π and γ .
- How does Nash Eq. depend on π & γ ?



Application



Summary of the talk

- * Strategic resource allocation (Blotto games):
We characterize equilibria of asymmetric Blotto game with
 - (i) Endogenous number of battlefields
 - (ii) Player alliances
- * Network design with security and reliability failures:
We find all equilibria for a game with
 - (i) General network topology and values of the nodes
 - (ii) Network operator knows only the relative frequencies of random faults and strategic attacks