Stochastic Hybrid Modeling of Flow Network Incidents

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FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Incident management

Network incidents

Water main breaks

Gas pipe bursts

Highway accidents

Our focus

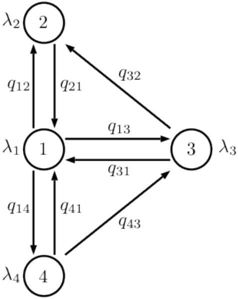
1. Stochastic hybrid dynamical models: combine nonlinear flow dynamics and random jumps due to state-dependent incidents;
2. Model identification: flow data in normal and incident modes;
3. Long-time properties: accessible sets, invariant measures, convergence under closed-loop dynamics

Stochastic incident model

Incident parameters: $(\lambda, \boldsymbol{\alpha})$

λ: occurrence/clearance rate

- Modeled as continuous-time Markov process;
- Incidents induce random jumps;
- Jump rates may depend on continuous state;
- Incident locations / types lead to different modes.



- λ_i=exiting rate from state i
- q_{ij}=transition rate from i to j
- $\lambda_i = \sum_j q_{ij}$
- λ_i and q_{ij} may depend on continuous state

Stochastic incident model

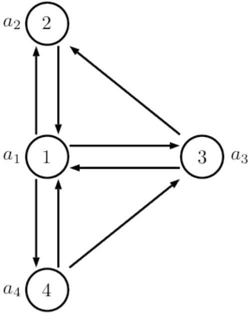
Incident parameters: $(\lambda, \boldsymbol{\alpha})$

$\boldsymbol{\alpha}^j$: Impact or intensity under incident mode j

Reduction in capacity of link i:

$$F_i^j = (1 - \alpha_i^j)F_i^0,$$

$\alpha_i^j \in (0,1)$: reduction factor
F_i^j : capacity in the jth incident mode
F_i^0 : nominal capacity



- Incident mode $j \rightarrow \boldsymbol{\alpha}^j = (\alpha_1, \alpha_2, \cdots, \alpha_N)$
- Switch in continuous dynamics and associated vector field
- A well known result: random switching behavior can lead to undesirable consequences.

Continuous dynamics

Network flow model

- Topology: directed link-node model with OD pair(s)
- Flow function: relationship between flow and density (load)
- Mass conservation differential equations
- Control / routing strategy

Known results for highway dynamics: Varaiya's Theorem

- Dynamics admit multiple equilibria.
- Structure of equilibria depend on system parameters and input.
- All trajectories converge to some stable equilibrium.

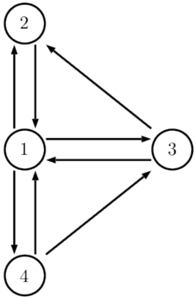
What happens when network is subject to (λ, α^j) type incidents?

Random incidents: Stochastic Hybrid System (SHS)

Incident model and Flow dynamics $\Rightarrow$ SHS

Under the $(\lambda, \boldsymbol{\alpha})$ model, incidents occur/clear randomly:

- State switches between α-dependent dynamics with transition rates λ;
- Deterministic flow dynamics between random switches.



- System switches between incident modes;
- Transition rate λ depends on continuous state (density or load);
- Evolution of the system is an alternation of deterministic motion and random transition.

Long-time properties: Accessible set

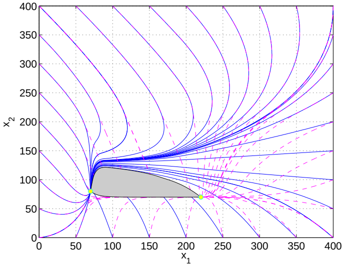
An SHS switching between a set of vector fields may not exhibit convergence. Two important questions arise:

- Set of accessible points and convergence;
- Existence and characterization of invariant measure.

Theorem [Jin and Amin, submitted]

For a given control input and (λ, α) incident model, the set of accessible points exists and is unique. The set supports an invariant measure.

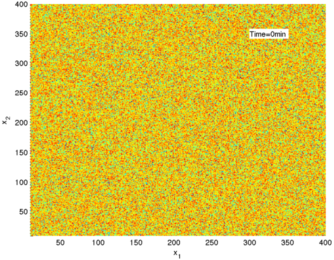
- Starting from any initial condition, the state ends up in the accessible set;
- Shape of the accessible set is determined only by *a*-dependent vector fields.



Long-time properties: Invariant measure

Accessible set supports an invariant measure:

- Distribution of system state after sufficiently long time;
- Depends on both transition rates and deterministic dynamics;
- Useful to
  - Identify the probability of different scenarios
  - Compute expected performance of the SHS
  - Evaluate different control inputs.



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