Auction-Based Coordination of Electric Vehicle Charging

Ian Hiskens
Vennema Professor of Engineering
University of Michigan

In conjunction with:
Zhongjing Ma
Beijing Institute of Technology

FORCES Retreat
June 16-17, 2014

Overview

Objective

- Coordinate the charging of a large number of electric vehicles (EVs).
- EVs act autonomously, distributed decision-making.
- Trade-offs:
  - Each EV desires a certain energy delivery over its charging period.
  - Minimize battery degradation.
  - Minimize total energy cost.

Approach

- Auction-based game.
- Progressive second-price (PSP) auction mechanism.
  - This form of auction achieves incentive compatibility.
  - All participants fare best when they truthfully reveal their private information.

EV charging model

Coordinate charging of a population of EVs, $\mathcal{N} \triangleq \{1, \cdots, N\}$, over a finite charging horizon $\mathcal{T} \triangleq \{0, \cdots, T-1\}$.

- For each EV, $n \in \mathcal{N}$, the energy delivered over the t-th time period is denoted x_{nt}.
- The battery state of charge (SoC) evolves according to

$$s_{n,t+1} = s_{nt} + \frac{1}{\Theta_n} x_{nt}$$

where Θ_n is the battery capacity and s_{nt} is the normalized SoC for the n-th EV at time t.

EV charging model (continued)

An *admissible charging strategy*, $\boldsymbol{x}_n \equiv (x_{nt}, t \in \mathcal{T})$, satisfies the constraints:

$$x_{nt} \begin{cases} \geq 0, & \text{when } t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \quad \text{with} \sum_{t=0}^{T-1} x_{nt} \leq \Gamma_n$$

- $\mathcal{T}_n \subset \mathcal{T}$ denotes the charging interval of the n-th EV.
- $\Gamma_n = \Theta_n(s_n^{max} - s_{n0})$ gives the maximum energy that can be received by the n-th EV.
- $0 \leq s_{n0} \leq s_n^{max} \leq 1$ describes the (normalized) minimum and maximum SoC.

The set of admissible charging strategies is denoted by $\mathcal{X}_n$.

EV utility

The utility function of the n-th EV is given by:

$$w_n(\boldsymbol{x}_n) = -\sum_{t=0}^{T-1} f_n(x_{nt}) - \delta_n \left(\sum_{t=0}^{T-1} x_{nt} - \Gamma_n \right)^2$$

- $f_n(\cdot)$ denotes the battery degradation cost of the n-th EV.
  - A measure of the cost associated with the decrease in the battery capacity due to battery resistance growth.
- The second term captures the cost of not fully charging the EV.
- δ_n is a fixed parameter that weights the relative importance of delivering the full charge.

The utility function $w_n(\boldsymbol{x}_n)$ establishes the tradeoff between the battery degradation cost and the benefit derived from delivering the full charge.

System cost

The system cost is given by:

$$J_s(\boldsymbol{x}) = \sum_{t=0}^{T-1} c\Big(D_t + \sum_{n=1}^{N} x_{nt}\Big) - \sum_{n=1}^{N} w_n(\boldsymbol{x}_n),$$

- Subject to a collection of admissible charging strategies $\boldsymbol{x}$.
- $c(\cdot)$ denotes the generation cost.
- D_t is the aggregate inelastic background demand at time t.
- $D_t + \sum_{n=1}^{N} x_{nt}$ is the total demand at time t.

The centralized EV charging coordination problem can be formulated as:

$$\boldsymbol{x}^{**} = \operatorname*{argmin}_{\boldsymbol{x}\in\mathcal{X}} J_s(\boldsymbol{x})$$

The collection of efficient charging strategies is given by $\boldsymbol{x}^{**}$.

Assumptions

(A1) $c(\cdot)$ is monotonically increasing, strictly convex and differentiable.

(A2) $f_n(\cdot)$, for all $n \in \mathcal{N}$, is monotonically increasing, strictly convex and differentiable.

- The generation cost $c(\cdot)$ is widely assumed to be a convex function of total generation.
- The battery degradation cost $f_n(\cdot)$ is governed by the chemical processes inherent in charging.
  - It has been shown that the growth of battery resistance, hence the fade of battery energy capacity, is generally increasing and convex with respect to charging rate.

Bid profiles of individual EVs

Each EV submits a $2T$-dimensional bid profile:

$$b_{nt} = (\beta_{nt}, d_{nt}), \quad \text{with } d_{nt} \begin{cases} \geq 0, & \text{when } t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \quad \text{and } \sum_{t=0}^{T-1} d_{nt} \leq \Gamma_n$$

- β_{nt} is the price that the n-th EV is willing to pay for energy at time t.
- d_{nt} is the maximum electrical energy that is desired at that time, so $0 \leq x_{nt} \leq d_{nt}$ for all $t \in \mathcal{T}$.
- Let $\boldsymbol{b}_n \equiv (b_{nt}, t \in \mathcal{T})$ and $\mathcal{B}_n$ denote the allowable set of bids for the n-th EV, with $\boldsymbol{b}_n \in \mathcal{B}_n$.

Each EV's revealed utility function is defined as:

$$\widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n) \triangleq \sum_{t=0}^{T-1} \beta_{nt} \min(x_{nt}, d_{nt}) = \sum_{t=0}^{T-1} \beta_{nt} x_{nt}$$

Revealed system cost

The revealed system cost with respect to a collection of bid profiles $\boldsymbol{b} \equiv (\boldsymbol{b}_n, n \in \mathcal{N})$ is given by:

$$J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) = \sum_{t=0}^{T-1} c(D_t + \sum_{n=1}^{N} x_{nt}) - \sum_{n=1}^{N} \widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n)$$

Auction-based EV charging allocation can be written as the optimization problem:

$$J^*(\boldsymbol{b}) = \min_{\boldsymbol{0} \leq \boldsymbol{x} \leq \boldsymbol{d}} J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b})$$

The objective of the auctioneer is to assign an optimal allocation $\boldsymbol{x}^*(\boldsymbol{b})$ with respect to bid profiles $\boldsymbol{b}$ to minimize the revealed system cost J.

Efficient allocation

Lemma

Consider a collection of bid profiles,

$$b_{nt}^* = (\beta_{nt}^*, d_{nt}^*) = \left(\frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n^{**}), x_{nt}^{**} \right)$$

for all $n \in \mathcal{N}$ *and* $t \in \mathcal{T}$. *Then, under Assumptions (A1,A2),* $\boldsymbol{x}^*(\boldsymbol{b}^*) = \boldsymbol{x}^{**}$, *i.e., the optimal charging allocation* $\boldsymbol{x}^*$ *of the auction with respect to* $\boldsymbol{b}^*$ *is efficient. Also,*

$$\beta_{nt}^* \begin{cases} = c'(D_t + \sum_{k=1}^N d_{kt}^*), & \text{if } x_{nt}^* > 0 \\ \leq c'(D_t + \sum_{k=1}^N d_{kt}^*), & \text{if } x_{nt}^* = 0 \end{cases}, \quad \text{for all } n \in \mathcal{N}, t \in \mathcal{T}$$

i.e., all EVs with an allocation larger than zero share the same marginal price as the generation.

Incentive compatibility

Incentive compatibility holds under the PSP auction mechanism.

- A bid profile with price satisfying $\beta_{nt} = \frac{\partial}{\partial d_{nt}} w_n(\boldsymbol{d}_n)$, as is the case in the previous Lemma, is the best choice among all possible bid profiles.
- It follows from the EV utility function that the truth-telling bid profile of the n-th EV is given by:

$$\beta_{nt} = -f_n'(d_{nt}) + 2\delta_n \left(\Gamma_n - \sum_{t=0}^{T-1} d_{nt} \right)$$

- An EV's marginal valuation at each time-step is determined by both its electrical energy request d_{nt} at that time and its total energy request $\sum_t d_{nt}$ over the entire multi-period charging horizon.

EV payment

Each EV's payment is the externality imposed on the system through its participation in the auction.

- For the n-th EV, this is given by the system-wide utility when the n-th EV does not join the auction process, minus the system-wide utility (but excluding the contribution of the n-th EV itself) when the n-th EV joins the auction.
- To express this payment, it is convenient to write the collection of bid profiles as $\boldsymbol{b} \equiv (\boldsymbol{b}_n, \boldsymbol{b}_{-n})$.
- The payment of the n-th EV is given by:

$$\tau_n(\boldsymbol{b}) = -J^*(\mathbf{0}_n, \boldsymbol{b}_{-n}) - \left(-J^*(\boldsymbol{b}) - \sum_{t=0}^{T-1} \beta_{nt} x^*_{nt}(\boldsymbol{b}) \right)$$

  - $(\mathbf{0}_n, \boldsymbol{b}_{-n})$ denotes the bid profile without the n-th EV's participation.
  - $\boldsymbol{x}^*(\boldsymbol{b})$ is the optimal charging allocation with respect to $\boldsymbol{b}$.

EV payoff

The *payoff function* of the *n*-th EV is given by the difference between the EV's utility and its payment:

$$u_n(\boldsymbol{b}) = w_n(\boldsymbol{x}_n^*(\boldsymbol{b})) - \tau_n(\boldsymbol{b})$$

- This payoff function provides the basis for defining a Nash equilibrium for the PSP auction game.

Definition

A collection of bid profiles $\boldsymbol{b}^0$ is a *Nash equilibrium* for the EV charging allocation auction if:

$$u_n(\boldsymbol{b}_n^0, \boldsymbol{b}_{-n}^0) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^0)$$

for all $\boldsymbol{b}_n \in \mathcal{B}_n$ and for all $n \in \mathcal{N}$. That is, no EV can benefit by unilaterally deviating from its bid profile $\boldsymbol{b}_n^0$.

Nash equilibrium

Theorem

Under Assumptions (A1,A2), the efficient bid profile $\boldsymbol{b}^* \equiv (\boldsymbol{b}_n^*; n \in \mathcal{N})$ satisfies the property:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^*), \qquad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n$$

and is therefore a Nash equilibrium for the underlying auction game.

Direct verification that the efficient bid profile is optimal for every individual EV is infeasible.

- This is a consequence of the cross elasticity arising from the summation term in each EV's truth-telling marginal valuation.

Cross elasticity

To proceed, the set of bid profiles $\mathcal{B}_n$ can be partitioned into a collection of subsets:

$$\mathcal{B}_n(A) \triangleq \left\{ \boldsymbol{b}_n \in \mathcal{B}_n; \ \text{s.t.} \ \sum_{t=0}^{T-1} d_{nt} = A \right\}$$

- This eliminates cross elasticity for bid profiles in $\mathcal{B}_n(A)$.

It is sufficient to show that $\boldsymbol{b}^*$ is a Nash equilibrium, if for every fixed $A \in [0, \Gamma_n]$:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*), \quad \text{for all } \widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A),$$

and for all $n \in \mathcal{N}$.

Analysis considers two cases: $A \geq \sum_{t=0}^{T-1} d_{nt}^*$ and $0 \leq A < \sum_{t=0}^{T-1} d_{nt}^*$.

Verification when $A \geq \sum_{t=0}^{T-1} d_{nt}^*$

Let $\boldsymbol{x}^*$ and $\widehat{\boldsymbol{x}}$ denote the optimal allocations with respect to $\boldsymbol{b}^*$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)$, where $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$.

Lemma

If $A \geq \sum_{t=0}^{T-1} d_{nt}^*$ then $\widehat{x}_{mt} = d_{mt}^*$, for all $m \in \mathcal{N} \setminus \{n\}$, i.e., each of the EVs $m \in \mathcal{N} \setminus \{n\}$ is fully allocated.

Using this result, it can be shown that:

$$\Delta u_n \triangleq u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*) \geq 0$$

- The n-th EV cannot benefit by unilaterally changing its bid profile $\boldsymbol{b}_n^*$ to any other bid profile $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$.

Verification when $0 \leq A < \sum_{t=0}^{T-1} d_{nt}^*$

Lemma

Consider a bid profile $\widehat{\boldsymbol{b}}_n^* \equiv \widehat{\boldsymbol{b}}_n^*(A) \equiv \left((\widehat{\beta}_{nt}^*, \widehat{d}_{nt}^*), t \in \mathcal{T}\right)$, *with* $A \in [0, \sum_{t=0}^{T-1} d_{nt}^*)$, *such that*

$$\widehat{\boldsymbol{b}}_n^* = \underset{\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)}{\text{argmax}} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)$$

Let $\widehat{\boldsymbol{x}}^* \equiv (\widehat{x}_{kt}^*, k \in \mathcal{N}, t \in \mathcal{T})$ *denote the optimal allocations with respect to* $(\widehat{\boldsymbol{b}}_n^*, \boldsymbol{b}_{-n}^*)$. *Then,*

$$\widehat{\boldsymbol{x}}_n^* = \widehat{\boldsymbol{d}}_n^*, \qquad \widehat{\boldsymbol{x}}_m^* = \boldsymbol{d}_m^* \text{ for all } m \in \mathcal{N} \setminus \{n\}$$

i.e., all EVs are fully allocated.

It can then be shown that $\Delta u_n \triangleq u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}_n^*, \boldsymbol{b}_{-n}^*) \geq 0$.

Conclusions

- This work considers the tradeoff between:
  - Economics (cost of energy)
  - Resilience (battery degradation)

  in the context of electric vehicle charging.
- Use of the progressive second price (PSP) auction mechanism ensures incentive compatibility.
- The efficient solution is a Nash equilibrium.