FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Operations of Freeways with Capacity Disruptions

Li Jin and Saurabh Amin
Massachusetts Institute of Technology
[jnl, amins]@mit.edu

Berkeley UNIVERSITY OF CALIFORNIA
Massachusetts Institute of Technology
NSF
MICHIGAN
VANDERBILT UNIVERSITY

Motivation: Disruptions in Transportation Systems

* Disruptions: any capacity-reducing events.
* Accidents, bad weather, work zone, etc.

* Freeway capacity: intrinsically a random variable [Brilon 05]
* Major impact: disruption-induced delay

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Main question

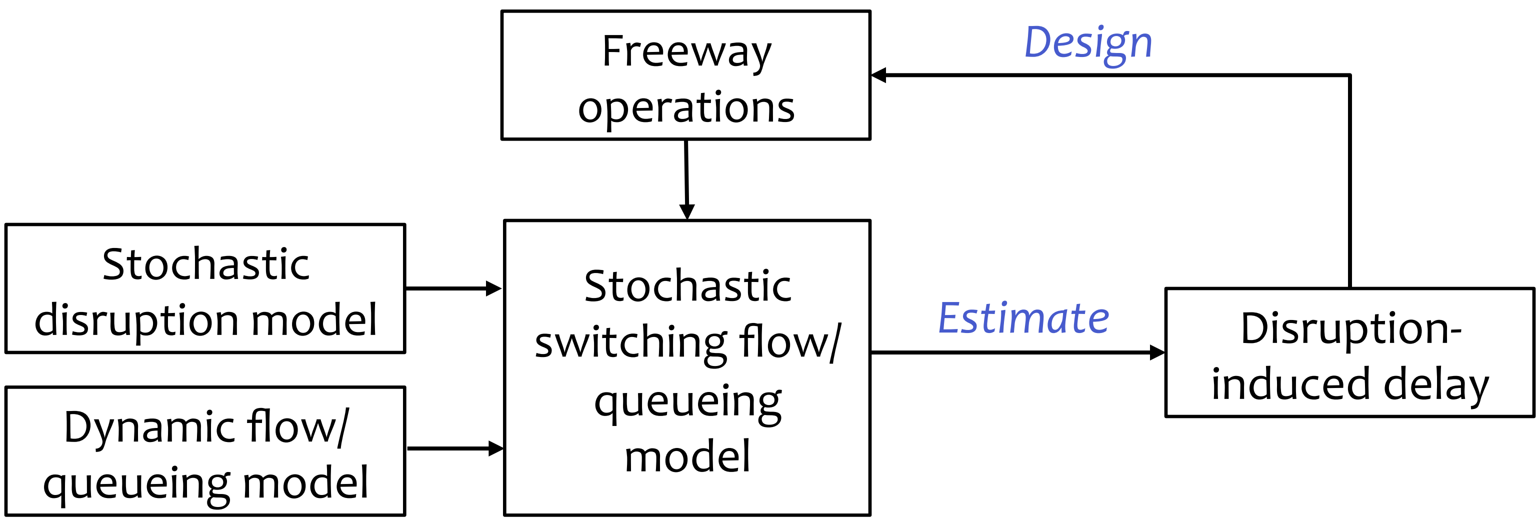
* How to incorporate disruptions into freeway operations design?

* Two steps:
  * Estimate disruption-induced delay.
  * Design freeway operations that minimizes disruption-induced delay.

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Our Method

* Disruption model: Markov chain [Skabardonis 97, Jin 16c]
  * Occurs as Poisson arrivals, duration exponentially distributed
* Traffic model: cell transmission model [Jin 15a], deterministic queueing model [Jin 16b], etc.

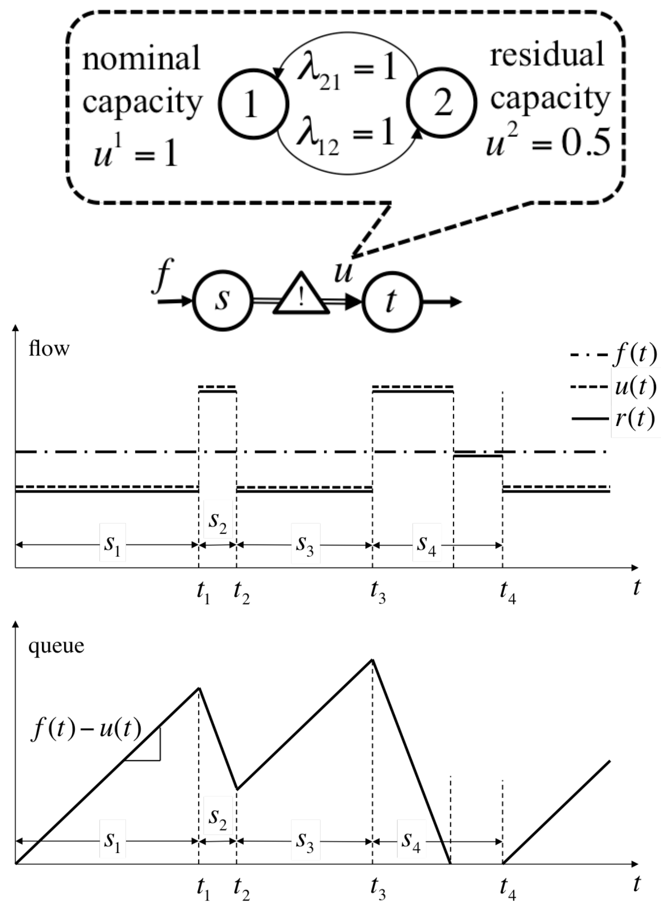


Starting Point: Single Link

* Comprehensive analysis: [Jin 16b]
* Constant inflow rate f = 0.6 per unit time.
* Saturation rate u:
  * Switches between 1.0 and 0.5
  * Markov transitions, $p_1=p_2=0.5$.
* Queue size q: change at rate $f - u$.

➔ Piecewise-deterministic queueing (PDQ) model [Jin 16b]

* Is the queue size q finite WP1?
* Steady-state distribution of q?



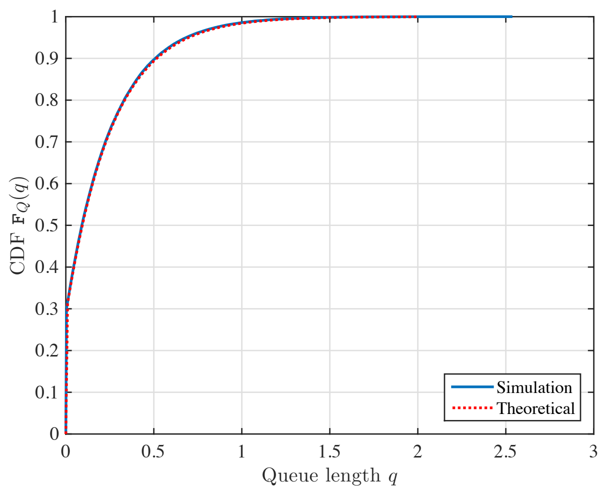
FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

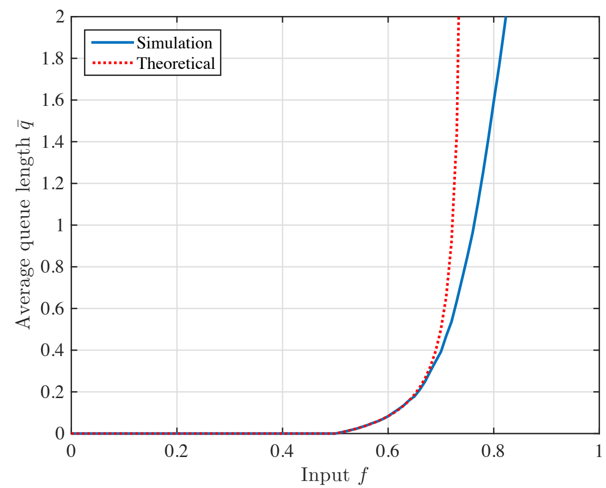
Stability of a Single Link

- Necessary condition: inflow f < time-average saturation rate $\bar{u}$.
  - Based on mass conservation; proof in [Jin 16b].
  - $f = 0.6 < \bar{u} = 0.5(1.0) + 0.5(0.5) = 0.75$.
- $\bar{u}$: effective capacity of the link.
- Sufficient condition: Lyapunov function $V = k_i \exp(bq)$.
  - If there exist $k_1, \ldots k_z$, and b such that $LV < K - CV$, the link is stable.
  - Based on Harris' theorem [Cloez 15]; proof in [Jin 16b].
  - $f = 0.6$ ➔ Exist such as Lyapunov function.
- Necessary condition & sufficient condition generally not equivalent, but turn out to be equivalent in this example.

Queue Size in a Single Link

* Closed-form solution for two-mode systems [Jin 16]
  * Distribution: probability mass at 0 + scaled exponential
  * Expected value: blows up as f approaches $\bar{u}$.

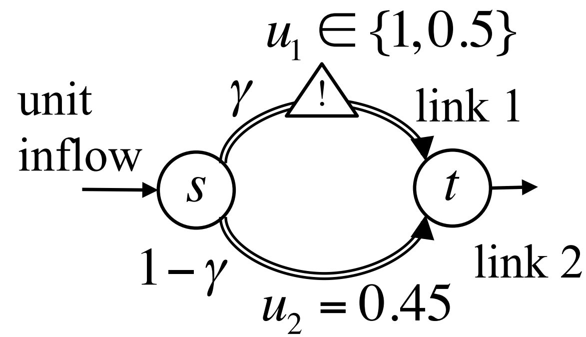




* Queue size (delay) very sensitive to capacity drop and duration.

FORCES FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

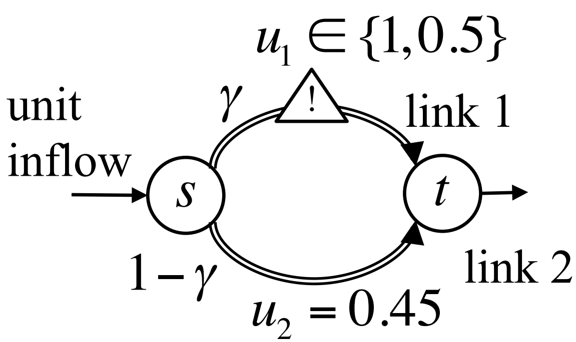
Network Operations: Diversion

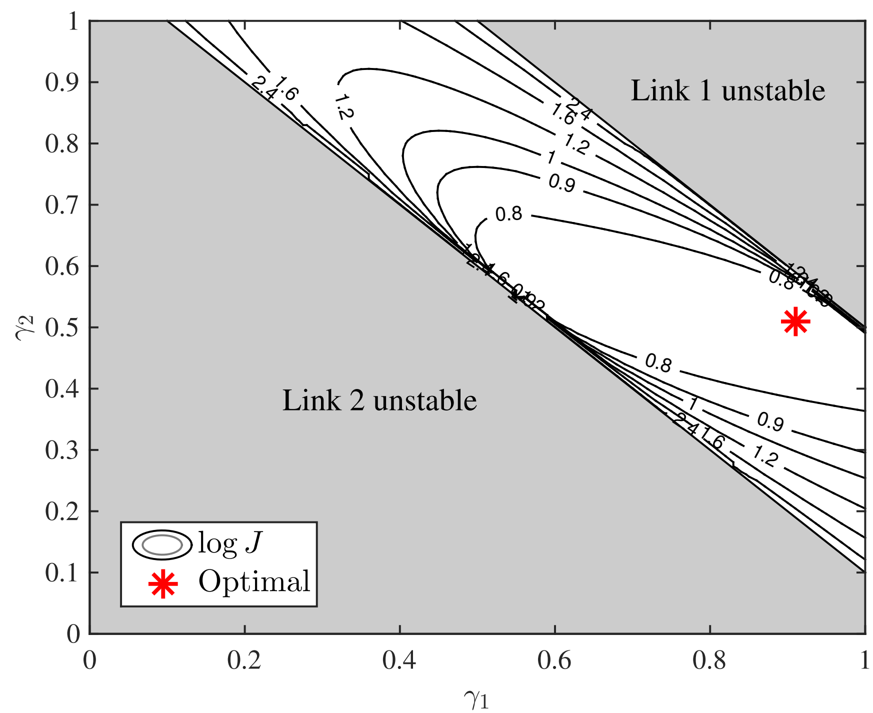


- Two parallel links
  - Link 1: u_1 switches between 1.0 and 0.5.
  - Link 2: no disruptions, $u_2 = 0.45$.
- Diversion during disruption:
  - When $u_1 = 1.0$ (nominal condition), send γ_1 traffic to link 1.
  - When $u_1 = 0.5$ (disruption), send γ_2 traffic to link 1.
- Queues in both links: tradeoff needed.
- Set of diversion policies: $[0,1]^2$
  - What is the set of stable diversion policies?
  - What is the optimal (minimum total queue size) diversion policy?

Design of Diversion during Disruptions

- Set of stable policies:
  - $\gamma_1 + \gamma_2 > 1.1$
  - $\gamma_1 + \gamma_2 < 1.5$
- Optimal policy:
  - $J = q_1 + q_2$
  - $\gamma^* = (0.91, 0.51)$





FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

References

* [Skabardonis 97] Skabardonis, Alexander, et al. "I-880 field experiment: analysis of incident data." *Transportation Research Record: Journal of the Transportation Research Board* 1603 (1997): 72-79.
* [Brilon 05] Brilon, Werner, Justin Geistefeldt, and Matthias Regler. "Reliability of freeway traffic flow: a stochastic concept of capacity." *Proceedings of the 16th International symposium on transportation and traffic theory*. Vol. 125143. Suite 1C, Joseph's Well, Hanover Walk, Leeds LS3 1AB, UK: Maney Publishing, 2005.
* [Cloez 15] Cloez, Bertrand, and Martin Hairer. "Exponential ergodicity for Markov processes with random switching." *Bernoulli* 21.1 (2015): 505-536.
* [Jin 16a] Jin, Li, and Saurabh Amin. "Analysis of a Stochastic Switched Model of Freeway Traffic Incidents." *arXiv preprint arXiv:1601.00204* (2016).
* [Jin 16b] Jin, Li, and Saurabh Amin. "Stability and Control of Piecewise-Deterministic Queueing Systems." *arXiv preprint arXiv:1604.02008* (2016).
* [Jin 16c] Jin, Li, and Saurabh Amin. "Modeling freeway saturation rates as stochastic switching signals." *working paper* (2016).