Informational Braess Paradox: The Effect of Information on Traffic Congestion

Daron Acemoglu, Ali Makhdoumi, Azarakhsh Malekian, Asu Ozdaglar

Department of Economics
Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

Rotman School of Management, University of Toronto

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Motivation

- A variety of GPS-based apps such as Waze now provide real-time traffic information.
- Such apps, which are set to become more widespread in the years to come, promise to improve traffic flows or at the very least reduce congestion by their users.
- In this paper, we study the efficiency implications of one aspect of such apps — to provide more information (about possible routes) to a subset of users.
- Our approach is based on a generalization of the classic traffic flows model of Beckmann et al. using Wardrop equilibrium.
- Our approach complements the important work [Amin 15], studying welfare implications of greater information in a strategic incomplete information two-player game.

Model Outline

- We study traffic flows over an arbitrary network linking a single origin and a single destination.
  - Each edge represented by a cost function.
  - There are two types of motorists, each aware of a subset of edges – information set (each travel through paths linking the origin-destination pair consisting of the edges in their information set).
- We characterize the constrained Wardrop equilibrium in this environment — where each motorist chooses the lowest-cost path among those in his information set.
- Key Question: Consider an expansion of the information set of one type of motorists (say group 2). Does this necessarily lead to reduced travel time for this group (or overall congestion)?
  - Although intuitive answer yes, several examples in game theory/ informational economics indicating why this may not be the case.
  - For instance, Hirshleifer (1979) showed that extra information available before entering into co-insurance arrangements may reduce welfare because it destroys insurance opportunities.

Informational Braess Paradox

- We relate this question to well-known Braess Paradox in traffic routing, whereby decreasing cost functions in the network can increase travel times.
- We say that there is an informational Braess paradox if providing information about additional edges to the group of motorists increases their travel time.
    - note that it would not be surprising for such information to increase the travel times of other motorists (who may see greater congestion in the paths second group was not able to use previously).
    - the surprising outcome would be for the group that is becoming more informed to experience worse outcomes.

Main Results

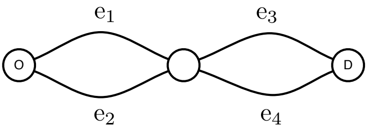
- We provide a sharp characterization of network topologies that lead to informational Braess paradox.
- In particular, we show the following:
  - if the network is series-parallel, there cannot be informational Braess paradox.
  - if the network is not series-parallel, then there exists an assignment of latency functions which will lead to the informational Braess paradox.
- We also investigate conditions on the additional information provided to the motorists for the emergence of the informational Braess paradox.
- Our result relates to the seminal paper [Milchtaich 06], which provided characterization of network structures that can lead to Braess paradox.
  - Differs both in terms of substance (by considering different types of users) and in analysis.
- Our ongoing work extends these results to a network consisting of multiple origin-destination pairs.

Related Literature

- Our work is related to the literature on user/traffic/Wardrop equilibrium:
  - [Beckmann, McGuire, Winsten 56], [Dafermos and Nagurney 84]
  - [Roughgarden and Tardos 00], [Correa, Schulz, Stier-Moses 06]: on the inefficiency of Wardrop equilibria over networks.
  - [Milchtaich 06]: on the structure of networks leading to Braess paradox.
  - [Acemoglu and Ozdaglar 07], [Acemoglu, Ozdaglar, Xin 08]: on decentralized pricing as a solution to network flow inefficiencies.
  - [Acemoglu, Johari, Ozdaglar 09]: on the inefficiency of hybrid routing schemes.
- Most closely related is the complementary work [Amin 15], which studies the impact of game theoretic/strategic interactions and incomplete information.
  - Here the focus is on an environment with "small" users that take the level of congestion on all edges as given.

Traffic Network With Multiple Information Types

- Consider a network represented by an undirected graph together with a single distinct pair of nodes, an origin O and a destination D.
  - Assume that each node and edge belongs to one path from O to D.
- Suppose there are two types of motorists.
  - Type i motorist has a prescribed amount of traffic $r_i \in [0, 1]$.
  - Type i motorist knows a subset of edges $\mathcal{E}_i$ called information set of type i.
  - $\mathcal{P}_i$: set of paths from O to D using edges in $\mathcal{E}_i$ (we focus on admissible information sets, in the sense that the resulting $\mathcal{P}_i$ is nonempty).
- Example: Information sets: $\mathcal{E}_1 = \{e_1, e_3, e_4\}$ and $\mathcal{E}_2 = \{e_1, e_2, e_4\}$.
  - These lead to $\mathcal{P}_1 = \{e_1 e_3, e_1 e_4\}$ and $\mathcal{P}_2 = \{e_1 e_4, e_2 e_4\}$.



Traffic Network With Multiple Information Types

- We refer to $(G, \mathcal{E}_1, \mathcal{E}_2, r_1, r_2)$ as a traffic network with multiple information types.
- A feasible flow is a flow vector $(f^{(1)}, f^{(2)})$ such that for $i = 1, 2$:

$$f^{(i)} : \mathcal{P}_i \to \mathbb{R}^+,$$
$$\sum_{p \in \mathcal{P}_i} f^{(i)}(p) = r_i.$$

- We assign to each edge $e \in E$ a cost function $c_e : \mathbb{R}^+ \to \mathbb{R}^+$ that is nonnegative, nondecreasing, and continuous.

Constrained Wardrop Equilibrium

- The total flow on each edge $e \in E$ is $f_e = \sum_{e \in p} f^{(1)}(p) + f^{(2)}(p)$.
- The cost of a path p for a given flow vector $(f^{(1)}, f^{(2)})$ is

$$c_p(f^{(1)}, f^{(2)}) = \sum_{e \in p} c_e(f_e).$$

Definition (Constrained Wardrop Equilibrium (CWE))

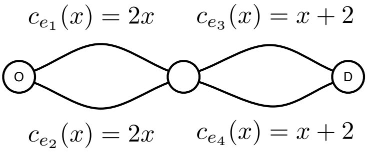
Consider a traffic network with multiple information types $(G, \mathcal{E}_1, \mathcal{E}_2, r_1, r_2)$ and an assignment of cost functions $\{c_e\}$. A flow vector $(f^{(1)}, f^{(2)})$ is a constrained Wardrop equilibrium if

- $(f^{(1)}, f^{(2)})$ is feasible.
- All of the paths used by type i have minimal cost, i.e.,
- $p, \hat{p} \in \mathcal{P}_i$ with $f^{(i)}(p) > 0$, then $c_p(f^{(1)}, f^{(2)}) \leq c_{\hat{p}}(f^{(1)}, f^{(2)})$.

- Equilibrium Cost of type i denoted by $c^{(i)}(f^{(1)}, f^{(2)})$: $c_p(f^{(1)}, f^{(2)})$ for $p \in \mathcal{P}_i$ such that $f^{(i)}(p) > 0$.

Constrained Wardrop Equilibrium

- Example: $\mathcal{E}_1 = \{e_1, e_3, e_4\}$, $\mathcal{E}_2 = \{e_2, e_4\}$, $r_1 = r$, $r_2 = 1 - r$, for $r \in (0, 1)$.
- There are two cases:
  - $r \leq \frac{1}{2}$:
    - $f^{(1)}(e_1 e_3) = r$, and $f^{(2)}(e_2 e_4) = 1 - r$.
    - Equilibrium costs: $c^{(1)} = 3r + 2$, $c^{(2)} = 5 - 3r$.
  - $r > \frac{1}{2}$:
    - $f^{(1)}(e_1 e_3) = \frac{1}{2}$, $f^{(1)}(e_1 e_4) = r - \frac{1}{2}$, and $f^{(2)}(e_2 e_4) = 1 - r$.
    - Equilibrium costs: $c^{(1)} = 2r + \frac{5}{2}$, $c^{(2)} = \frac{9}{2} - 2r$.



Potential Function of CWE

Proposition

A feasible flow $(f^{(1)}, f^{(2)})$ is a constrained Wardrop equilibrium if and only if it is a solution to

$$\min \sum_{e \in E} \int_0^{f_e} c_e(z) dz$$

$$f_e = \sum_{e \in p} f_p^{(1)} + f_p^{(2)},$$

$$\sum_{p \in \mathcal{P}_i} f_p^{(i)} = r_i, \text{ and } f_p^{(i)} \geq 0 \text{ for all } p \in \mathcal{P}_i.$$

We call $\Phi \triangleq \sum_{e \in E} \int_0^{f_e} c_e(z) dz$ the potential function.

- Proof: Since the objective function is convex, first order condition of the Lagrangian characterizes the solution. FOC coincides with the definition of CWE.

Existence and Uniqueness of CWE

Theorem

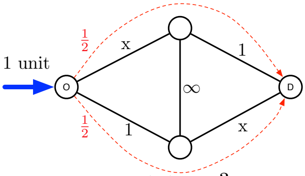
Consider a traffic network with multiple information types $(G, \mathcal{E}_1, \mathcal{E}_2, r_1, r_2)$ and an assignment of cost functions $\{c_e\}$.

- *(Existence)There exists a constrained Wardrop equilibrium $(f^{(1)}, f^{(2)})$.*
- *(Essential Uniqueness) If $(f^{(1)}, f^{(2)})$ and $(\tilde{f}^1, \tilde{f}^2)$ are both constrained Wardrop equilibria, then $c_e(f_e) = c_e(\tilde{f}_e)$ for every edge $e \in E$.*

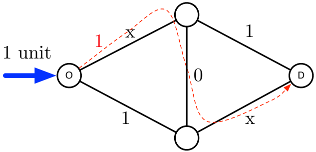
- Proof: Using Extreme Value Theorem of Weierstrass, the potential function Φ attains its minimum, showing the existence of CWE.
  Let $(f^{(1)}, f^{(2)})$ and $(\tilde{f}^1, \tilde{f}^2)$ be two equilibria. By convexity
  $$\Phi(\alpha(f^{(1)}, f^{(2)}) + (1-\alpha)(\tilde{f}^1, \tilde{f}^2)) \leq \alpha\Phi(f^{(1)}, f^{(2)}) + (1-\alpha)\Phi(\tilde{f}^1, \tilde{f}^2),$$
  for every $\alpha \in [0, 1]$. Since $\Phi(f^{(1)}, f^{(2)})$ and $\Phi(\tilde{f}^1, \tilde{f}^2)$ are both global minima of Φ, the functions $\int_0^{f_e} c_e(z)dz$ for any $e \in E$ must be linear between values of f_e and $\tilde{f}_e$. This shows that all cost functions c_e are constant between f_e and $\tilde{f}_e$.

Impact of Extra Information on Equilibrium Cost

- Key Question: Does expansion of information sets lead to improved equilibrium costs?
- Related questions in literature:
  - Effect of decreasing cost functions on equilibrium cost (with only one type of motorist).
  - Braess paradox: Equilibrium cost **increases** by decreasing cost funcs.
  - Braess paradox occurs in a Wheatstone graph [Braess 68, Arnott, Small 94].



$c(f) = \frac{3}{2}$



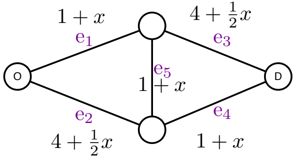
$c(\tilde{f}) = 2$

Informational Braess Paradox (IBP)

- Intuitively, more information should help the motorists.
- Consider a traffic network with multiple information types $(G, \mathcal{E}_1, \mathcal{E}_2, r_1, r_2)$ and an assignment of cost functions $\{c_e\}$.
- Suppose we provide extra information to type two motorists leading to information set $\tilde{\mathcal{E}}_2$ with $\mathcal{E}_2 \subset \tilde{\mathcal{E}}_2$.
- Denote the equilibrium cost of type 2 motorists under the two information structures by $c^{(2)}(f^{(1)}, f^{(2)})$ and $c^{(2)}(\tilde{f}^{(1)}, \tilde{f}^{(2)})$
- We say that Informational Braess paradox (IBP) occurs in G if the equilibrium cost of type two motorists increases after adding information, i.e.,
$$c^{(2)}(\tilde{f}^{(1)}, \tilde{f}^{(2)}) > c^{(2)}(f^{(1)}, f^{(2)}).$$

Informational Braess Paradox (IBP)

- Example: Consider Wheatstone graph with $r_1 = r$ and $r_2 = 1 - r$, $r \in (0,1)$ and the following cost functions:



- $\mathcal{E}_1 = \{e_1, e_2, e_3, e_4, e_5\}$, $\mathcal{E}_2 = \{e_1, e_2, e_3, e_4\}$.
  - CWE: $f^{(1)}(e_1 e_5 e_4) = r$ and $f^{(2)}(e_1 e_3) = f^{(2)}(e_2 e_4) = \frac{1}{2}(1 - r)$.
  - Equilibrium costs: $c^{(1)} = 4 + 2r$ and $c^{(2)} = \frac{23}{4} + \frac{1}{4}r$
- $\mathcal{E}_1 = \{e_1, e_2, e_3, e_4, e_5\}$, $\tilde{\mathcal{E}}_2 = \{e_1, e_2, e_3, e_4, e_5\}$.
  - CWE: $\tilde{f}^{(1)}(e_1 e_5 e_4) = r$ and $\tilde{f}^{(2)}(e_1 e_5 e_4) = (1 - r)$.
  - Equilibrium costs: $\tilde{c}^{(1)} = \tilde{c}^{(2)} = 6$.
- IBP occurs: cost of both types has increased after providing extra information to type two, i.e., $\tilde{c}^{(i)} > c^{(i)}$ for $i = 1, 2$.

Series-Parallel Graphs

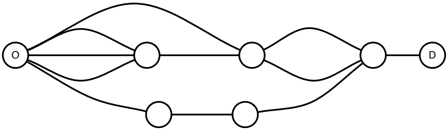
- Which network topologies lead to informational Braess paradox?
- Two networks G and G' with the same OD pair (but no other common nodes or edges) can be connected in parallel.
- Two networks G and G' with a single common node (destination in G and origin in G') can be connected in series.

Definition: We say that a graph G is Series-Parallel if:

(i) It only has a single edge.

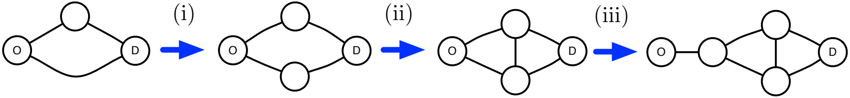
(ii) It is the result of attaching two series-parallel graphs in series.

(iii) It is the result of attaching two series-parallel graphs in parallel.



Characterization of Series-Parallel Graphs

- We say that a network G' is embedded in a network G if G is derived from G' by applying the following operations (any number of times in any order):
  - (i) Dividing an existing edge by replacing it with two edges with a single common end node.
  - (ii) Adding one edge between two nodes.
  - (iii) Extending origin or destination by one edge.



Proposition (Duffin, 65)

A graph G is series-parallel if and only if the Wheatstone graph is not embedded in G.

Network Topology and IBP

Theorem

Consider a traffic network with multiple information types $(G, \mathcal{E}_1, \mathcal{E}_2, r_1, r_2)$ *and assume* $\mathcal{E}_1 = E$. *Informational Braess paradox does not occur in* G *if and only if* G *is series-parallel. More precisely:*

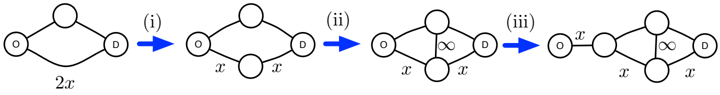
- *if* G *is not series parallel, then there exist information sets* $\mathcal{E}_1$, $\mathcal{E}_2$, $\tilde{\mathcal{E}}_2$, *and an assignment of cost functions for which informational Braess's paradox occurs.*
- *if* G *is series-parallel, then for any assignment of information sets* $\mathcal{E}_1$, $\mathcal{E}_2$, $\tilde{\mathcal{E}}_2$, *and for any assignment of cost functions, informational Braess paradox does not occur.*

Remarks:

- The assumption $\mathcal{E}_1 = E$ can be relaxed to $\mathcal{E}_1 \supset \tilde{\mathcal{E}}_2$.
- We conjecture that the second statement holds under the weaker assumption that the subgraph of G induced by $\mathcal{E}_1 \cup \tilde{\mathcal{E}}_2$ is series-parallel.

Proof Sketch

- Suppose G is not series parallel.
  - It follows from Duffin's characterization that Wheatstone graph G' is embedded in G.
  - Assign cost functions of the example to G'.
  - Construct G from G' using 3 operations:
    - When dividing an edge: assign half of its cost to each of the new edges (add these edges to an information set if the original edge was in it).
    - When adding an edge: assign infinity cost.
    - When extending origin or destination, assign cost function $c_e(x) = x$ (add this edge to all information sets).



- Prove the other direction by induction on the number of edges and using the recursive definition of a series-parallel graph.

Conclusions and Ongoing Work

- We presented a novel framework to study the effect of extra information on the equilibrium cost in traffic networks.
  - We introduced constrained Wardrop equilibrium and informational Braess paradox.
  - We showed informational Braess paradox does not occur in graph G if and only if the graph is series-parallel.
- Ongoing and Future Work:
  - Conditions on additional information for occurrence of IBP.
  - Extension to multiple OD pairs.
  - Extension to multiple (more than two) types.
  - Characterizing price of anarchy for constrained Wardrop equilibrium.