FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS

Control of Water Networks: Geometric Programming Approach

Lina Sela and Saurabh Amin
Dept. of Civil & Environmental Engineering, MIT, MA, USA

Berkeley UNIVERSITY OF CALIFORNIA
Massachusetts Institute of Technology
NSF
MICHIGAN
VANDERBILT UNIVERSITY

Resilient Water Networks

Cyber-Physical Systems

Network sensing

- Monitor pressure and water quality
- Data acquisition and telemetry

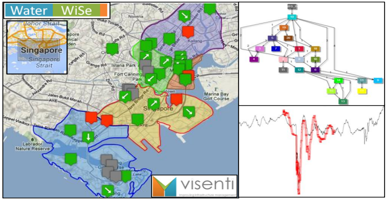
Analytics

- Hydraulic modeling and simulation
- Event detection, localization, and classification
- Data visualization

Research scope

- Network control
- Sensor placement
- System vulnerability
- Fault diagnostics

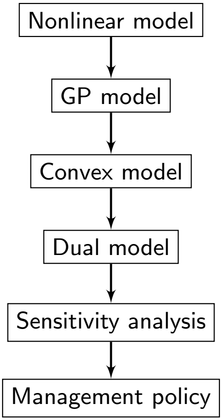
Water CPS, Singapore



MIT, SMART-CENSAM, Visenti Pte., PUB, NRF Singapore

Network Control Model

Workflow



Nonlinear Network Control Model

$$
\begin{array}{lcr}
\underset{H,\beta,\gamma,s,q}{\text{minimize}} & \displaystyle\sum_{i\in E_{pp}} c_{1i} q_i \beta_i^{m_{1i}} + \sum_{i\in E_v} c_{2i}\gamma_i^{-m_{2i}} & \\
\textit{(Cost)} & \displaystyle + \sum_{i\in N_D} c_{3i} H_i^{-m_{3i}} + \sum_{i\in N_D} c_{4i} d_i s_i^{-m_{4i}} & \\
\text{subject to} & & \\
\textit{(Pump)} & H_j = \beta_k H_i & \forall k \in E_{pp} \\
\textit{(Valve)} & H_j = \gamma_k H_i & \forall k \in E_v \\
\textit{(Headloss)} & H_j + R_k q_k^{\alpha} \leq H_i & \forall k \in E_p \\
\textit{(Flow)} & \sum q_{j,out} + d_i s_i \leq q_i & \forall i \in N_{D_{inter}} \\
\textit{(Shedding)} & q_i = d_i s_i & \forall i \in N_{D_{end}} \\
\textit{(Resource)} & q_i \beta_i \leq \overline{Po}_i & \forall i \in E_{pp} \\
\textit{(Pressure)} & \underline{H_i} \leq H_i \leq \overline{H_i} & \forall i \in N_D \\
\textit{(Power)} & 1 \leq \beta_i \leq \overline{\beta}_i & \forall i \in E_{pp} \\
\textit{(Relief)} & 0 \leq \gamma_i \leq 1 & \forall i \in E_v
\end{array}
$$

Methodology: GP approach

Geometric Program

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le 1 \quad \forall i \\ & h_j(x) = 1 \quad \forall j \\ & x \ge 0 \end{aligned}$$

Primal Convex

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & p_0(A_0 x + b_0) \\ \text{subject to} \quad & p_i(A_i x + b_i) \le 0 \quad \forall i \\ & A_J x + b_J \le 0 \\ & A_K x + b_K = 0 \end{aligned}$$

p_i – $LogSumExp$ function

Lagrange Dual

$$\underset{\lambda,\nu}{\text{maximize}} \quad \sum_{i=0}^{m}\sum_{j=1}^{k_0} \nu_{ij} b_{ij} + \nu_K^T b_K + \lambda_J^T b_J$$

$$- \sum_{j=1}^{k_0} \nu_{0j} \log \nu_{0j} - \sum_{i=1}^{m}\sum_{j=1}^{k_i} \nu_{ij} \log(\nu_{ij}/\lambda_i)$$

$$\text{subject to} \quad \sum_{i=0}^{m} A_i^T \nu_i + A_K^T \nu_j + A_J^T \lambda_j = 0$$

$$\nu_0 \succeq 0, \qquad \mathbb{1}^T \nu_0 = 1 \qquad \forall i$$

$$\nu_i \succeq 0, \qquad \mathbb{1}^T \nu_i = \lambda_i \qquad \forall i$$

$$\lambda_i \ge 0$$

Sensitivity Analysis

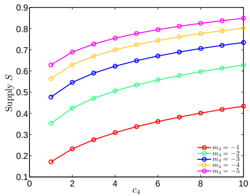
$$p^*(u, v) \ge p^*(0,0) - {\lambda^*}^T u - {\nu^*}^T v$$

$$\lambda_i^* > 0 \Rightarrow p_i(x^*) = 0 \quad \lambda_i^* = 0 \Rightarrow p_i(x^*) < 0$$

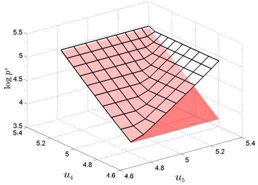
Application

Sensitivity analysis

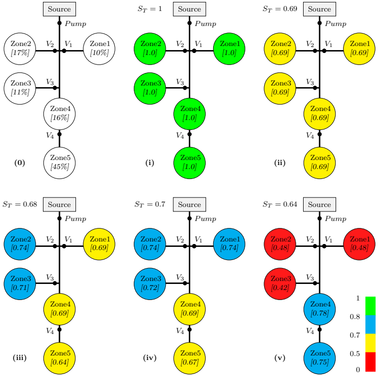
Tradeoff curves:
Demand shedding vs. cost



Perturbation analysis



Demand shedding



(i) Zero demand shedding
(ii) Limited resources and collective demand shedding
(iii) Limited resources and individual demand shedding with low penalties
(iv) As (iii) with high penalties
(v) As (iii) with mixed penalties

FORCES
FOUNDATIONS OF RESILIENT CYBER-PHYSICAL SYSTEMS