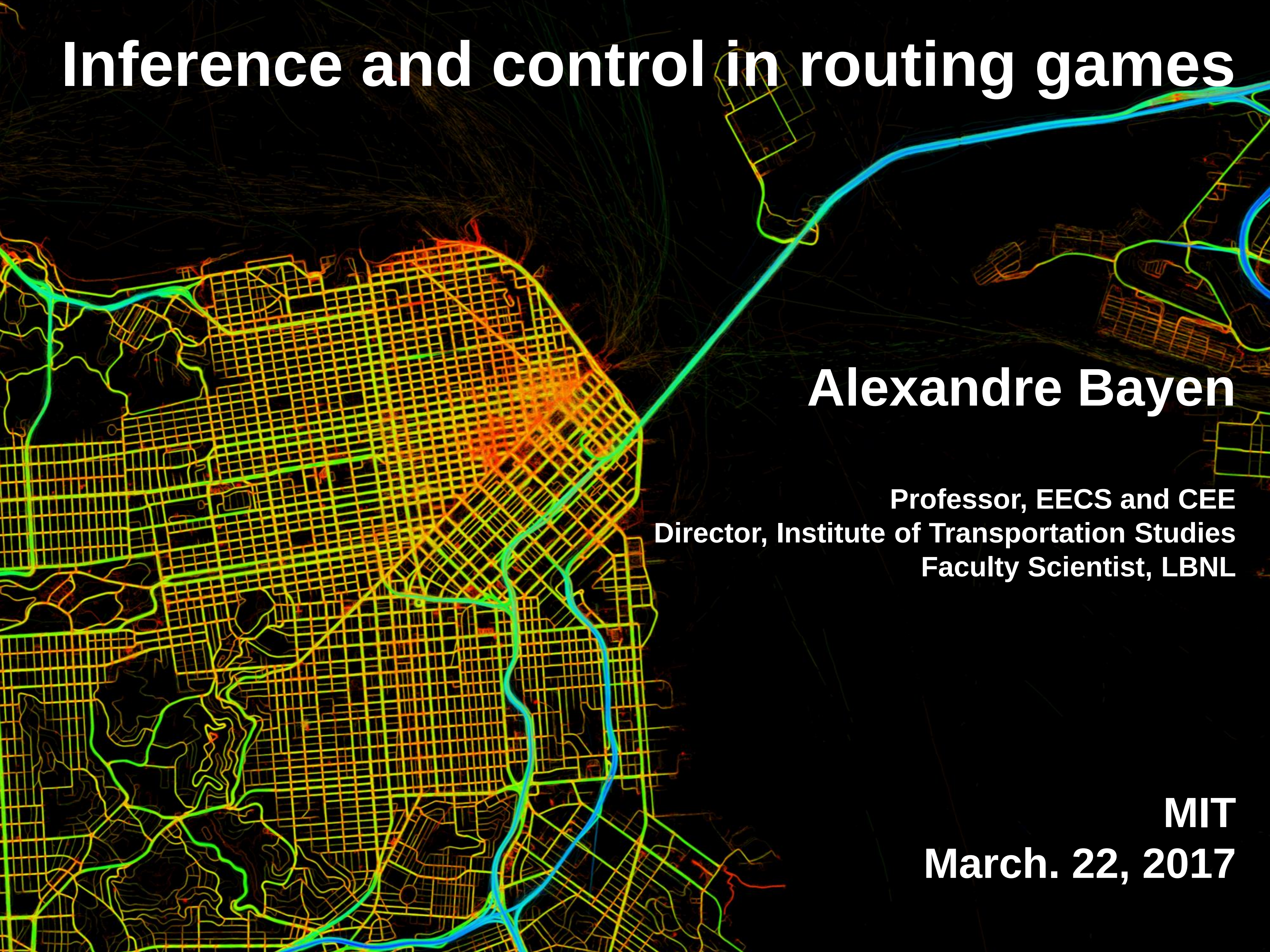


Inference and control in routing games

Alexandre Bayen

Professor, EECS and CEE
Director, Institute of Transportation Studies
Faculty Scientist, LBNL

MIT
March. 22, 2017

Outline

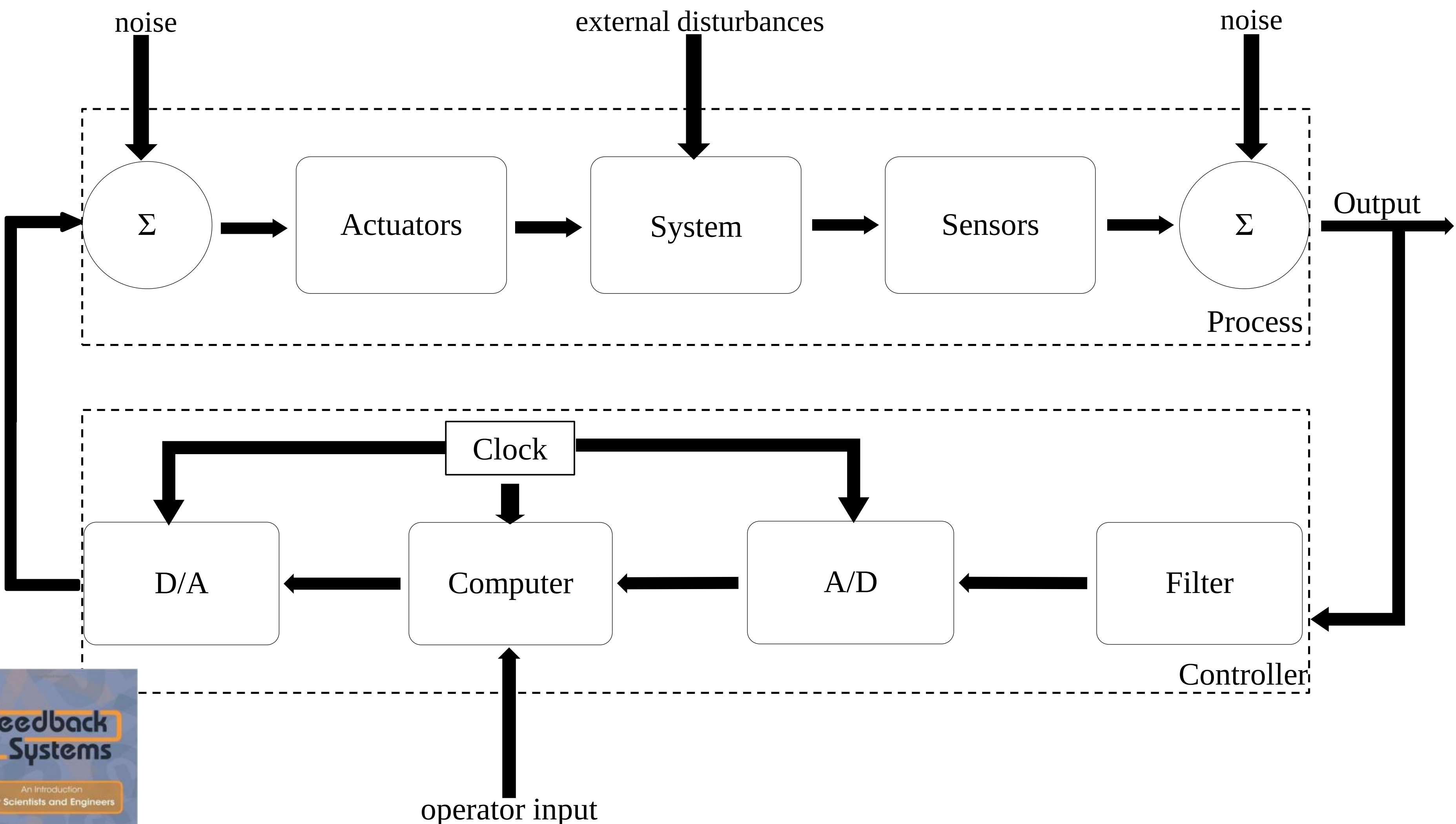
1. **General framework for traffic operations**

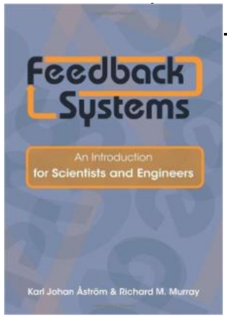
1. **Inference problems**
   1. **Demand inference**
   2. **Traffic estimation**

2. **Heterogeneous games**
   1. **Heterogeneous game, Nash-Stackelberg solutions**
   2. **Learning dynamics in repeated games**

3. **Other mobile sensor and data and CPS education**

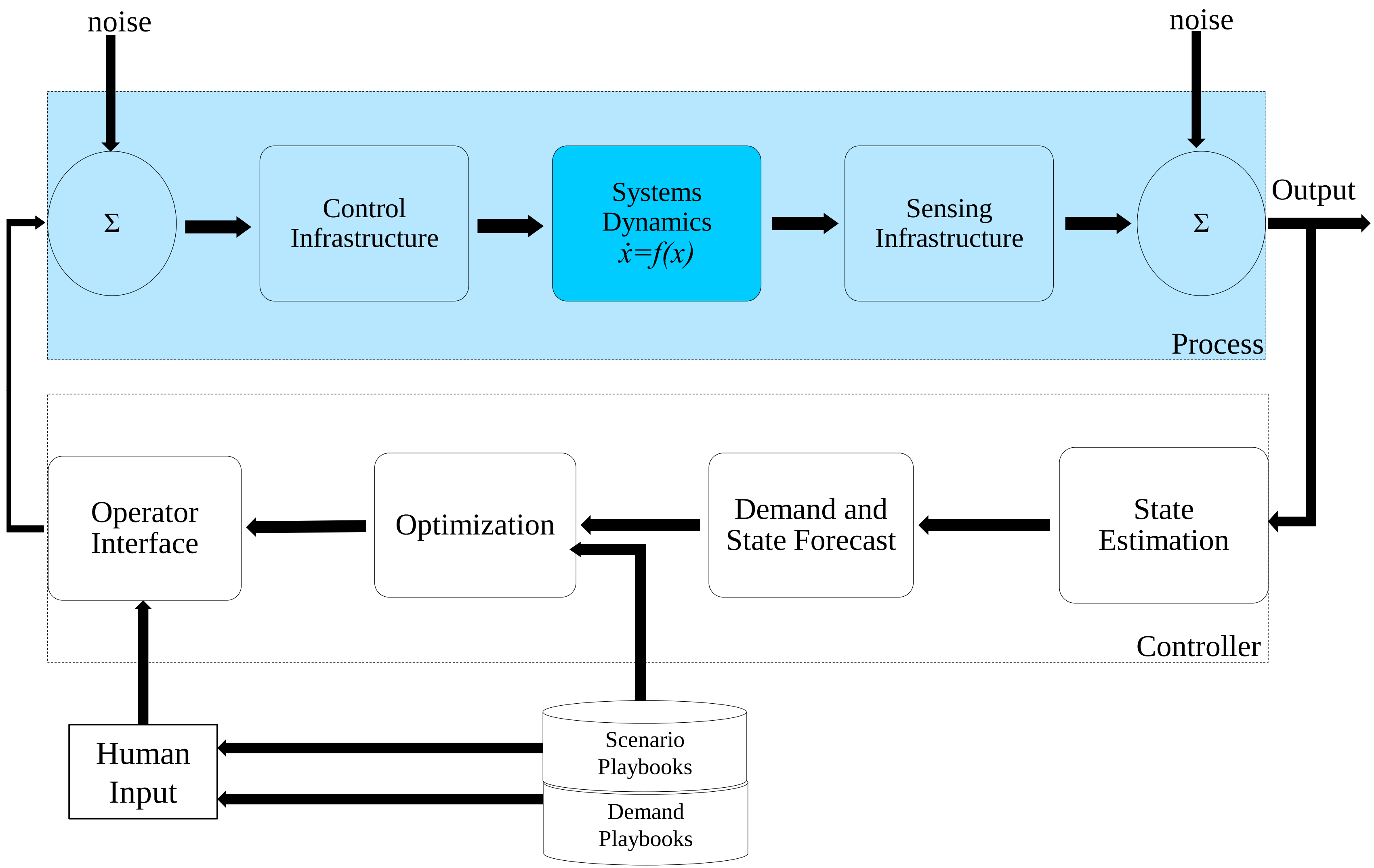
Classical control framework





[Astrom, Murray, 2008]

Classical operations framework in transportation

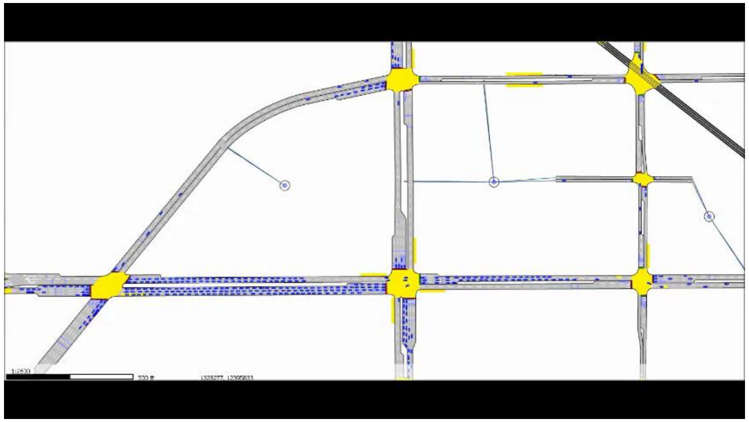


Nonlinear dynamics (1935 – present)

Freeway dynamics
- Lighthill-Whitham-Richards PDE
- Second order models (ARZ)
- Phase transition models
- Hamilton-Jacobi PDE

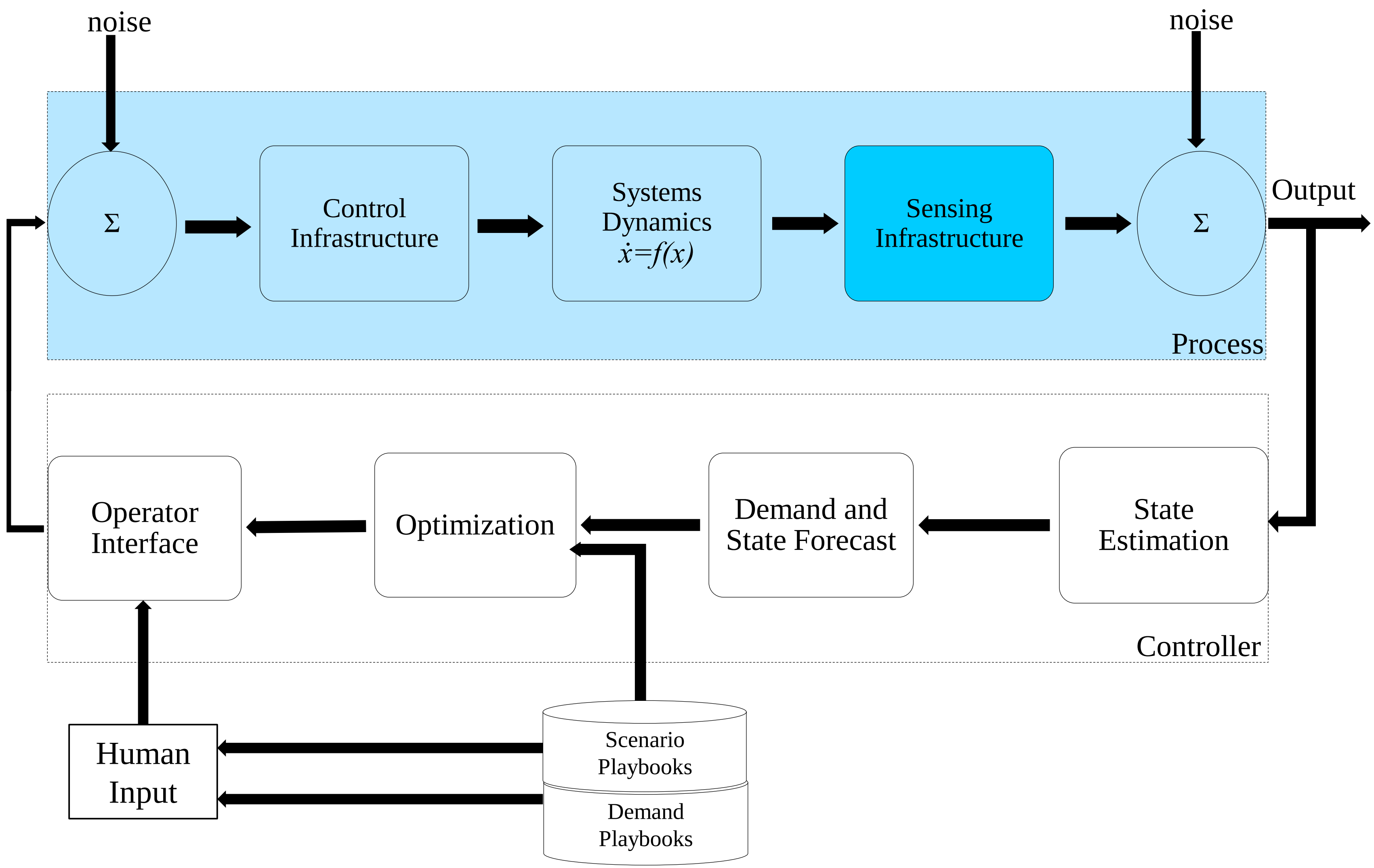
Arterial modeling
- Hamilton-Jacobi PDE
- Queuing systems

Routing
- Stochastic on time arrival (SOTA)
- Riemann solvers (junctions)
- Max-pressure controllers



[Bayen, Strub, IJRNC 2006, Work et al. AMRX 2010, Blandin et al., SIAM JMA, 2011, Delle Monache et al., SIAM JAM 2014]

Classical operations framework in transportation



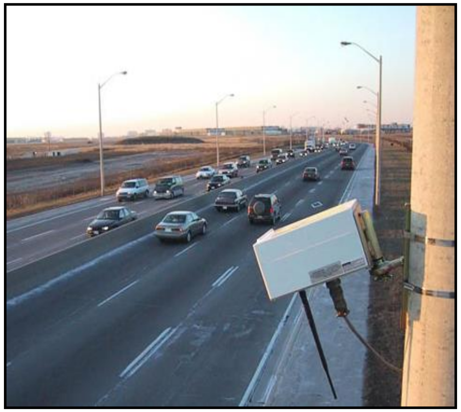
Classical sensing infrastructure (1960' – present)

Dedicated traffic monitoring infrastructure (since the 1960'):

- Self inductive loops
- Wireless pavement sensors
- FasTrak, EZ-pass transponders
- Cameras
- Radars
- License plate readers
- Traffic tubes

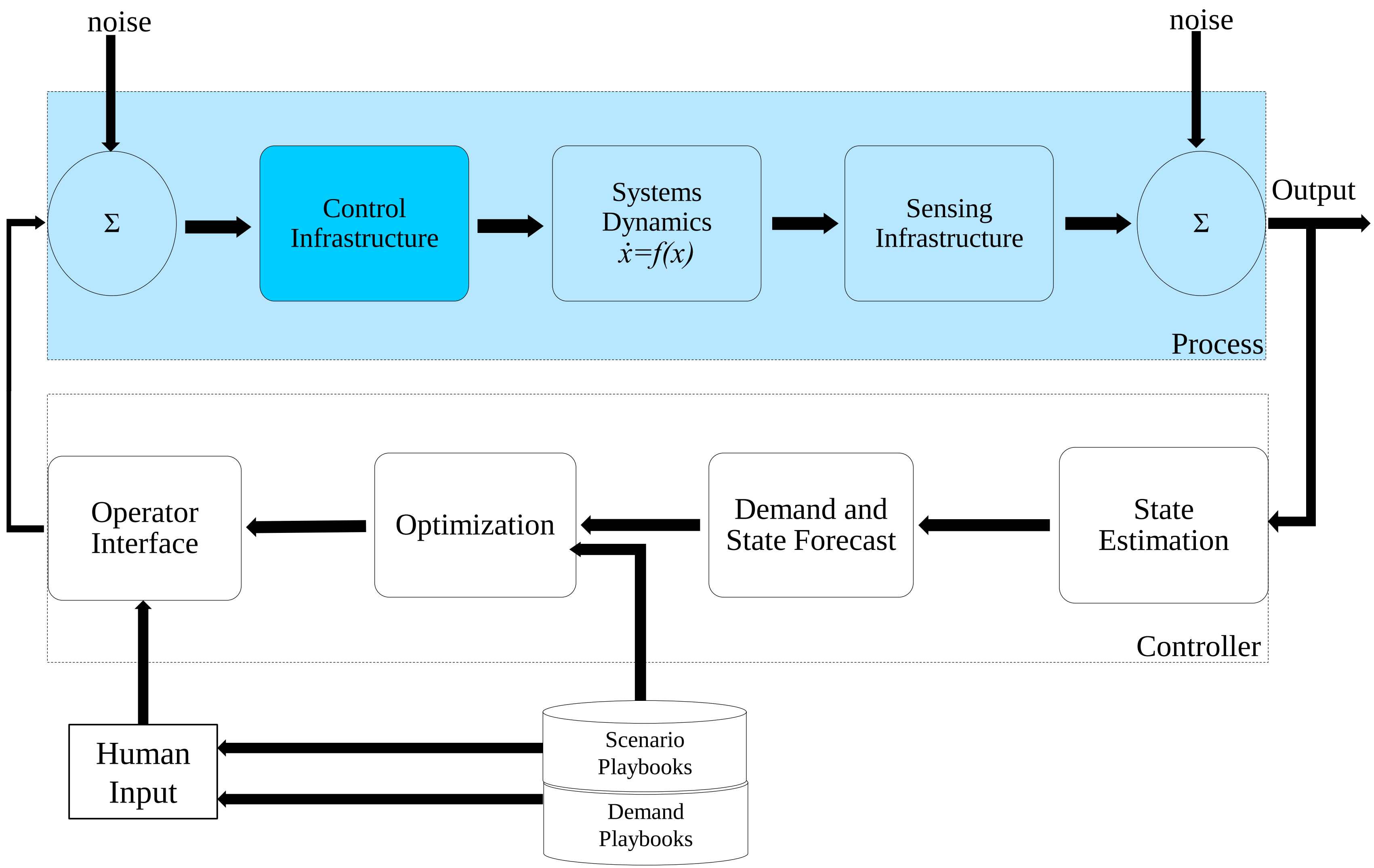
Issues with this traditional infrastructure

- Installation and maintenance costs
- Reliability
- Sparse coverage



[Hoh et al., IEEE TMC 2012, MobiSys 2008, Claudel, Bayen, Saint-Pierre HSCC 2007]

Classical operations framework in transportation



Classical control infrastructure (1960 – present)

Dedicated traffic control infrastructure (since the 1960'):

- City traffic lights
- Metering lights
- Changeable message signs
- HOV lanes, HOT lanes, reversible lanes
- Bridge metering
- Variable speed limits

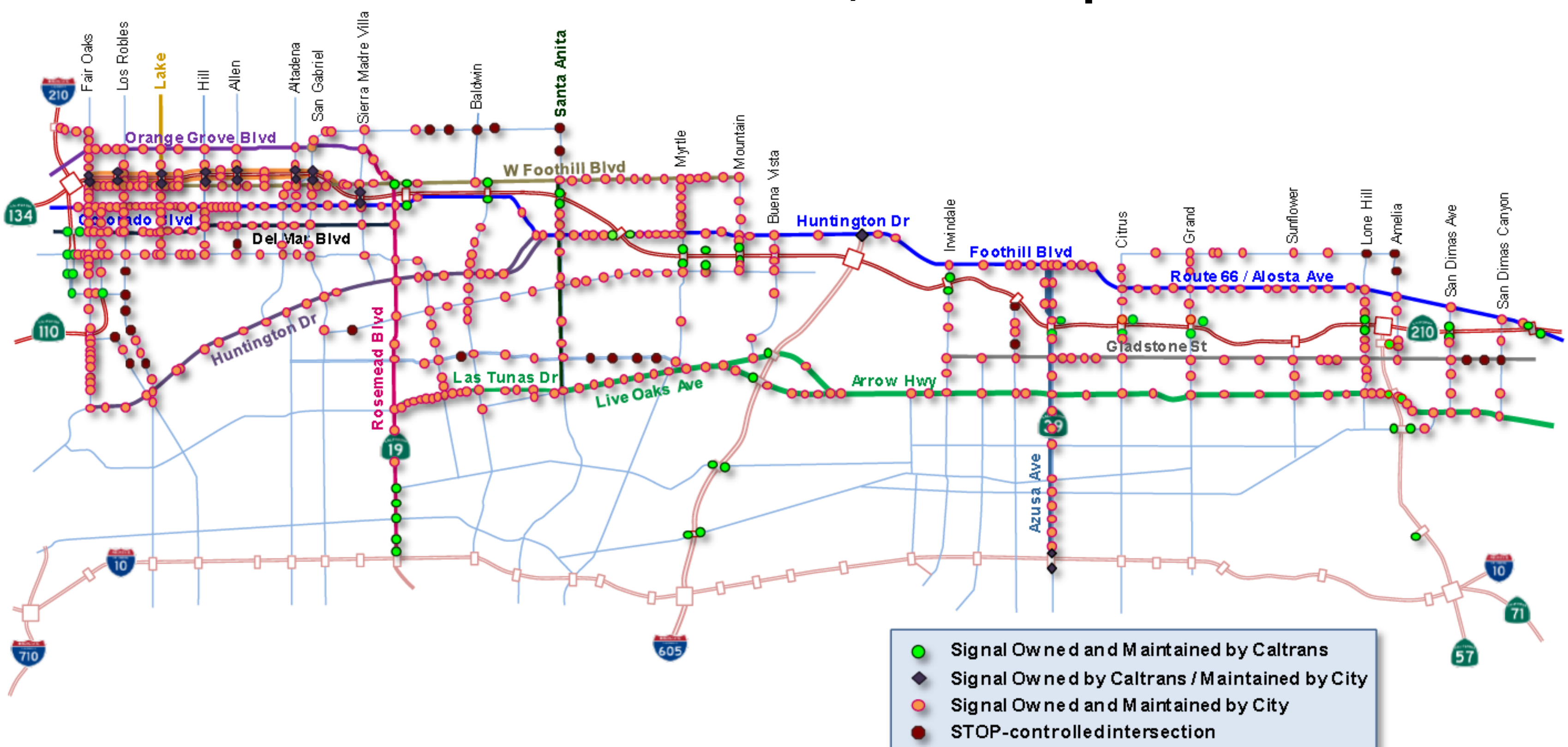
Issues with this traditional infrastructure

- Limited control over motorists
- Virtually no control over routing
- Limited availability of demand and forecast
- Fragmentation of systems

Example: the I210 Connected Corridors testbed

Asset inventory sample

- **Metering lights: 35, including I-650/I-210 freeway-to-freeway metering**
- **Instrumented intersections: 450 across all cities**
- **Changeable message signs: 4 existing + 6 Caltrans +12 Pasadena**
- **Wayfinding signs: 60 to be installed across corridor**
- **HOV lanes: 1 - On I-210 EB and WB, 2 on-ramp w. dedicated HOV lane**



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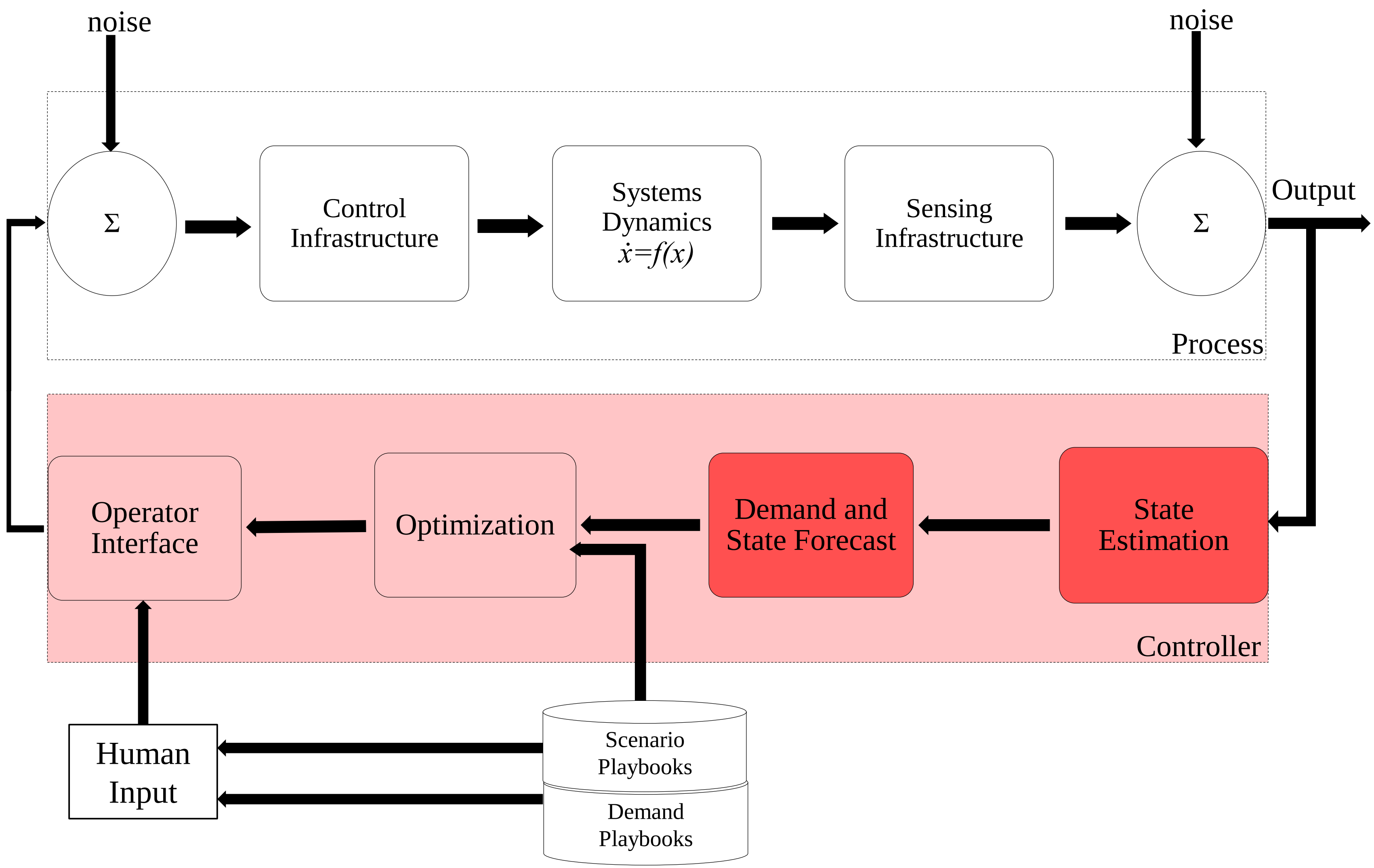
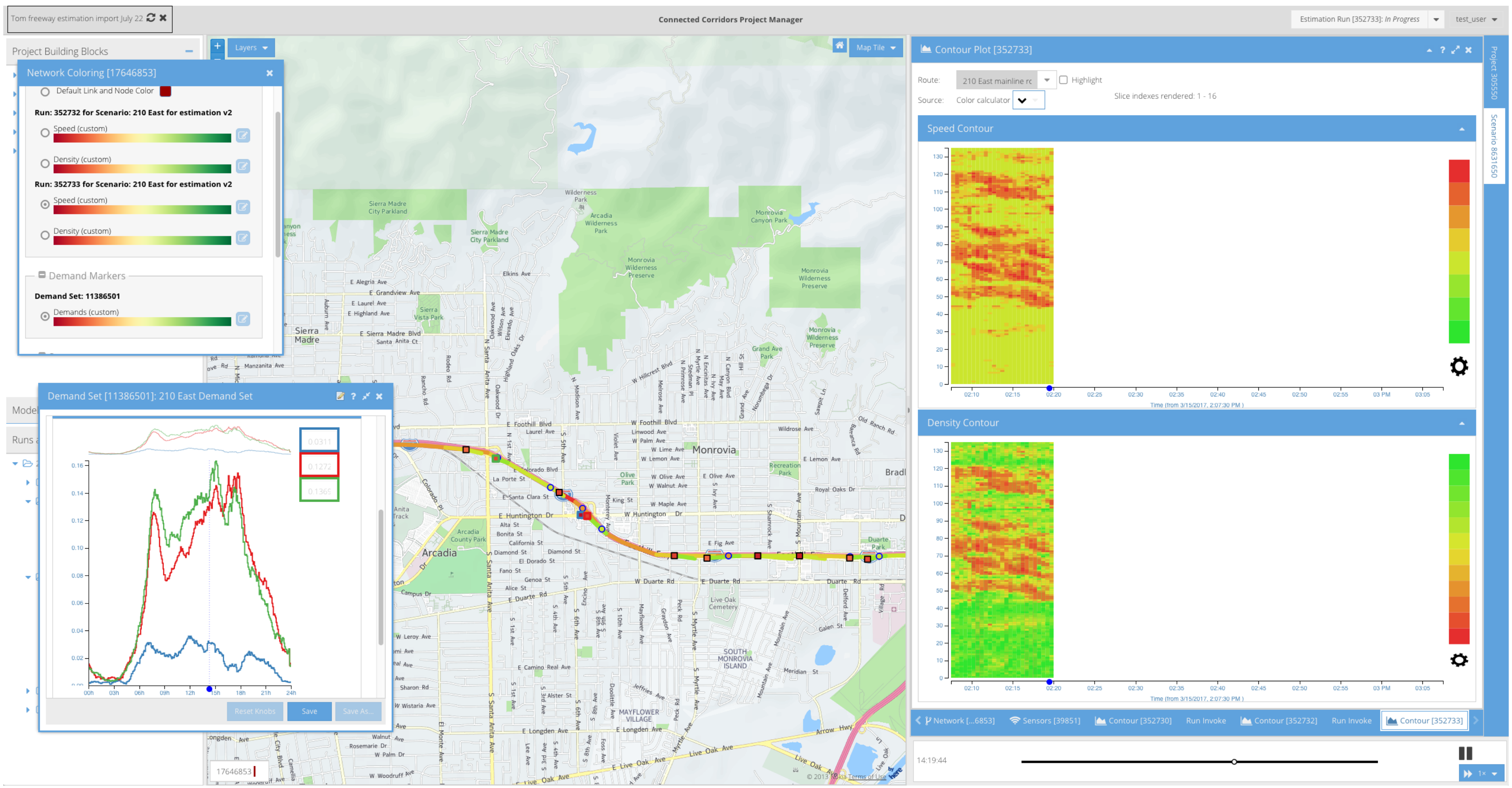


State estimation, demand and state forecast

Example: interface of the Connected Corridors decision support system

- Real-time demand forecast
- Real-time state estimation and state forecast



State estimation, demand and state forecast

Example: interface of the Connected Corridors decision support system

- **Real-time demand forecast**
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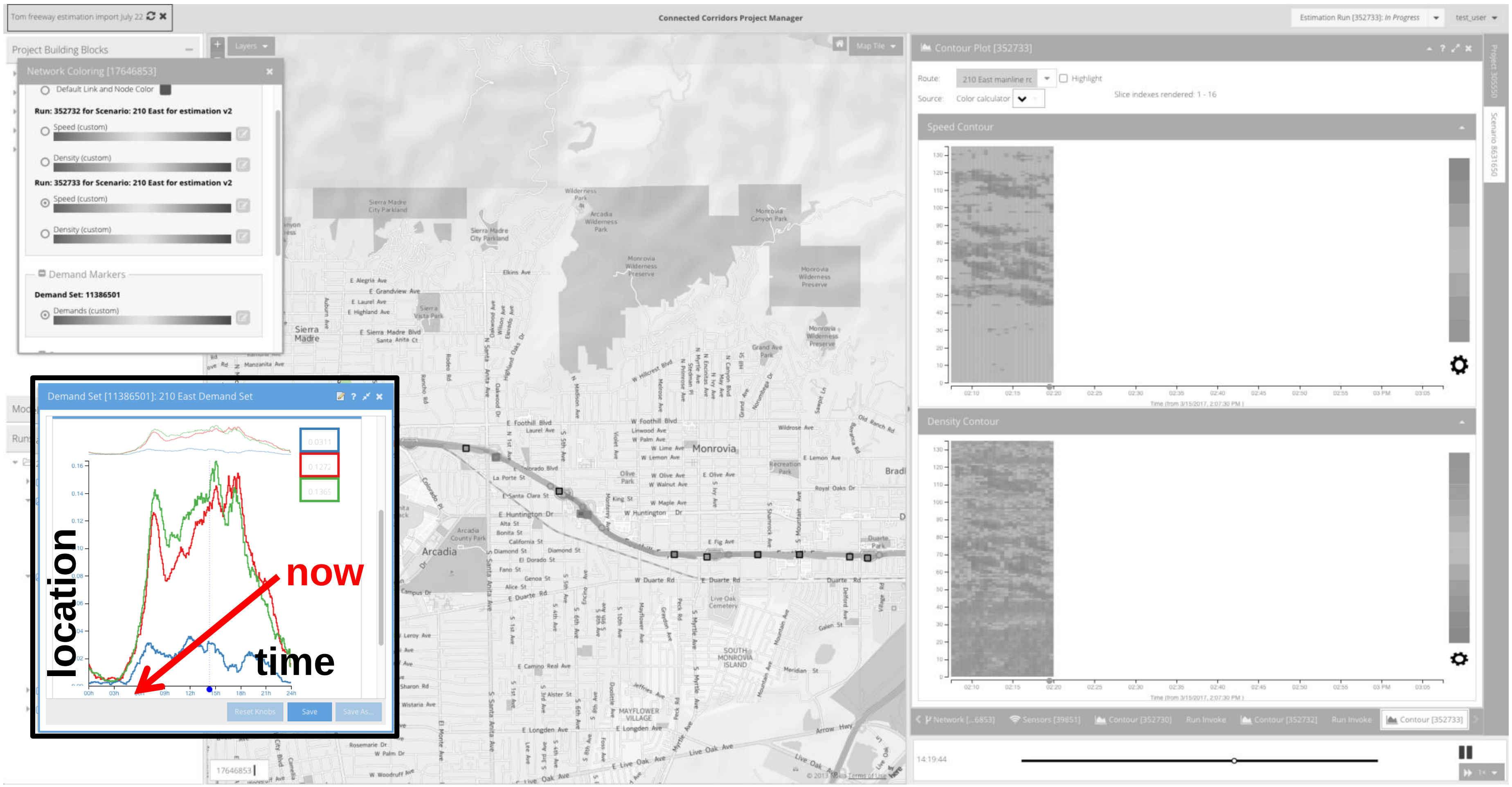
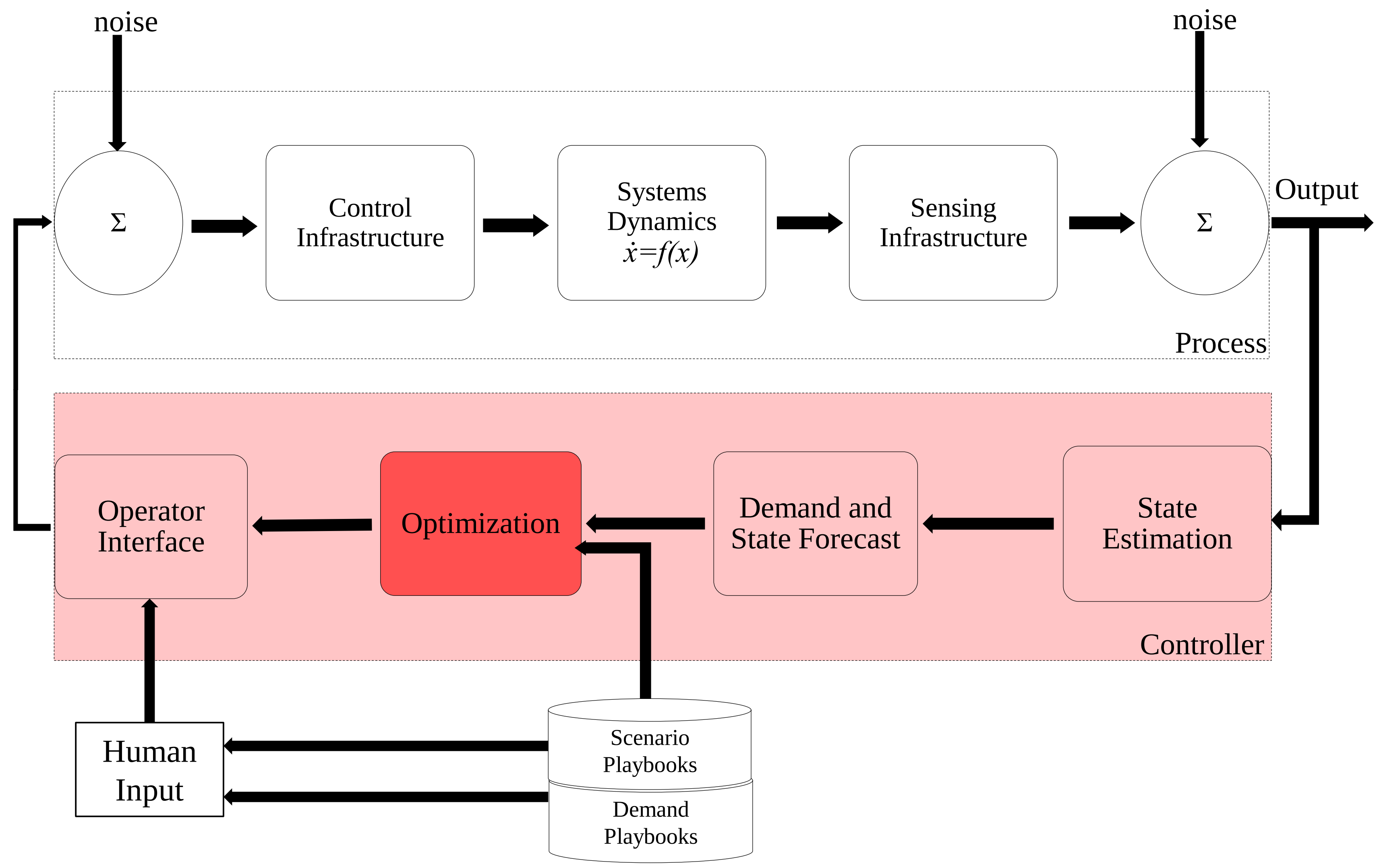


Classical operations framework in transportation



Optimization and control

Algorithms for traffic flow control and optimization

- Playbooks among scenarios
- In some cases: real-time (P, PID, MPC, etc.)

Optimization and control

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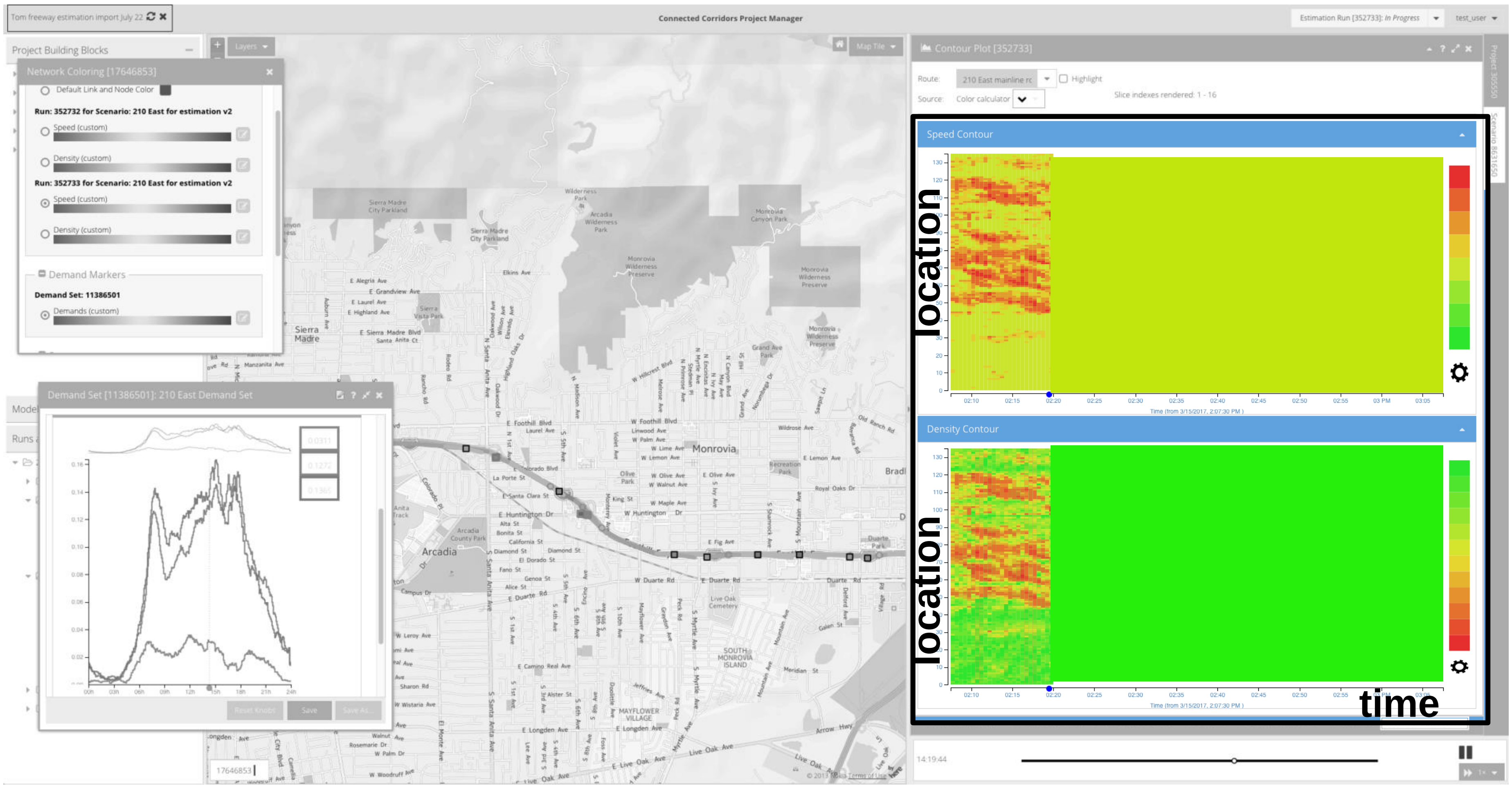
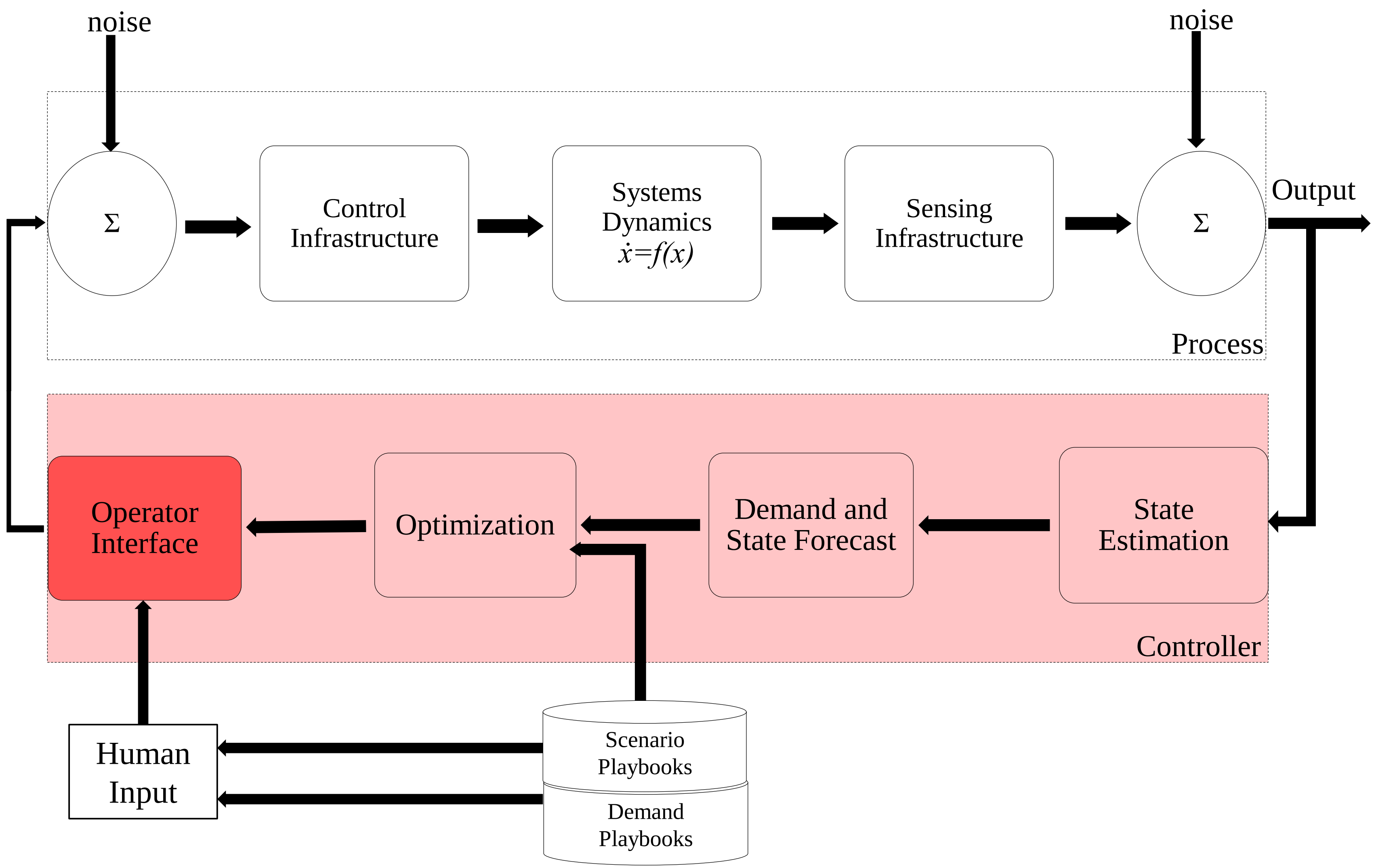


[Reilly et al., JOTA 2015, Reilly et al. IEEE TITS 2015]

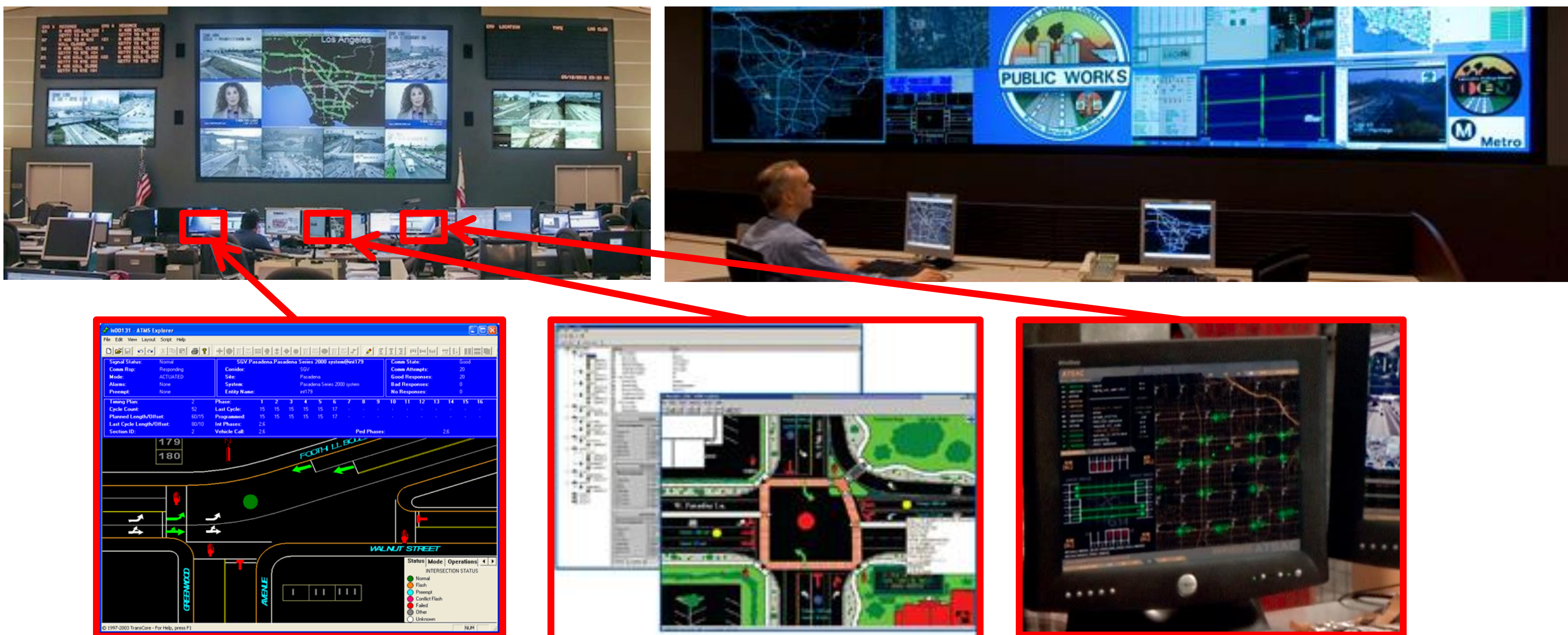
Classical operations framework in transportation



User interface

Human in the loop for infrastructure control

- **Down to single asset level (traffic light, CMS, etc.)**
- **Limited ability to actuate pre-planned scenarios (system-wide)**
- **Difficulties to coordinate across jurisdictions**



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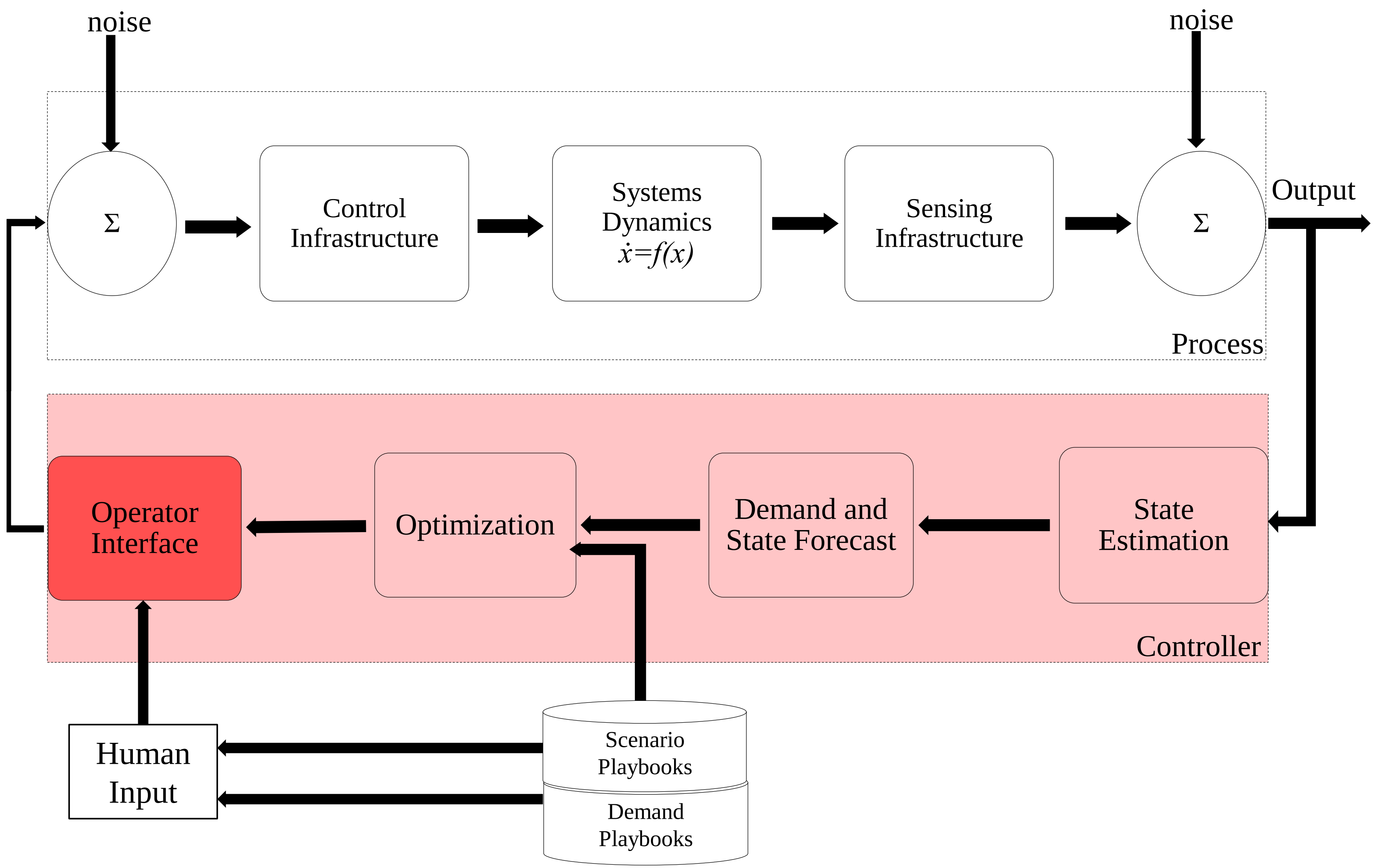

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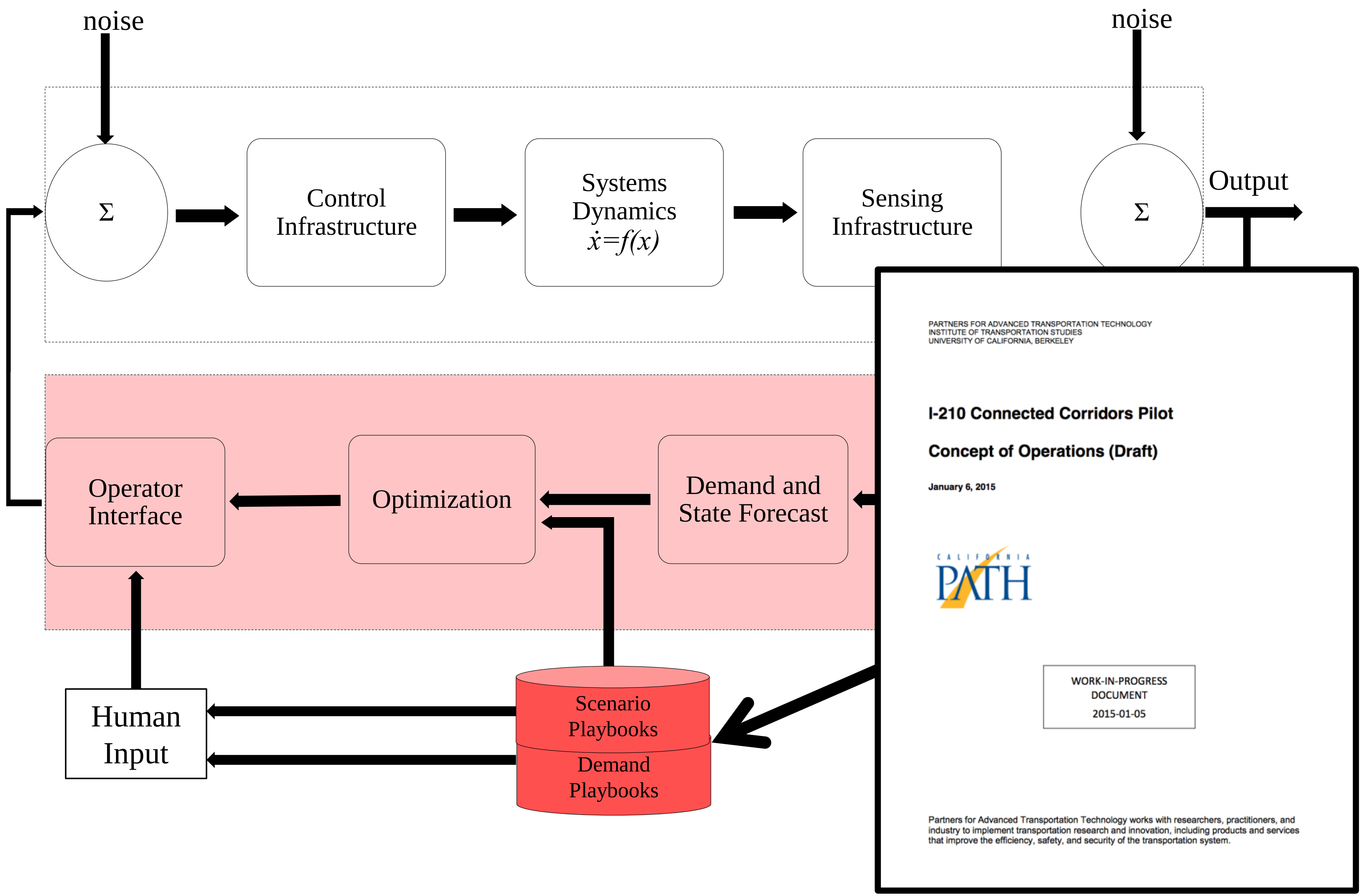

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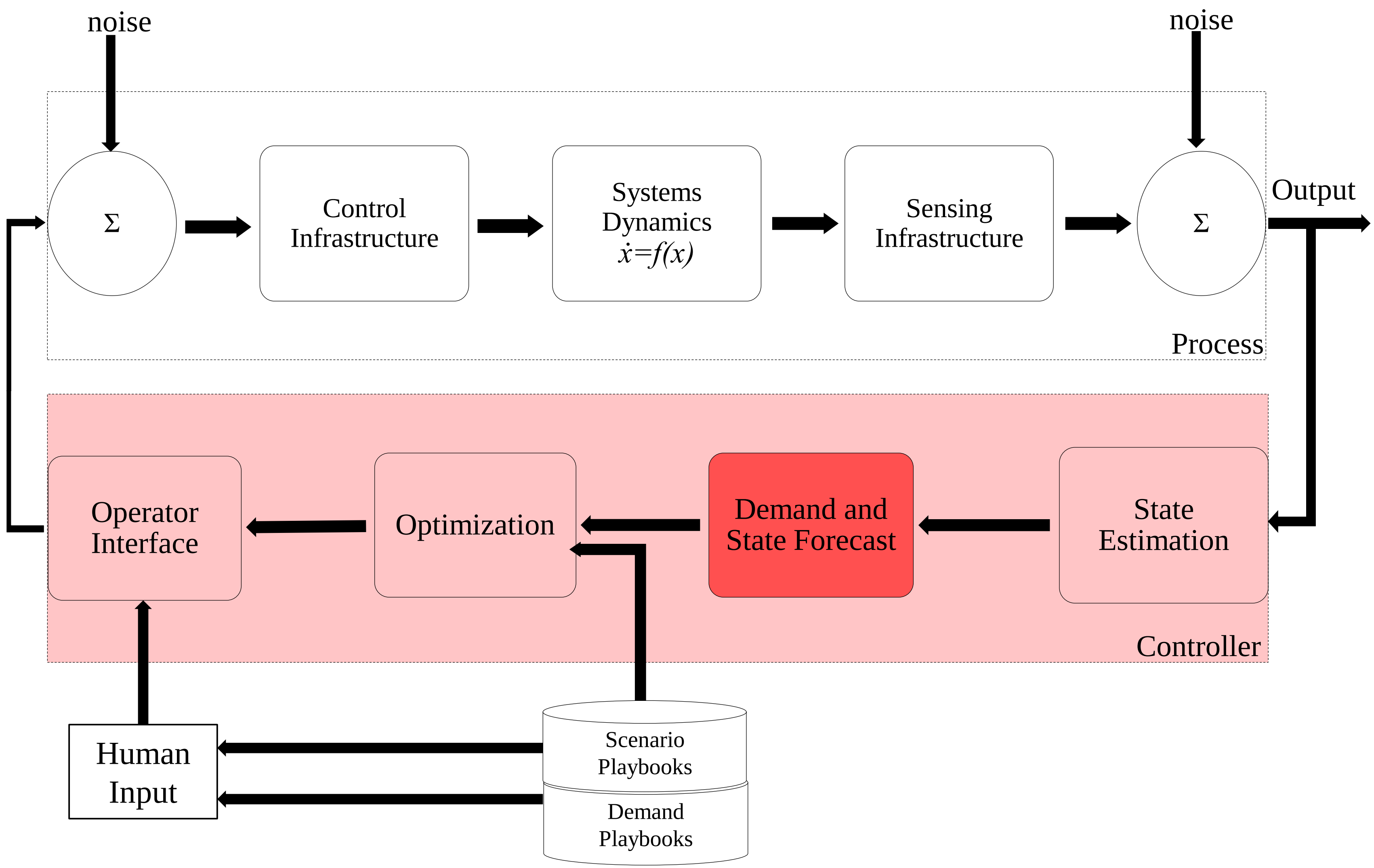
Demand Forecast



Problem statement: route flow estimation

Route flow estimation problem

Given

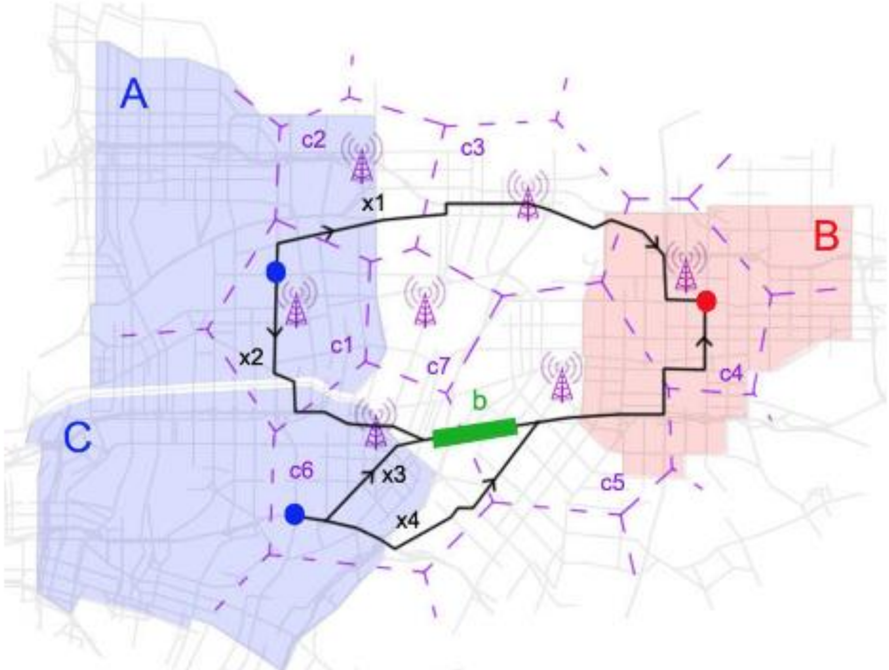
- Road network, origins, cells
- Top routes between OD pairs
- Cellpath flows, f
- OD flows, d
- Observed link flows, b

Recover

- Flow along routes, x

Cellpath flow

Flow along a sequence of cells



Assumptions

- Static, noiseless
- Cell partitioning = Voronoi
- Cellpaths contiguous
- Cellpaths well-posed

[Wu et al., ISTTT 2013, TR-C 2013, ACM TCPS, 2017]

Block simplex constrained quadratic programming

Cellpath + observed link flows	Cellpath + OD + observed link flows
$\begin{aligned} \min \quad & \frac{1}{2}\|Ax - b\|_2^2 \\ \text{s.t.} \quad & Ux = f,\ x \geq 0 \end{aligned}$	$\begin{aligned} \min \quad & \frac{1}{2}\left\| \begin{bmatrix} A \\ T \end{bmatrix} x - \begin{bmatrix} b \\ d \end{bmatrix} \right\|_2^2 \\ \text{s.t.} \quad & Ux = f,\ x \geq 0 \end{aligned}$

- link-route: $A_{lr} = \begin{cases} 1 & \text{if } l \in r \\ 0 & \text{else} \end{cases}$; cellpath-route: $U_{pr} = \begin{cases} 1 & \text{if } r \in p \\ 0 & \text{else} \end{cases}$

- $b \in \mathbb{R}_+^{|\mathcal{L}|}$ observed link flow vector, $b = (b_l)_{l \in \mathcal{L}}$
- $f \in \mathbb{R}_+^{|\mathcal{P}|}$ cellpath flow vector $f = (f_p)_{p \in \mathcal{P}}$
- $x \in \mathbb{R}_+^{|\mathcal{R}|}$ route flow vector, $x = (x_r)_{r \in \mathcal{R}}$
- $d \in \mathbb{R}_+^{|\mathcal{K}|}$ OD flow vector, $x = (x_k)_{k \in \mathcal{K}}$

OD-route:

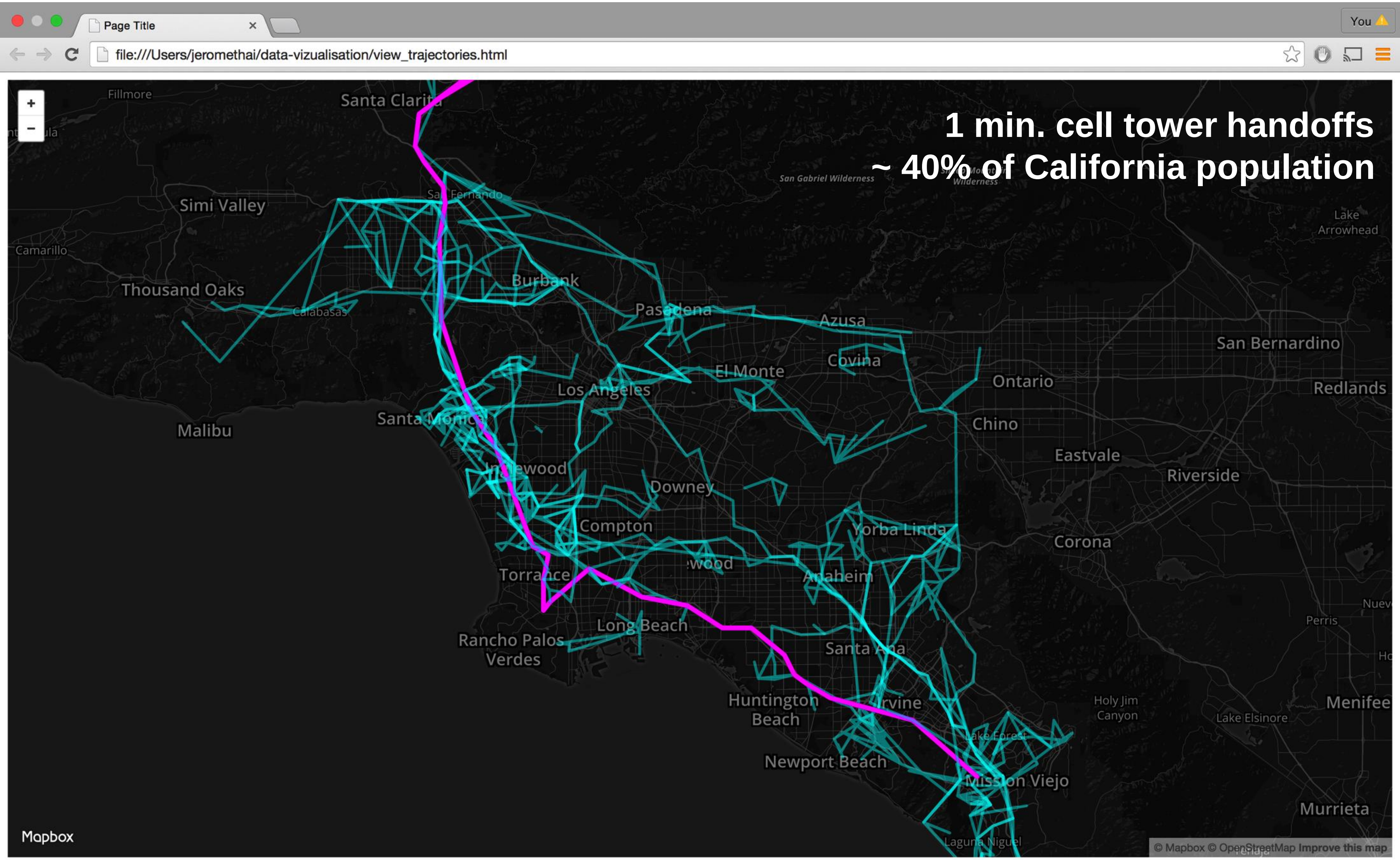
$$T_{kr} = \begin{cases} 1 & \text{if } r \in k \\ 0 & \text{else} \end{cases}$$

Theorem: Optimal solution to box-constrained isotonic regression

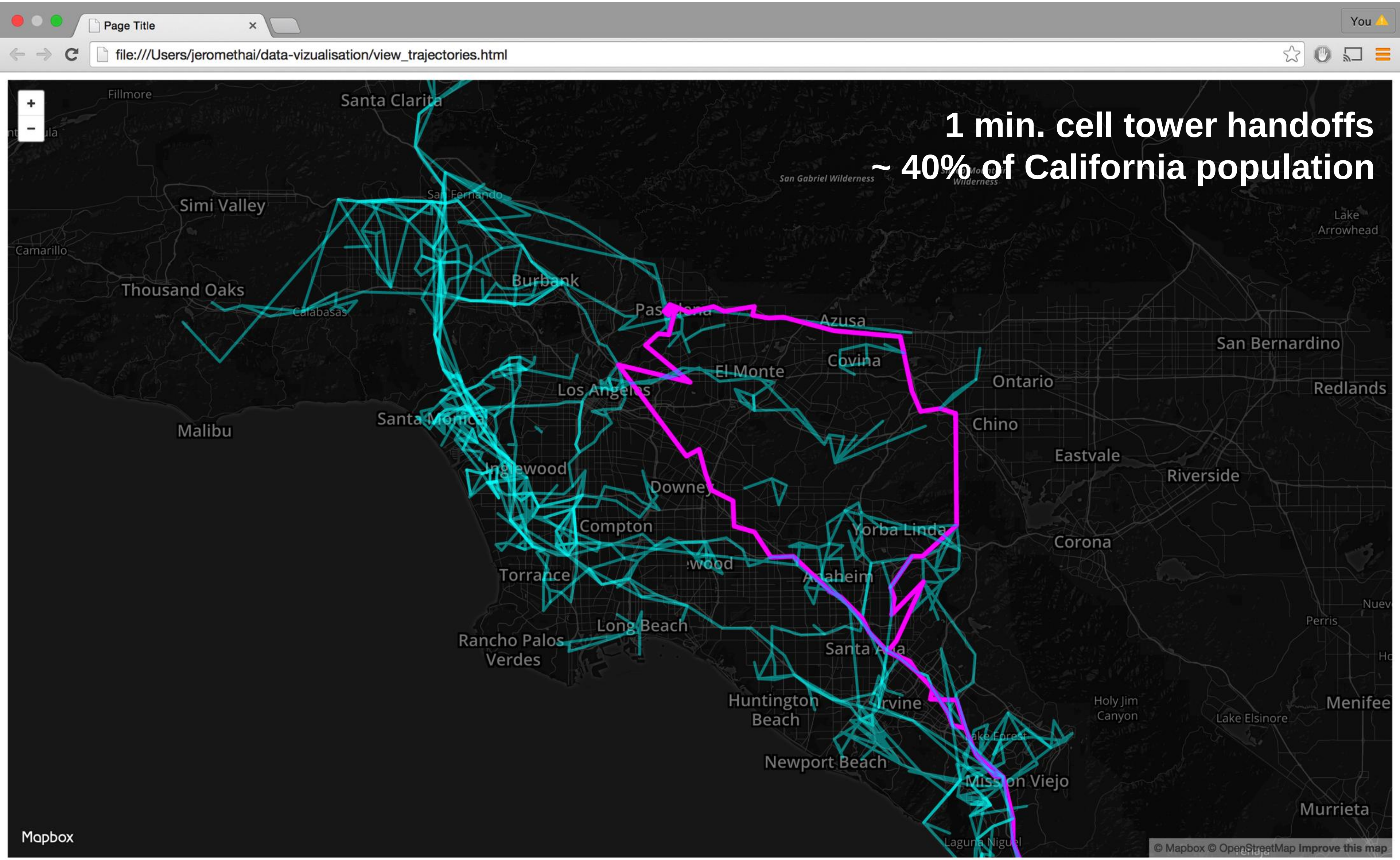
Solution x^* to block-constrained isotonic regression (BCIR) is the Euclidean projection of the solution x^{iso} to isotonic regression (IR) onto the box $[0, f_p]^{|r \in p|}$.

[Wu et al., ISTTT 2013, TR-C 2013, ACM TCPS, 2017]

Example implementation on STEM data



Example implementation on STEM data



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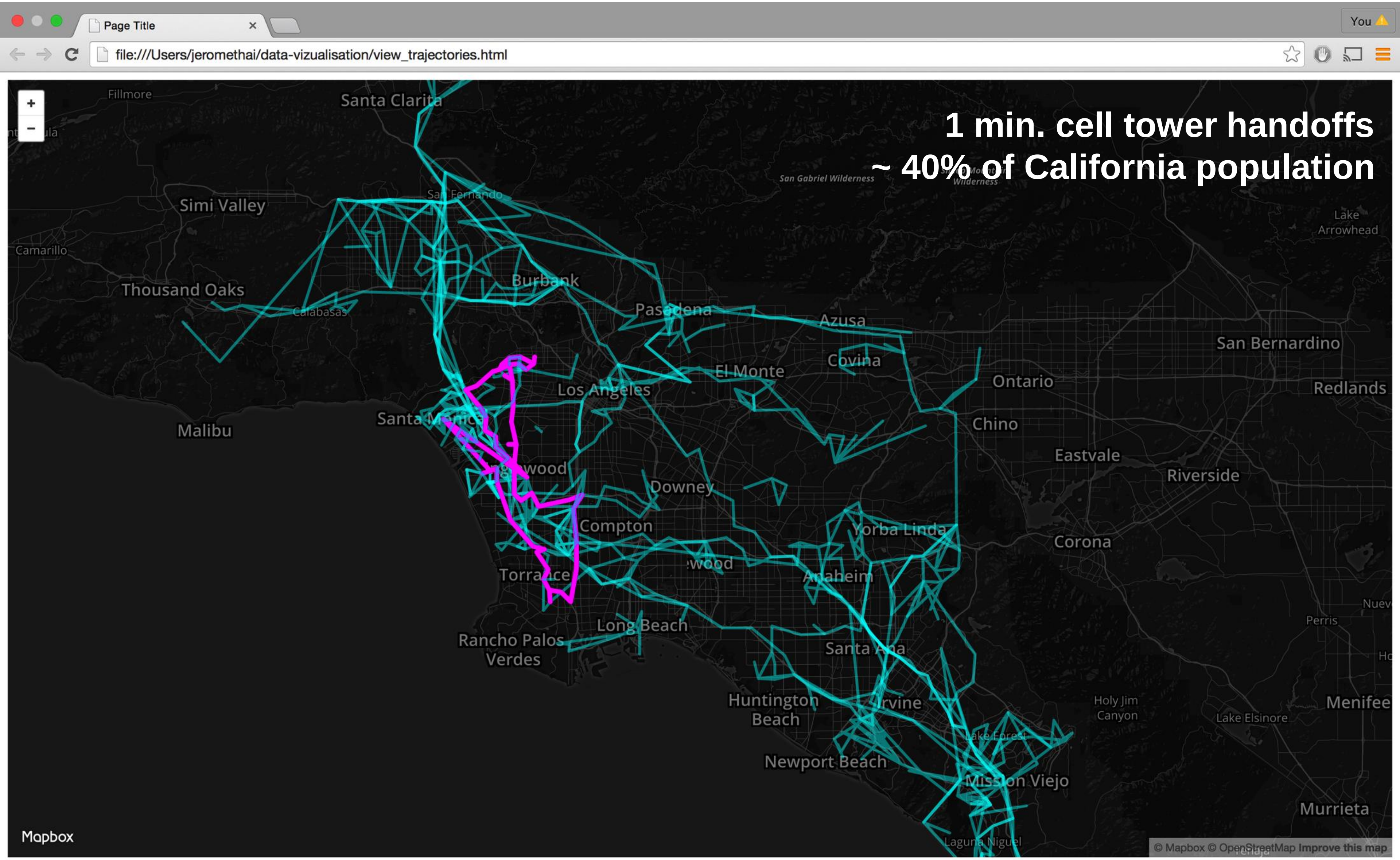

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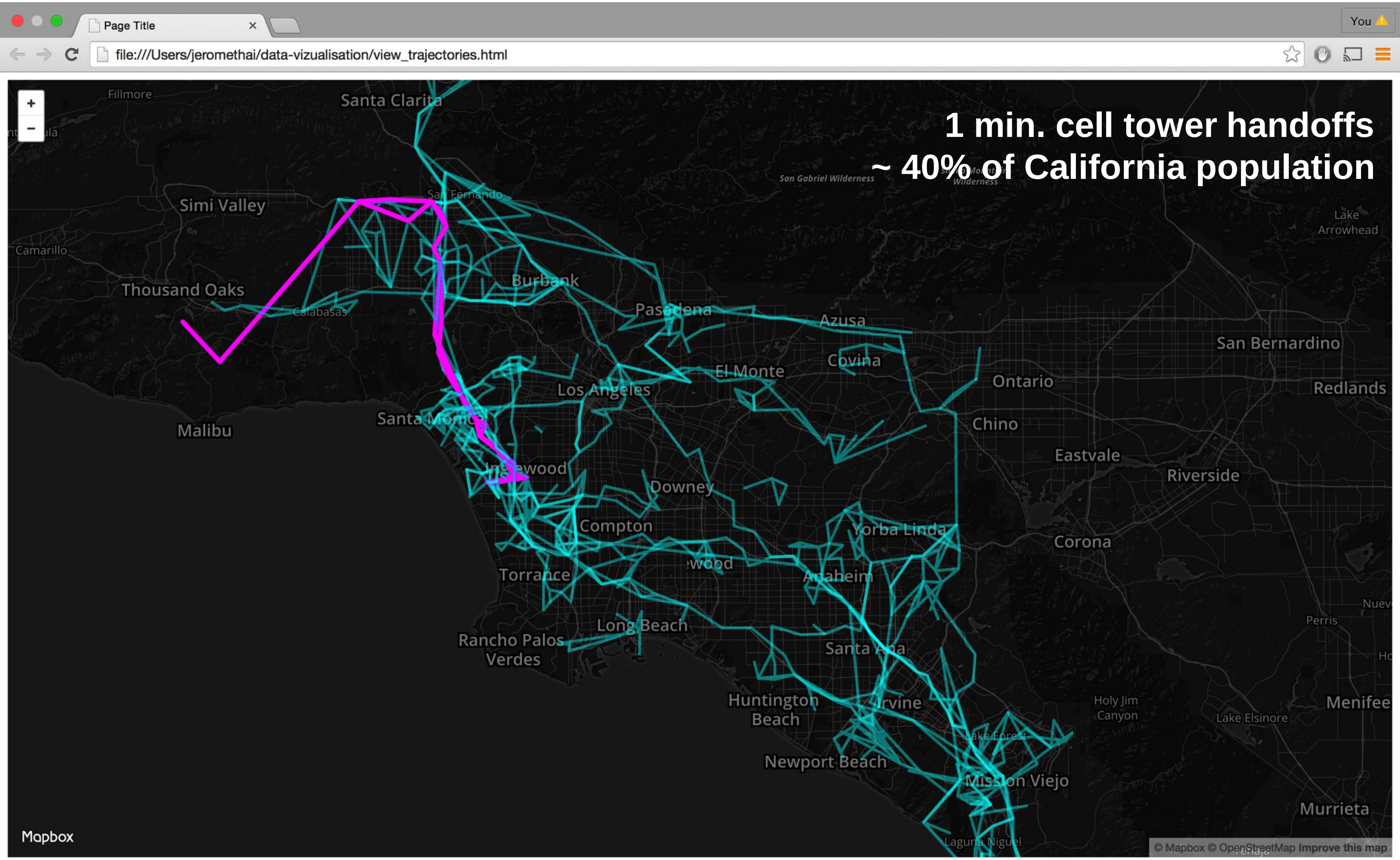

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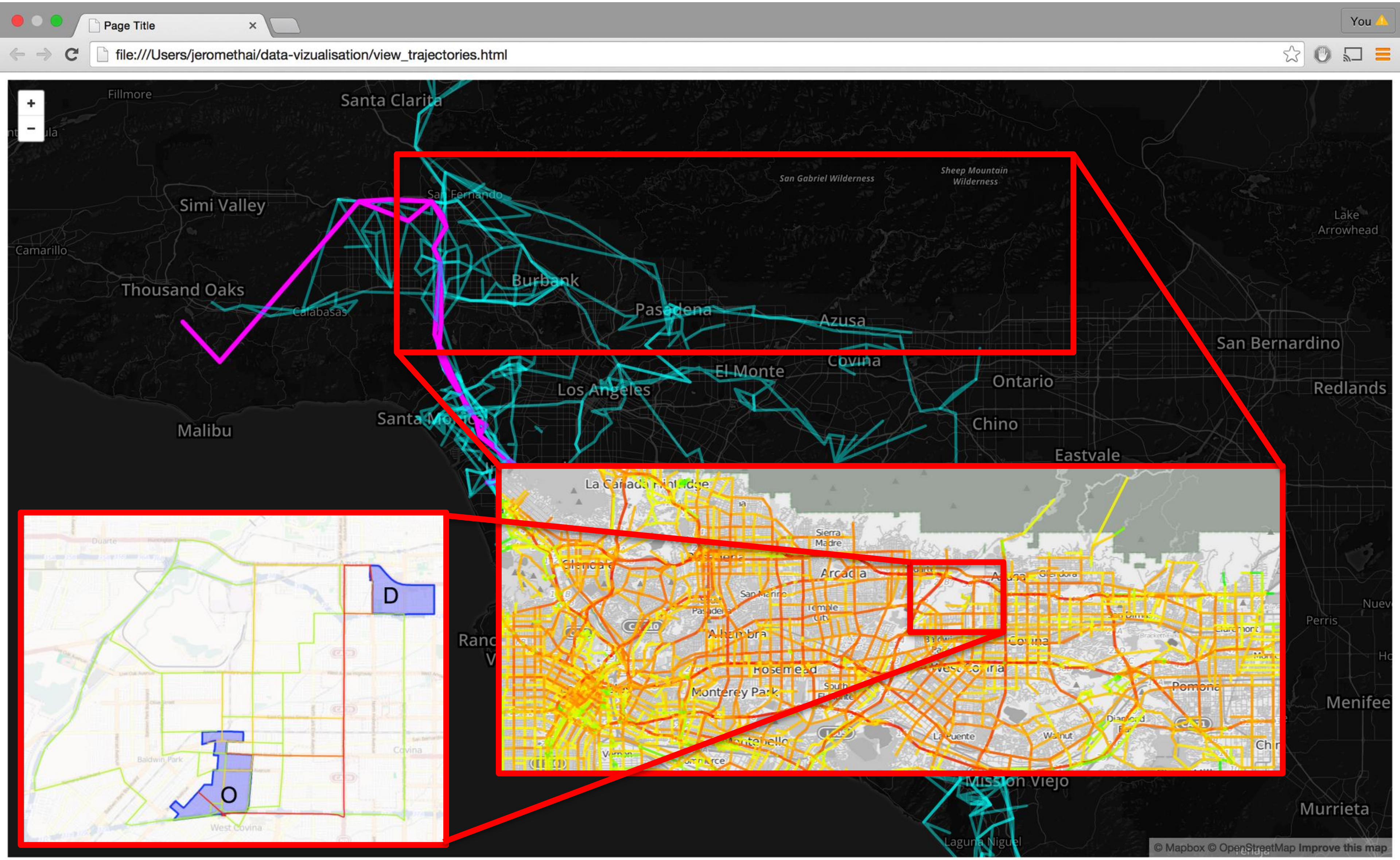
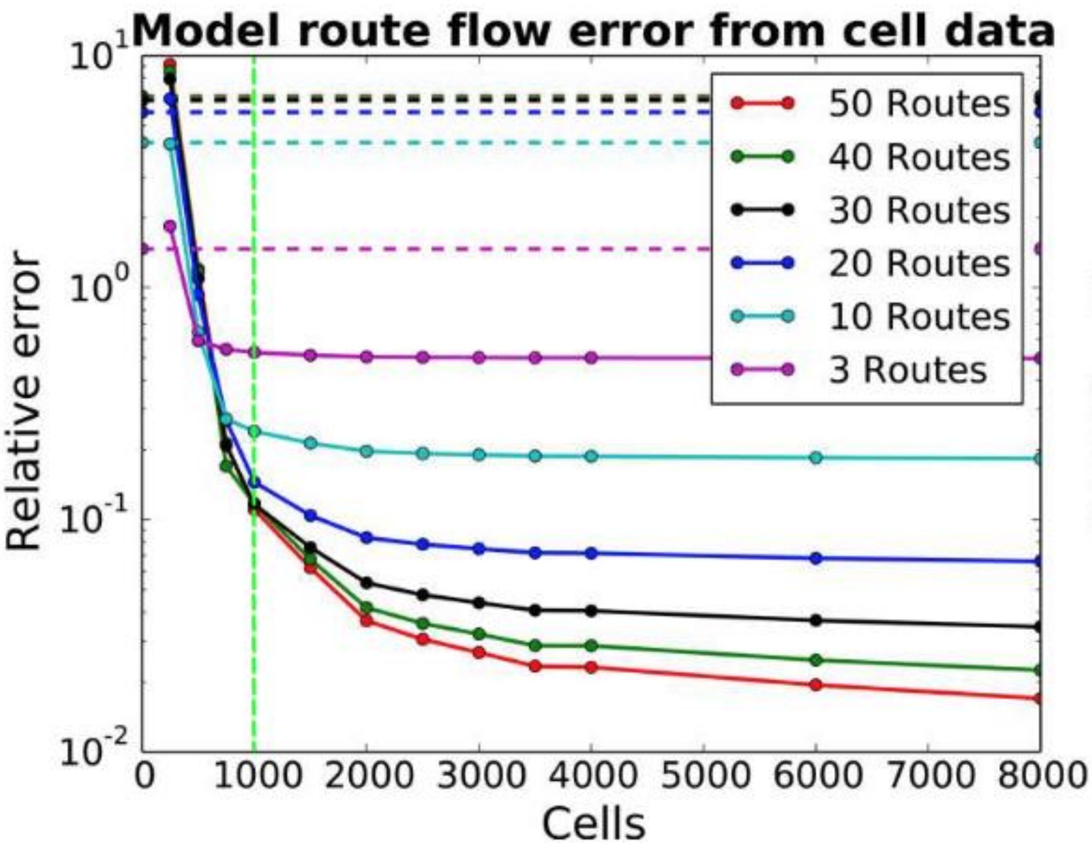
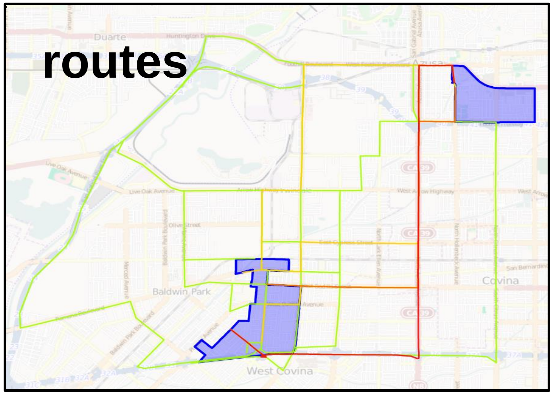


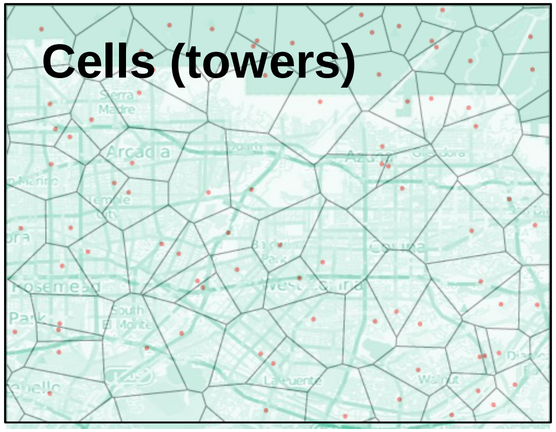
Results

Algorithm produces distribution of flows along the different OD paths

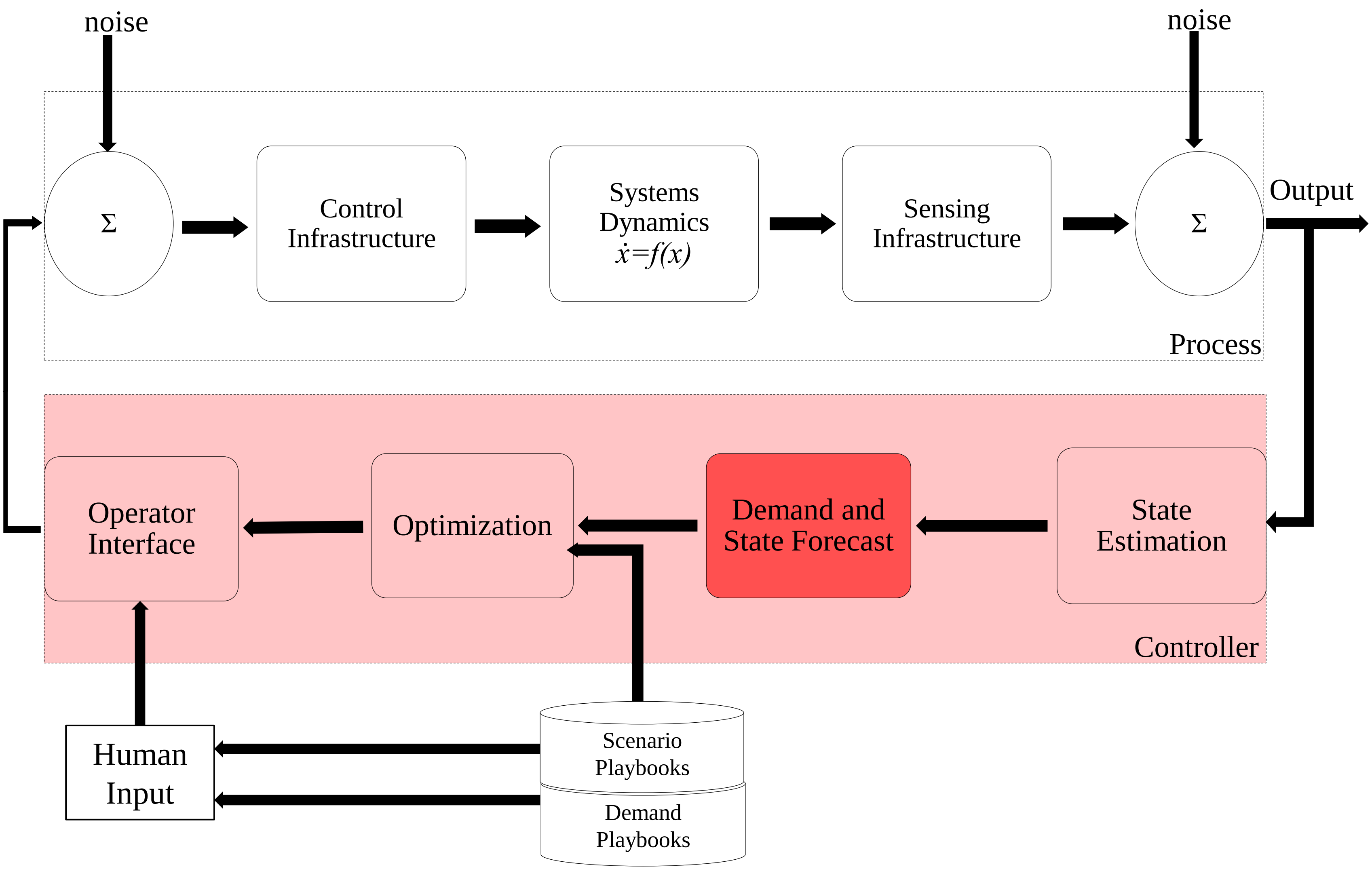
- **Approach is data driven, does not make assumption on the routing behavior of agents (Nash, UE, Social Optimum, etc.)**
- **Approach takes into account "potential" routes taken by users (which can be parametrized).**







Demand Forecast



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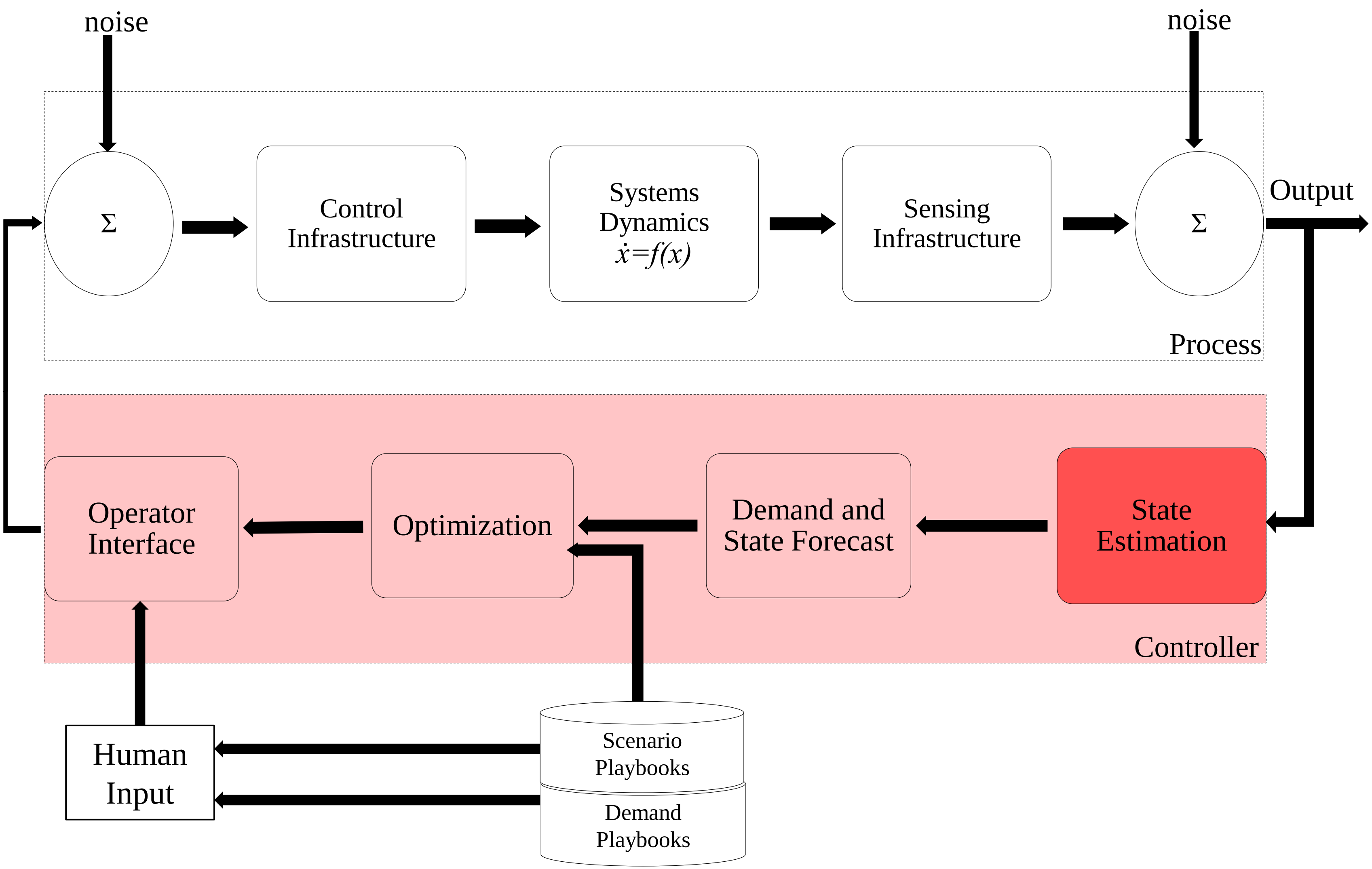
State estimation from heterogeneous sources



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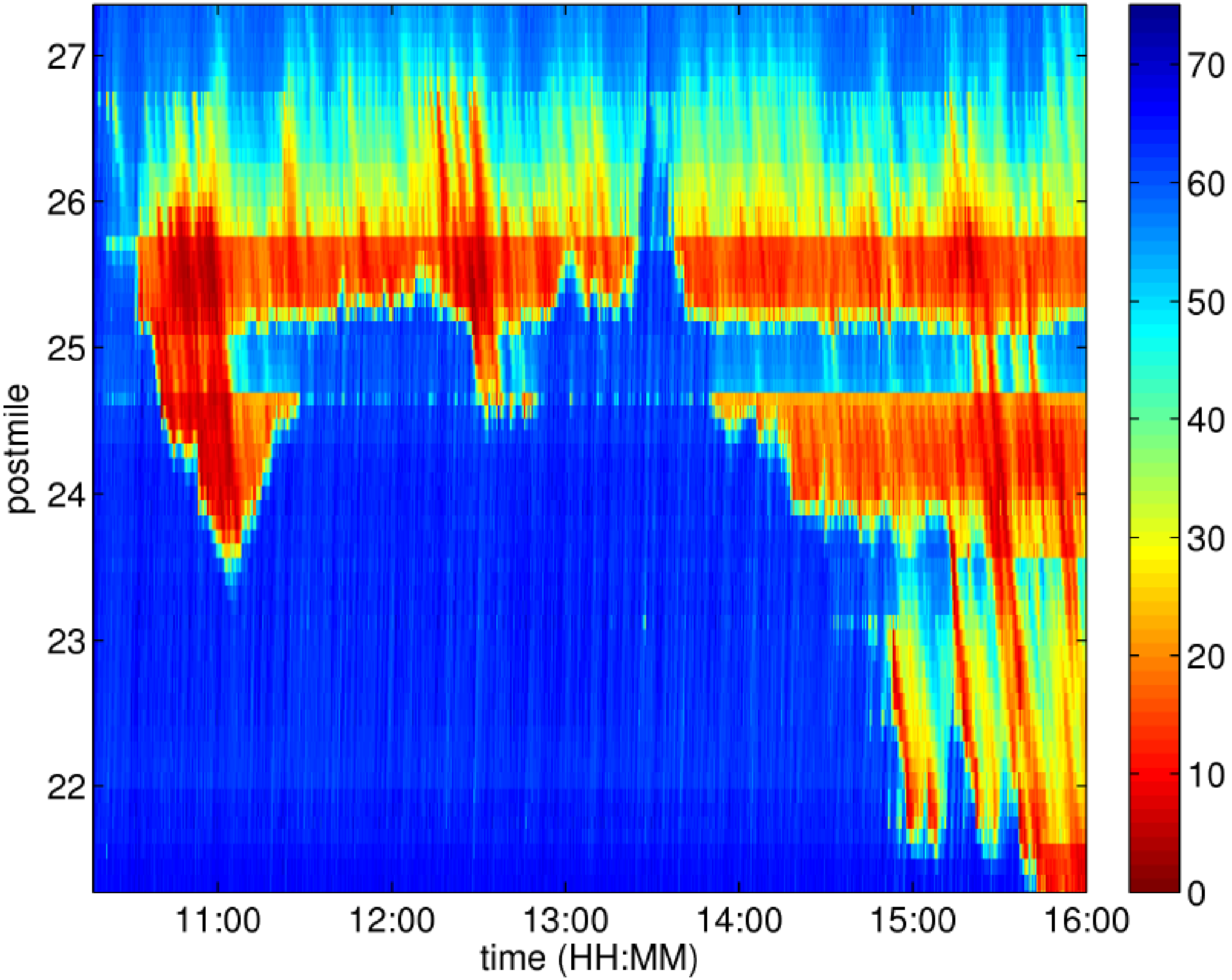

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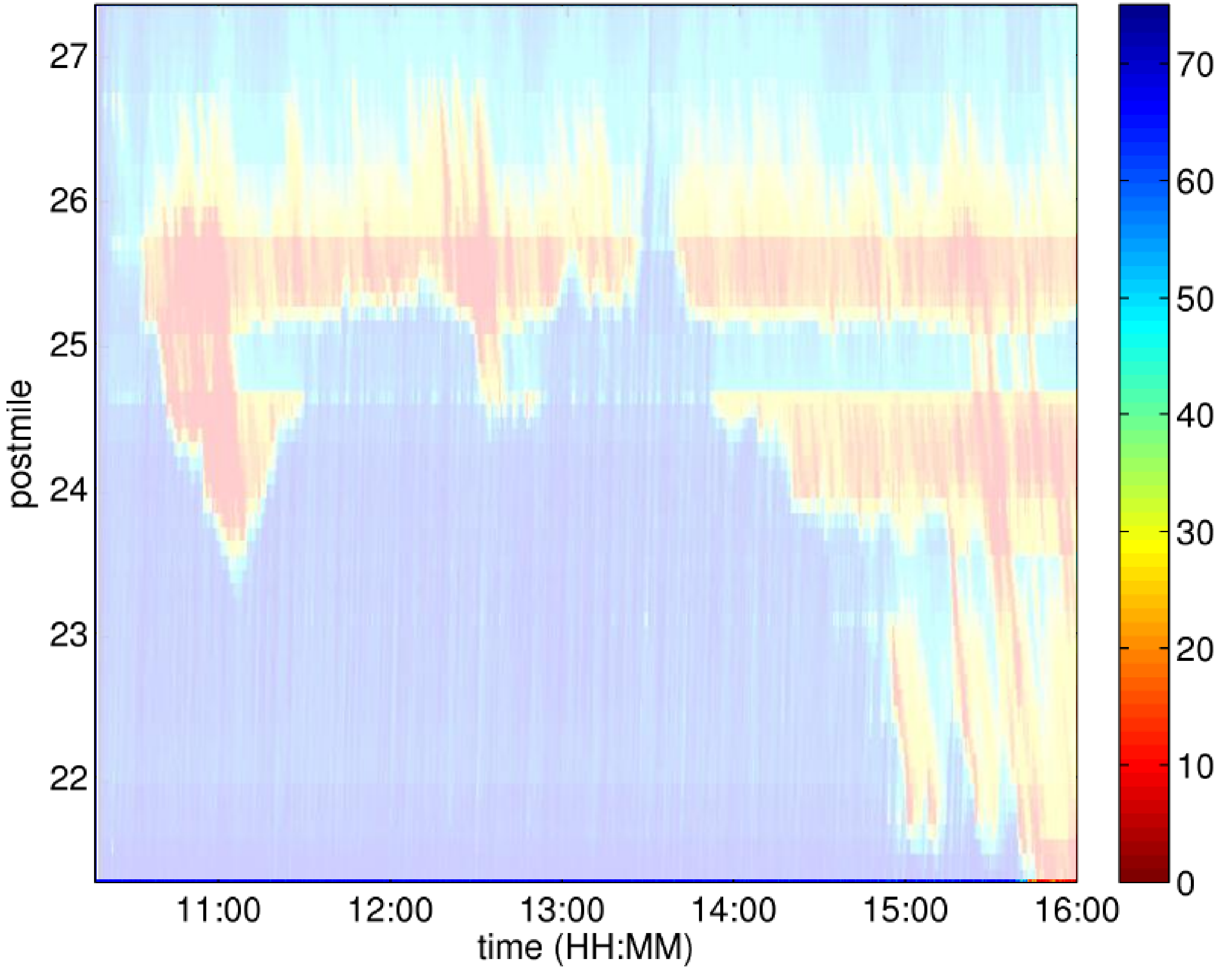
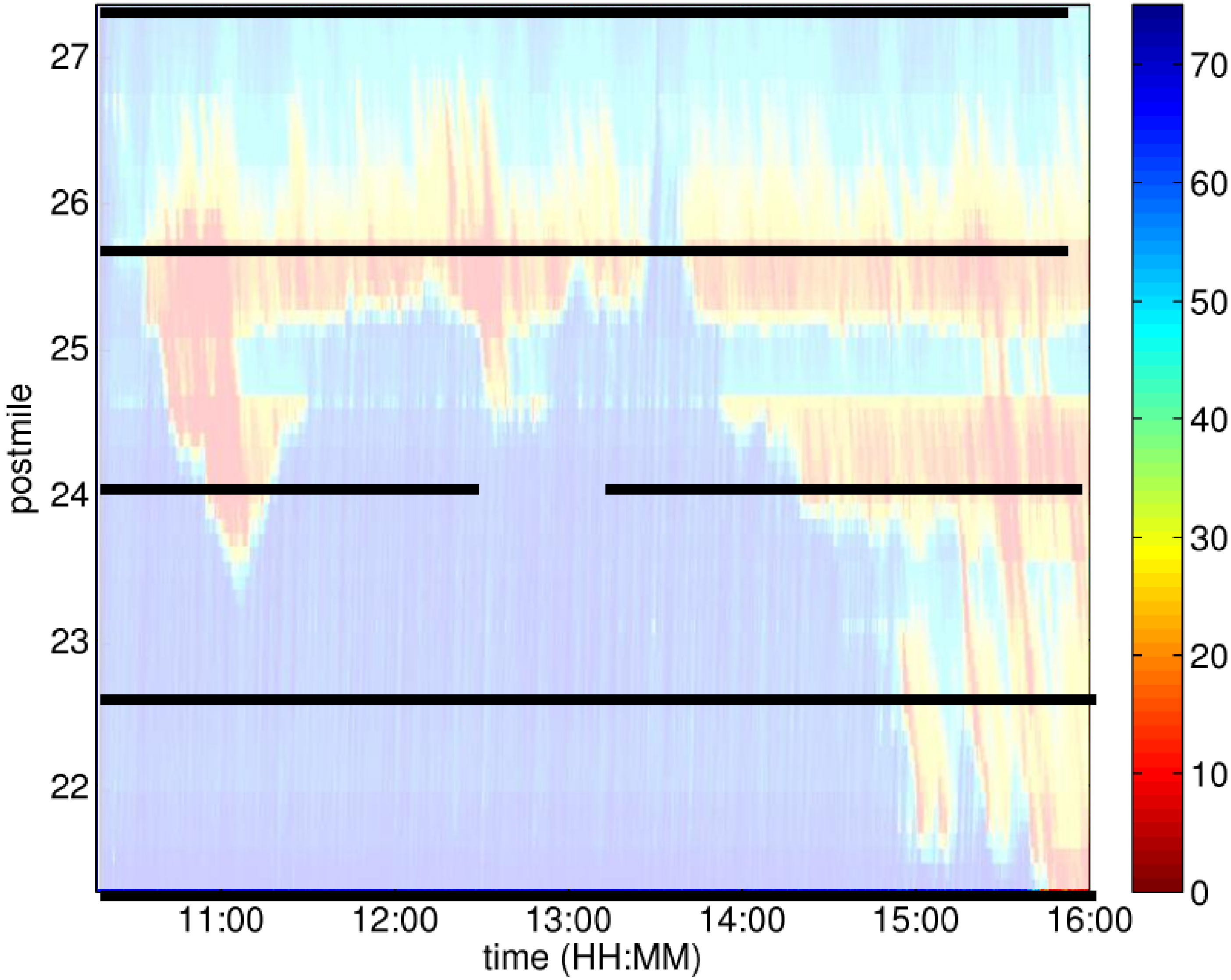


State estimation from heterogeneous sources

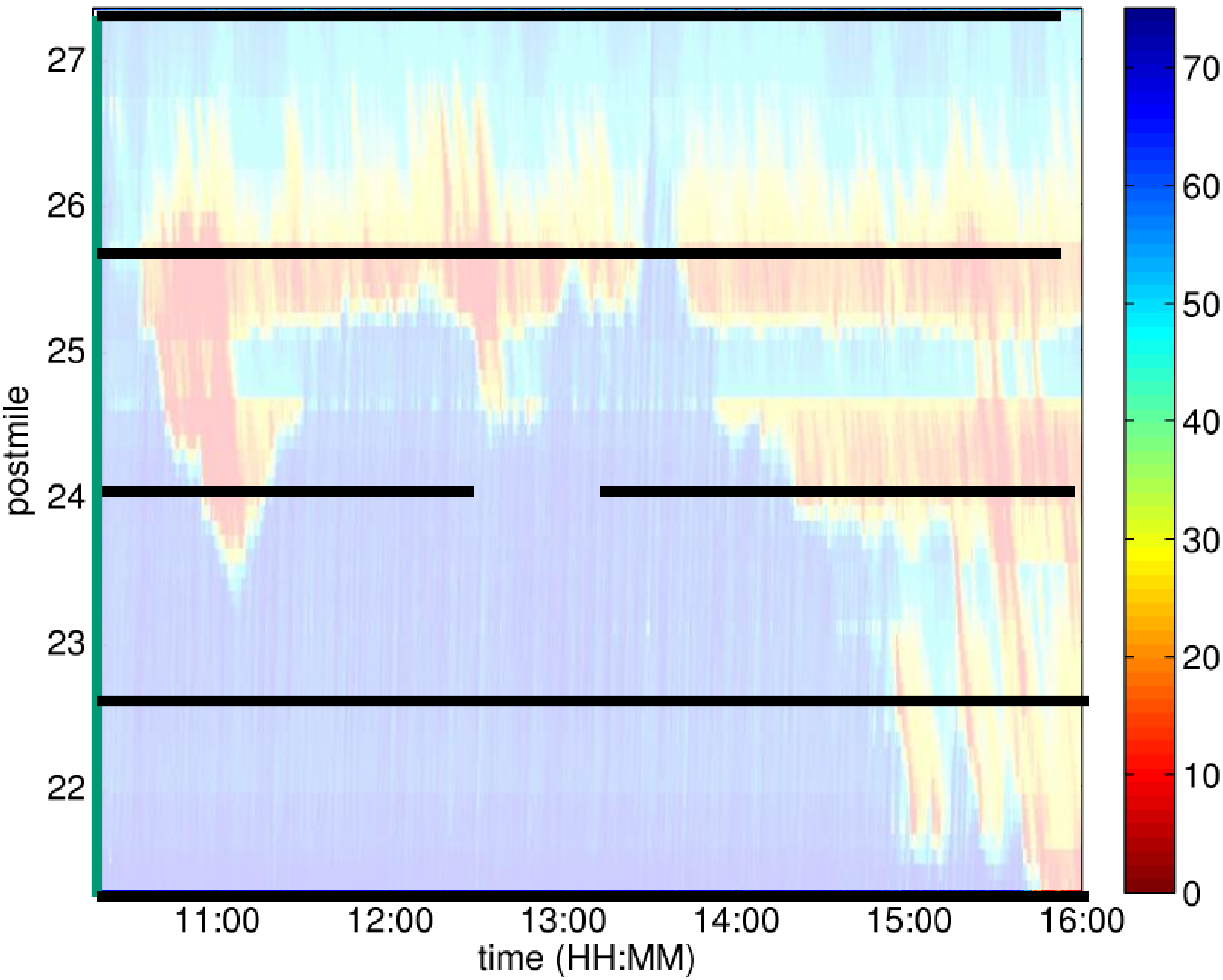
Loop detector



State estimation from heterogeneous sources

Loop detector

Initial condition (night reset)



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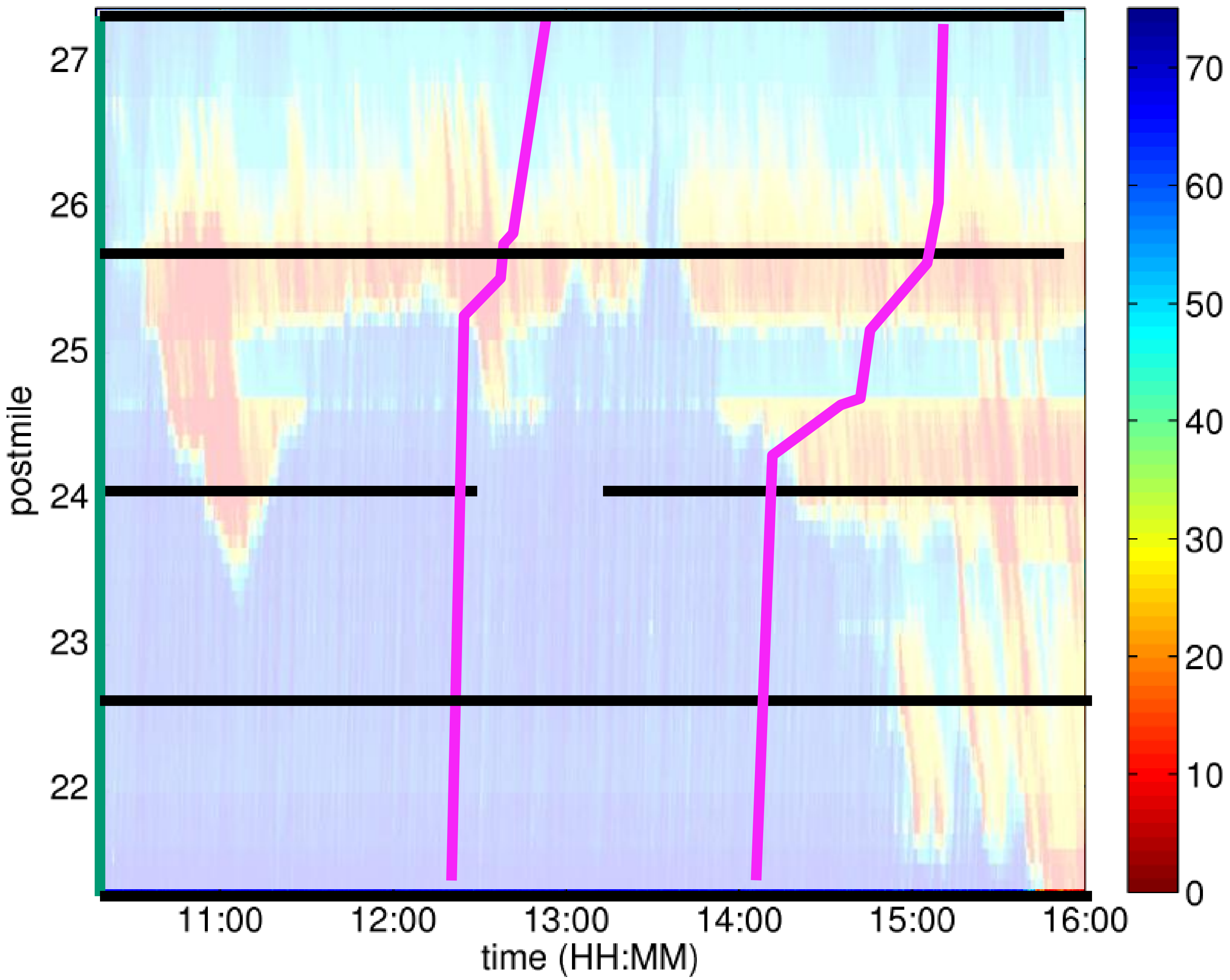
Initial condition (night reset)

Probe vehicle (phone)



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Loop detector

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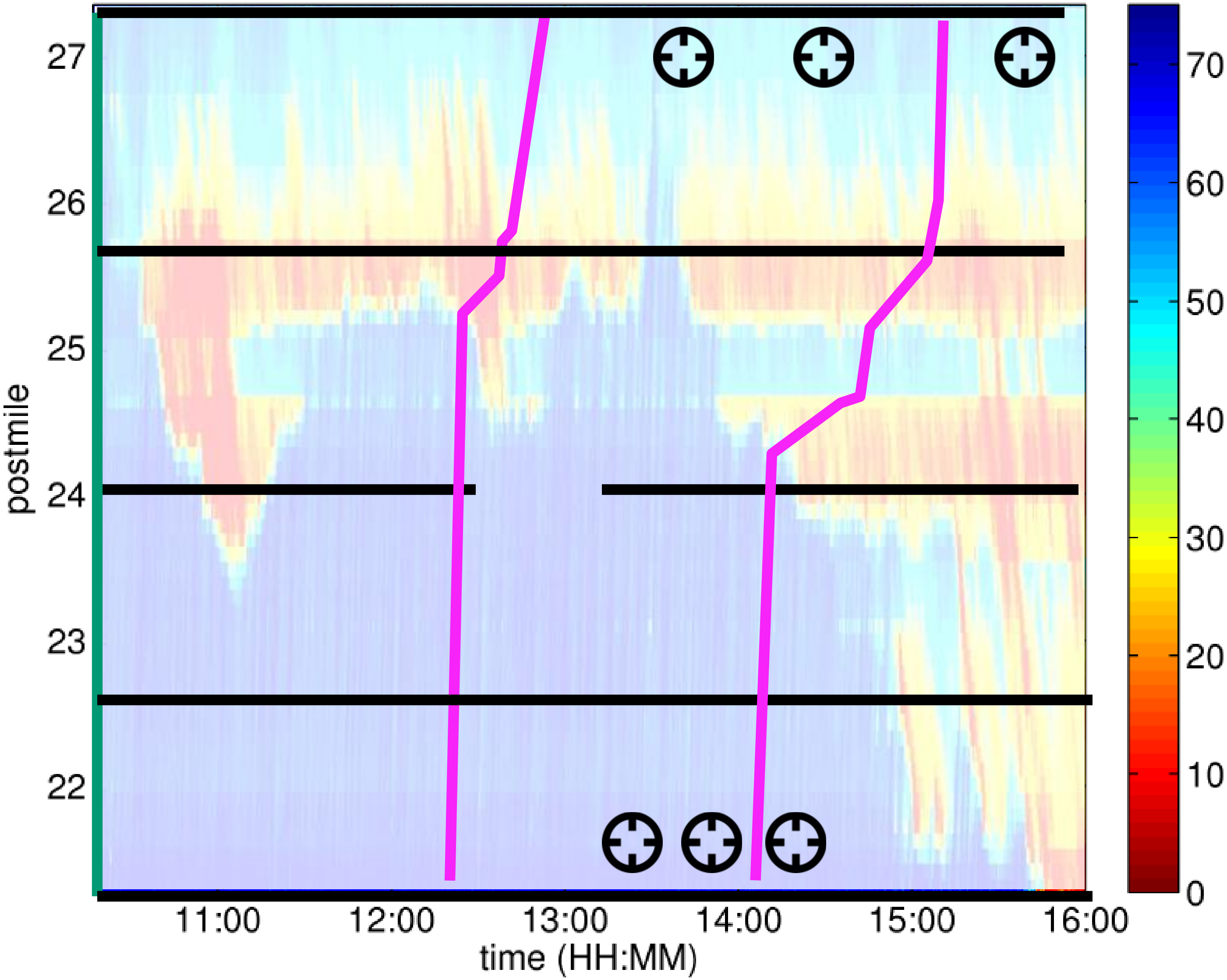
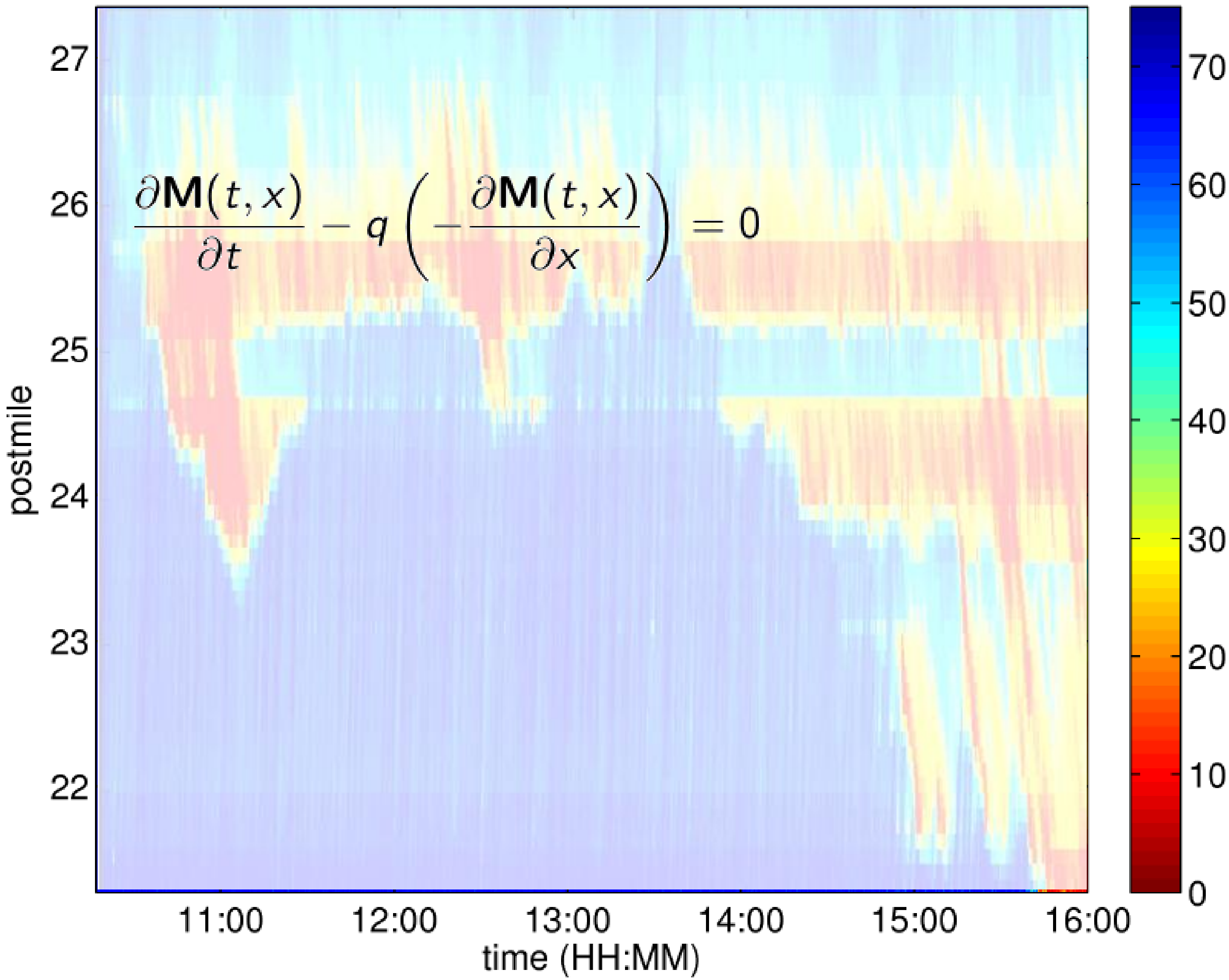
Probe vehicle (phone)

Re-ID (LPR, FasTrack)



State estimation from heterogeneous sources

Process: Hamilton-Jacobi PDE, with concave non smooth Hamiltonian



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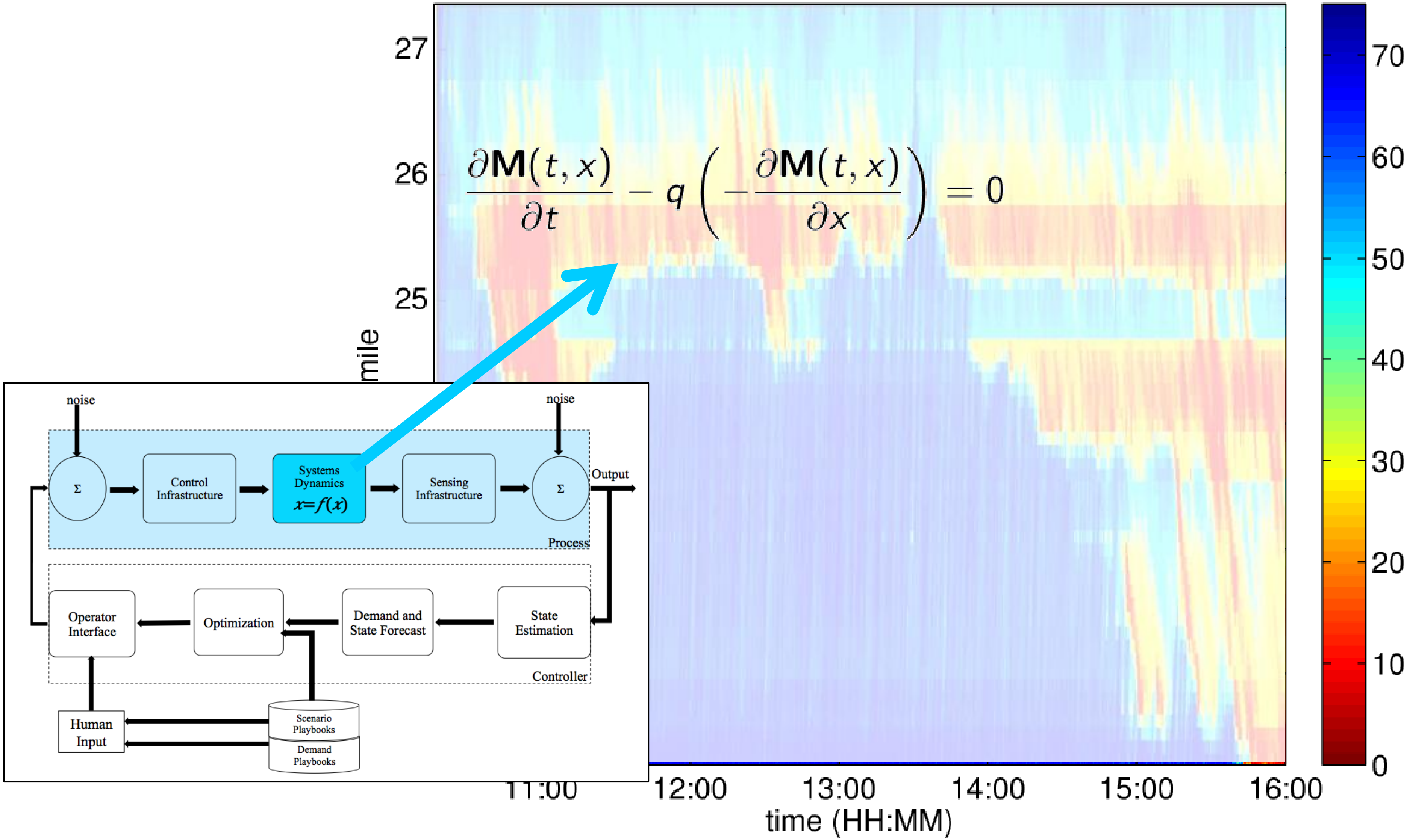
$$\frac{\partial \mathbf{M}(t,x)}{\partial t} - q\left(-\frac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0$$



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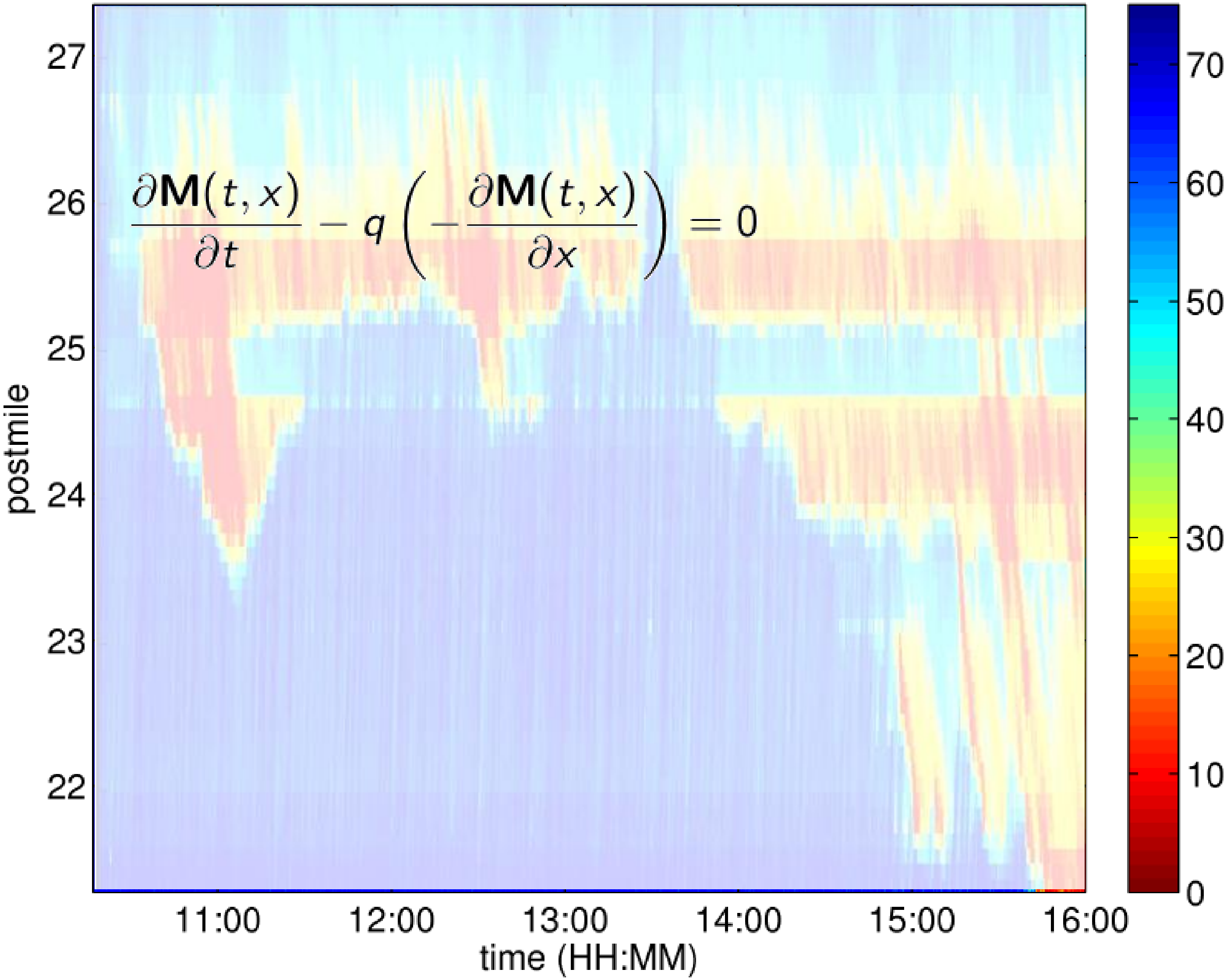

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$\mathbf{M}(t, x_{\text{loop}})$

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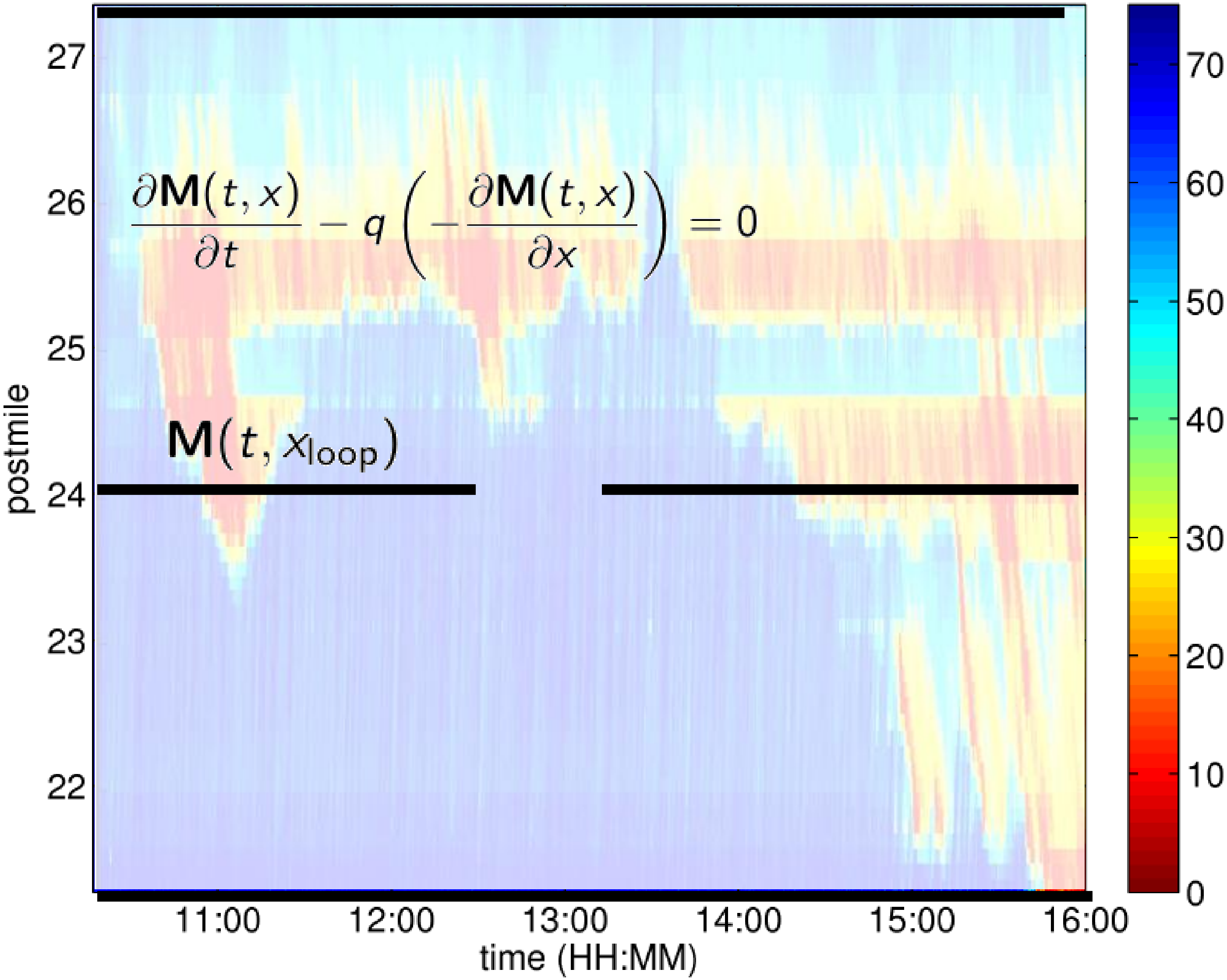

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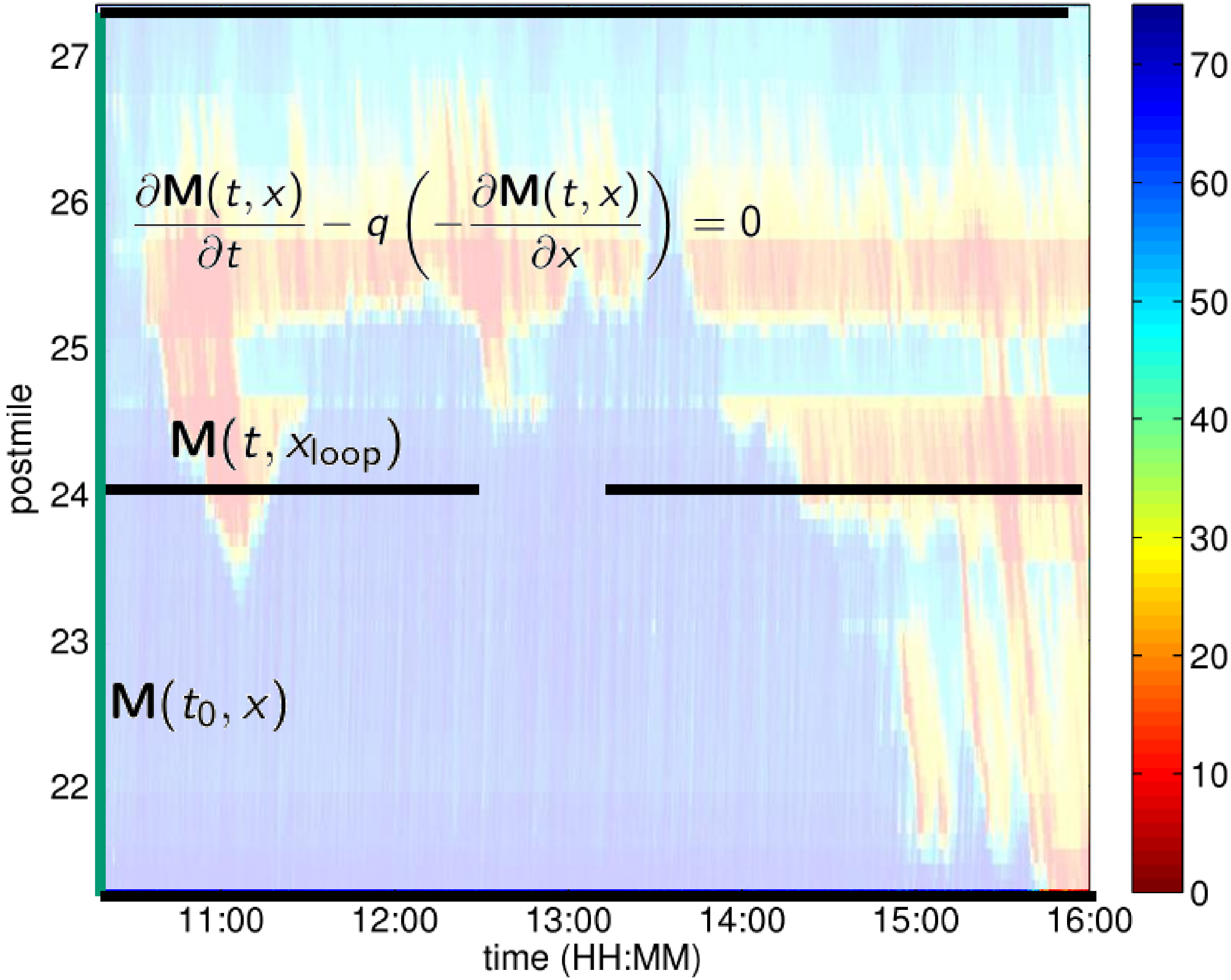

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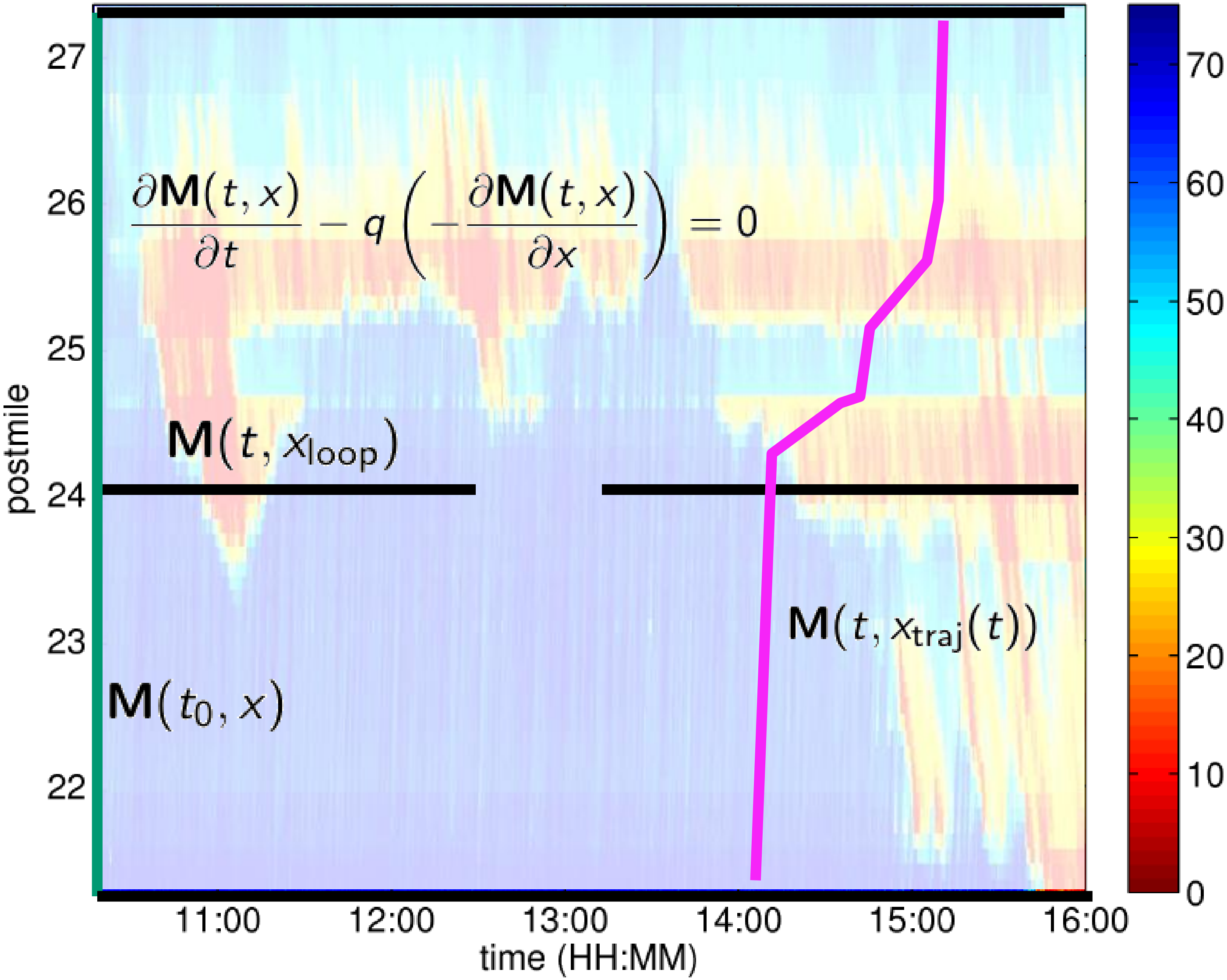

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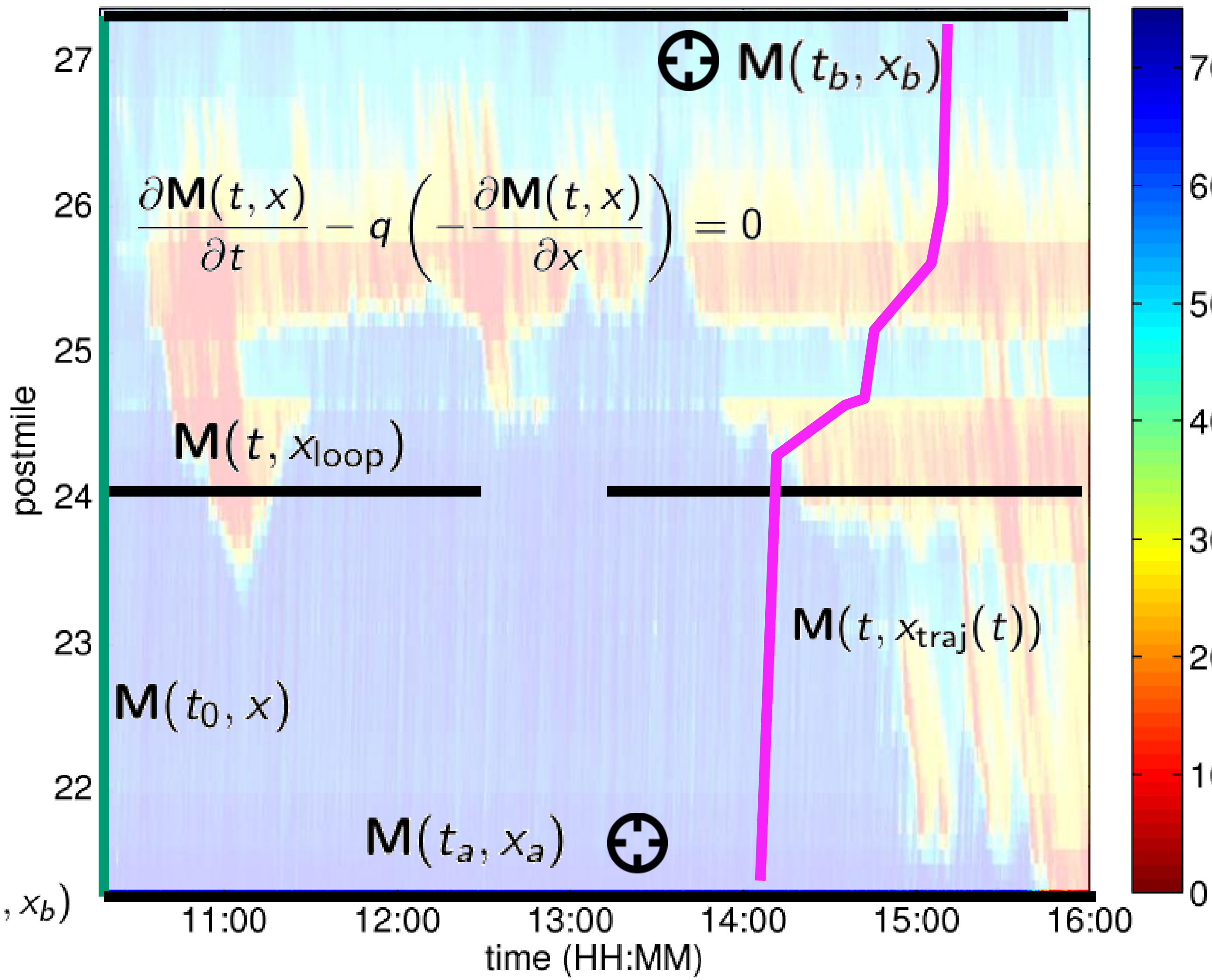
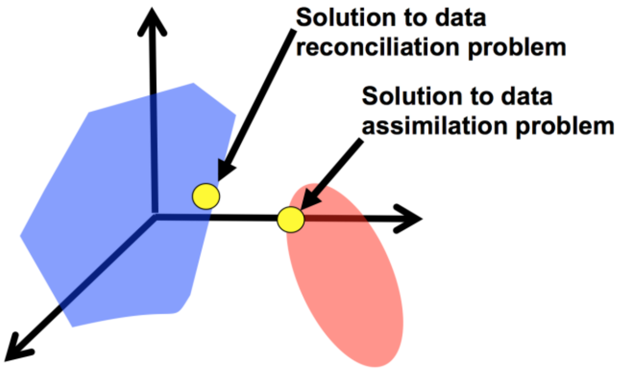
Re-ID (LPR, FasTrack)

$\mathbf{M}(t_a, x_a) = \mathbf{M}(t_b, x_b)$

Process: Hamilton-Jacobi PDE, with concave non smooth Hamiltonian



Data assimilation and reconciliation



Existence/uniqueness of solution of (1)
[SIAM SICON, 2008]

Lax Hopf formula
[IEEE TAC 2010a]

Internal / BC / IC sol. computation
[IEEE TAC 2010b]

Convex formulation for the estimation problem
[SIAM SICON 2011]

Theorem

For PWA boundary, internal, and initial conditions

$$\overline{\mathbf{M}}(t,x), \tag{1}$$

with parameters in acceptable intervals, model constraints induced by

$$\frac{\partial \mathbf{M}(t,x)}{\partial t} - q\left(-\frac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0 \tag{2}$$

are a set of convex inequalities in the unknown initial, boundary and internal condition coefficients. For measurement data error bounded in L_1, L_2 or L_∞ norm, constraints resulting from measurement error are convex.

Corollary

The data reconciliation problem and data assimilation problems can be posed in convex form as

$$\begin{array}{ll} \mathbf{min}_{\{m,n\}} & \|m-n\| \\ \mathbf{s.t.} & \text{m satisfies inequality constraints induced by (2)} \\ & \text{n satisfies inequality constraints from sensor specs. (1)} \end{array}$$

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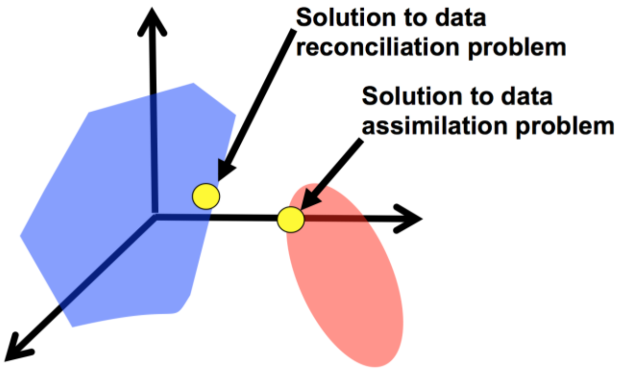

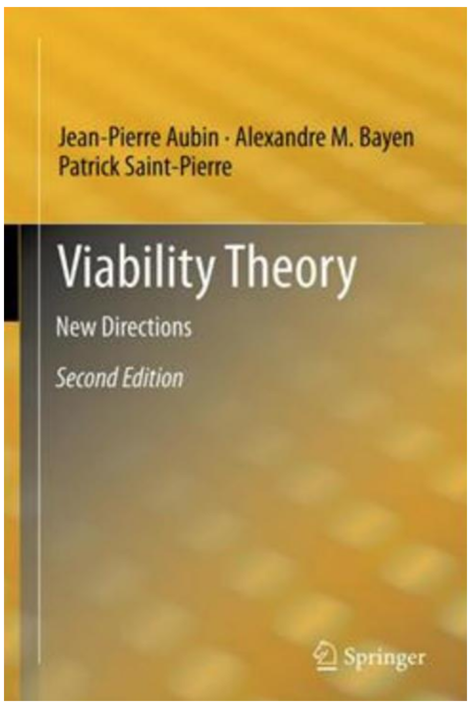

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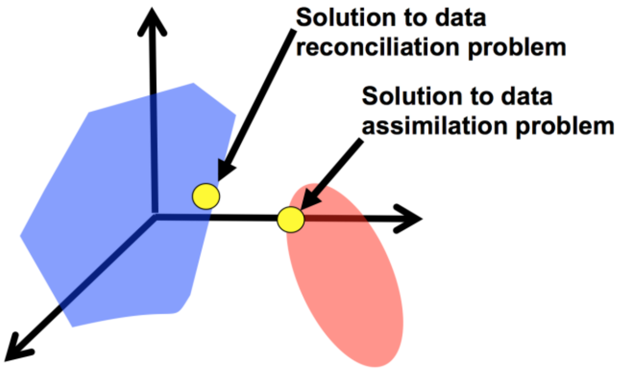
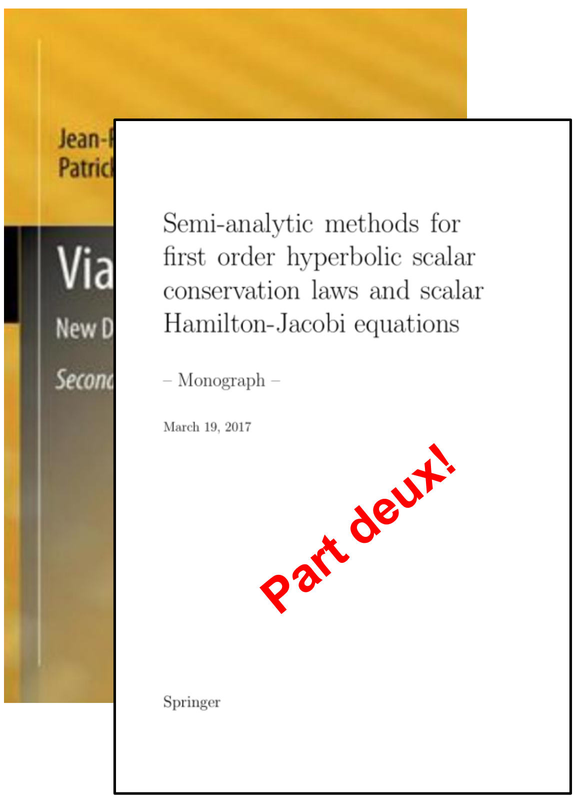
Data assimilation and reconciliation





Theorem

For PWA boundary, internal, and initial conditions

$$\overline{\mathbf{M}}(t,x), \quad (1)$$

with parameters in acceptable intervals, model constraints induced by

$$\frac{\partial \mathbf{M}(t,x)}{\partial t} - q\left(-\frac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0 \quad (2)$$

are a set of convex inequalities in the unknown initial, boundary and internal condition coefficients. For measurement data error bounded in L_1, L_2 or L_∞ norm, constraints resulting from measurement error are convex.

Corollary

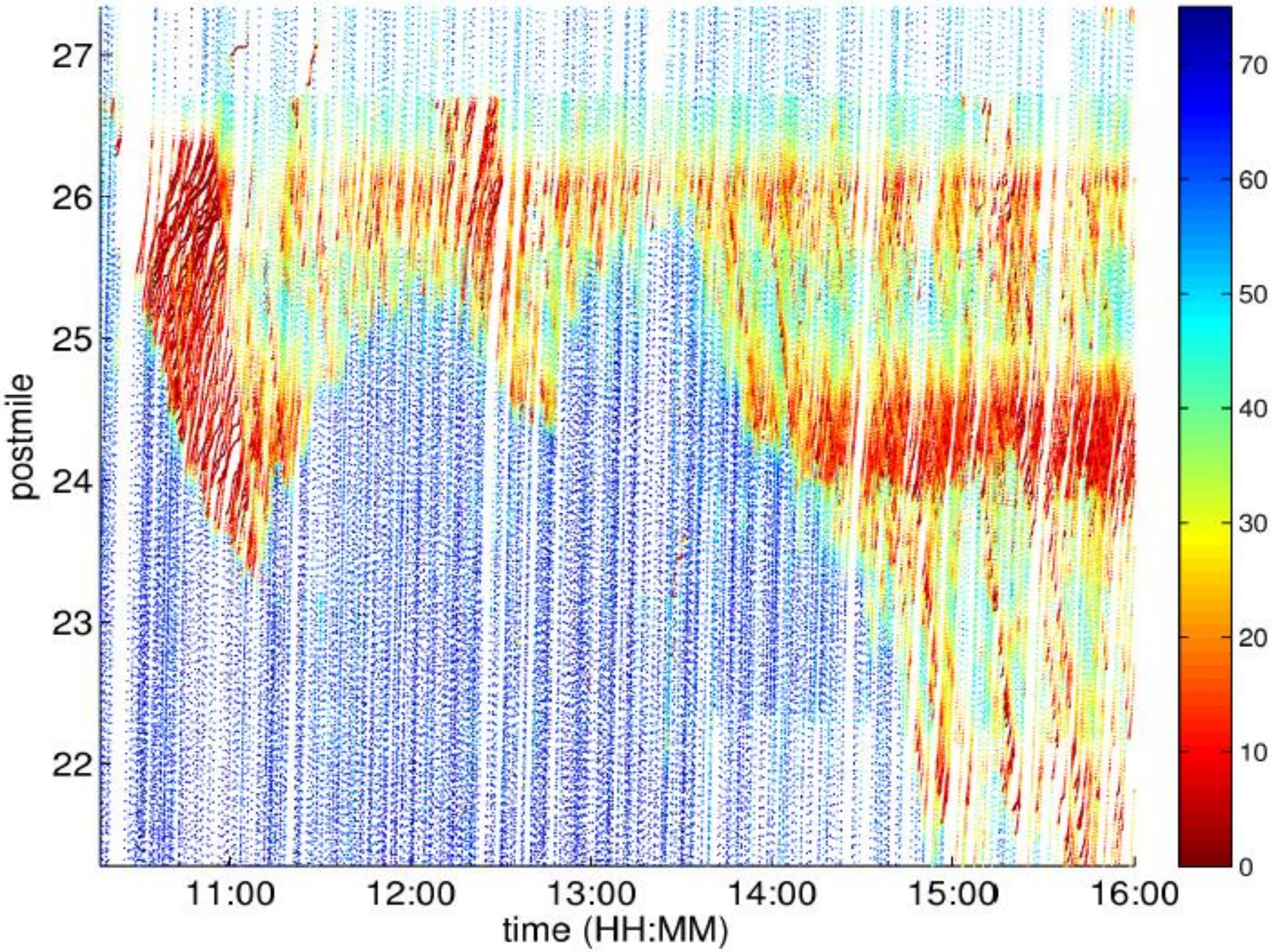
The data reconciliation problem and data assimilation problems can be posed in convex form as

$$\begin{array}{ll} \mathbf{min}_{\{m,n\}} & \|m-n\| \\ \mathbf{s.t.} & \text{m satisfies inequality constraints induced by (2)} \\ & \text{n satisfies inequality constraints from sensor specs. (1)} \end{array}$$

Implementation at 2% penetration rate

Paradise for data assimilation starts at 2% penetration rate

- **However, it is rare to have such penetration uniformly**
- **Algorithms often used at lower penetration**



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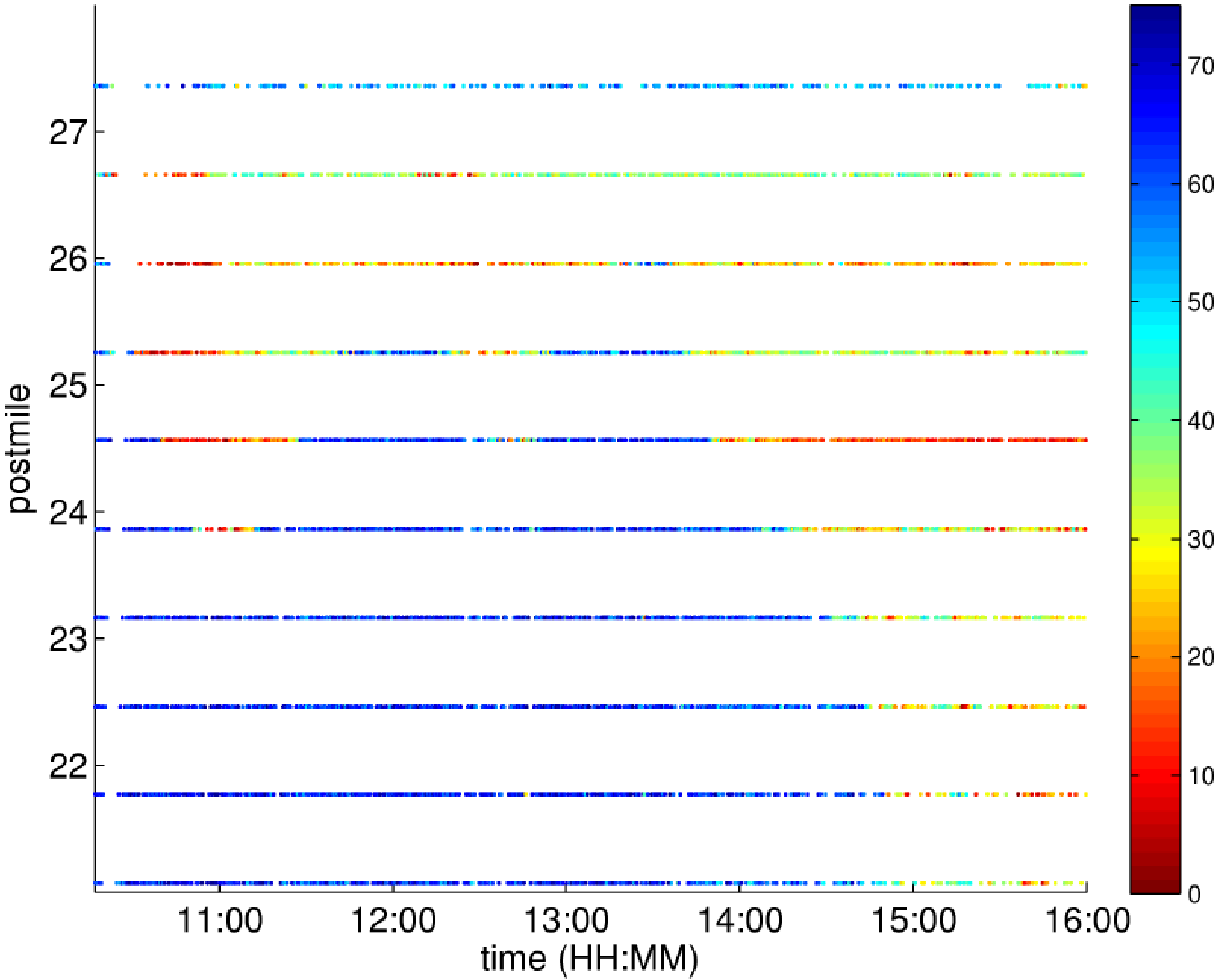
10 measurement points



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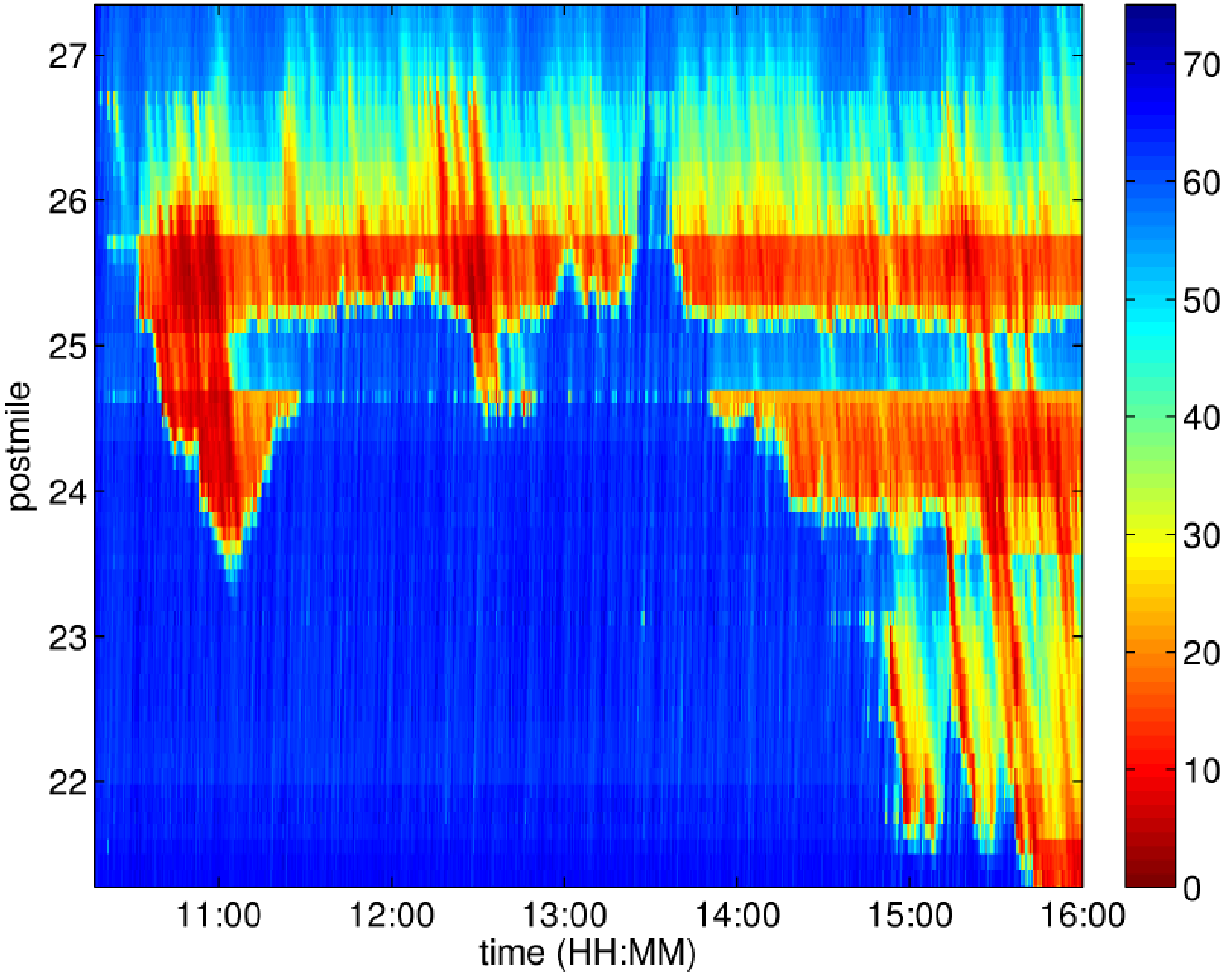
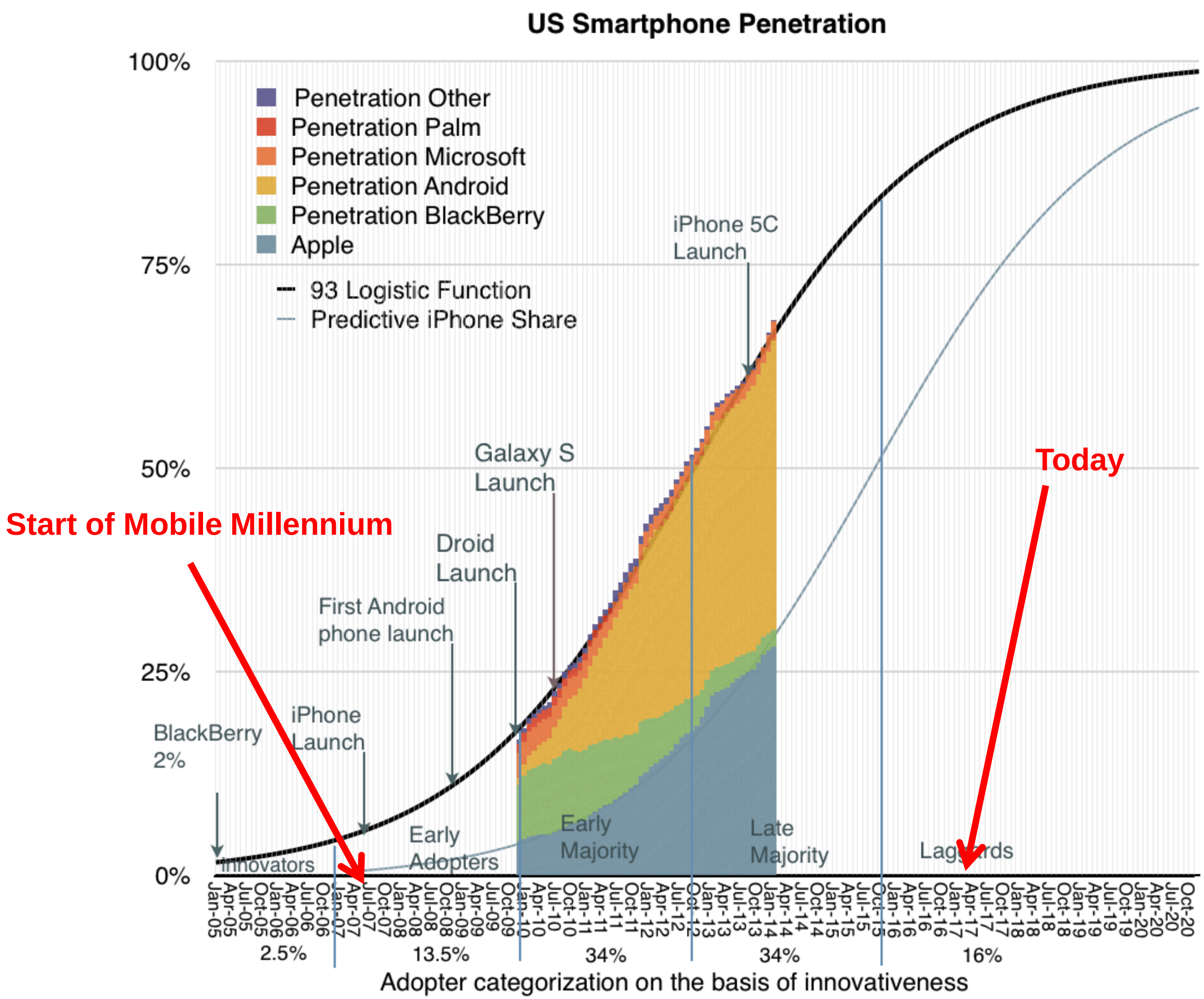


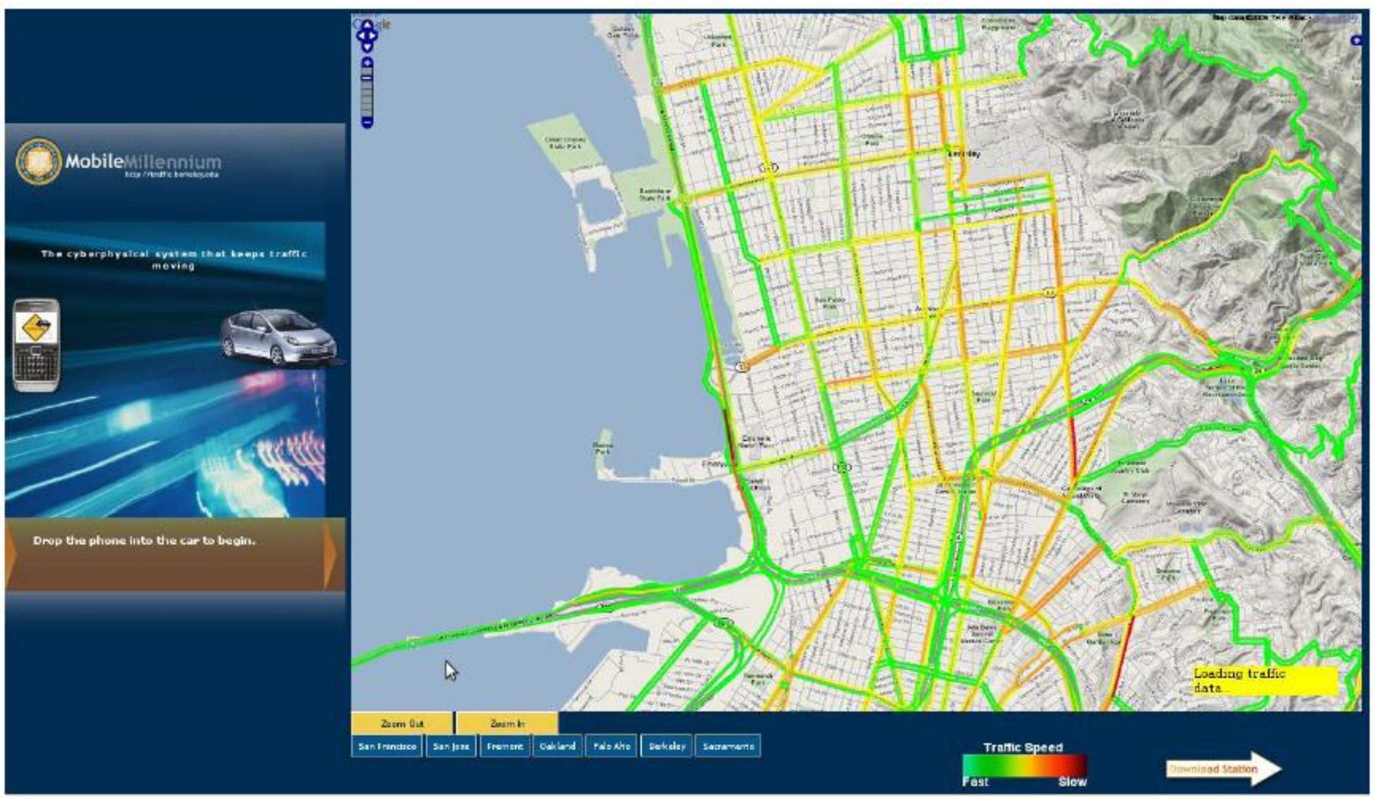
Historical perspective on mobile devices

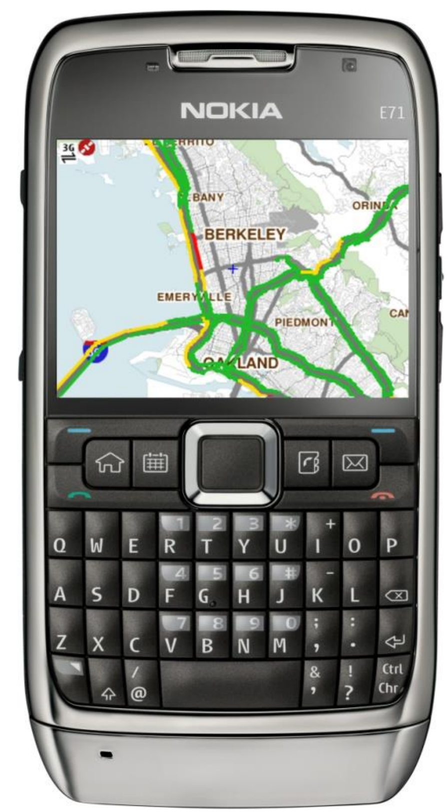


Mobile Millennium (2008-2010)

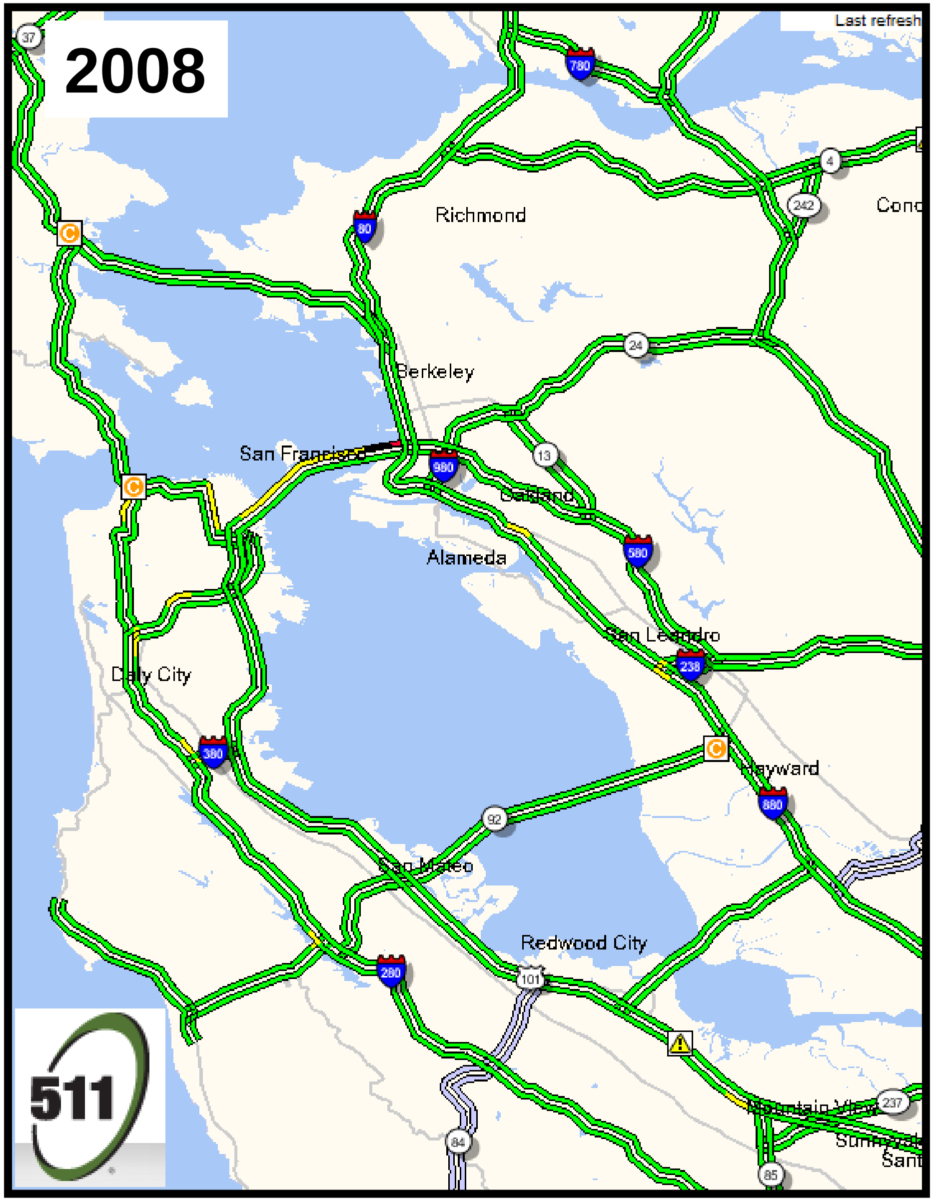
An early instantiation of participatory sensing

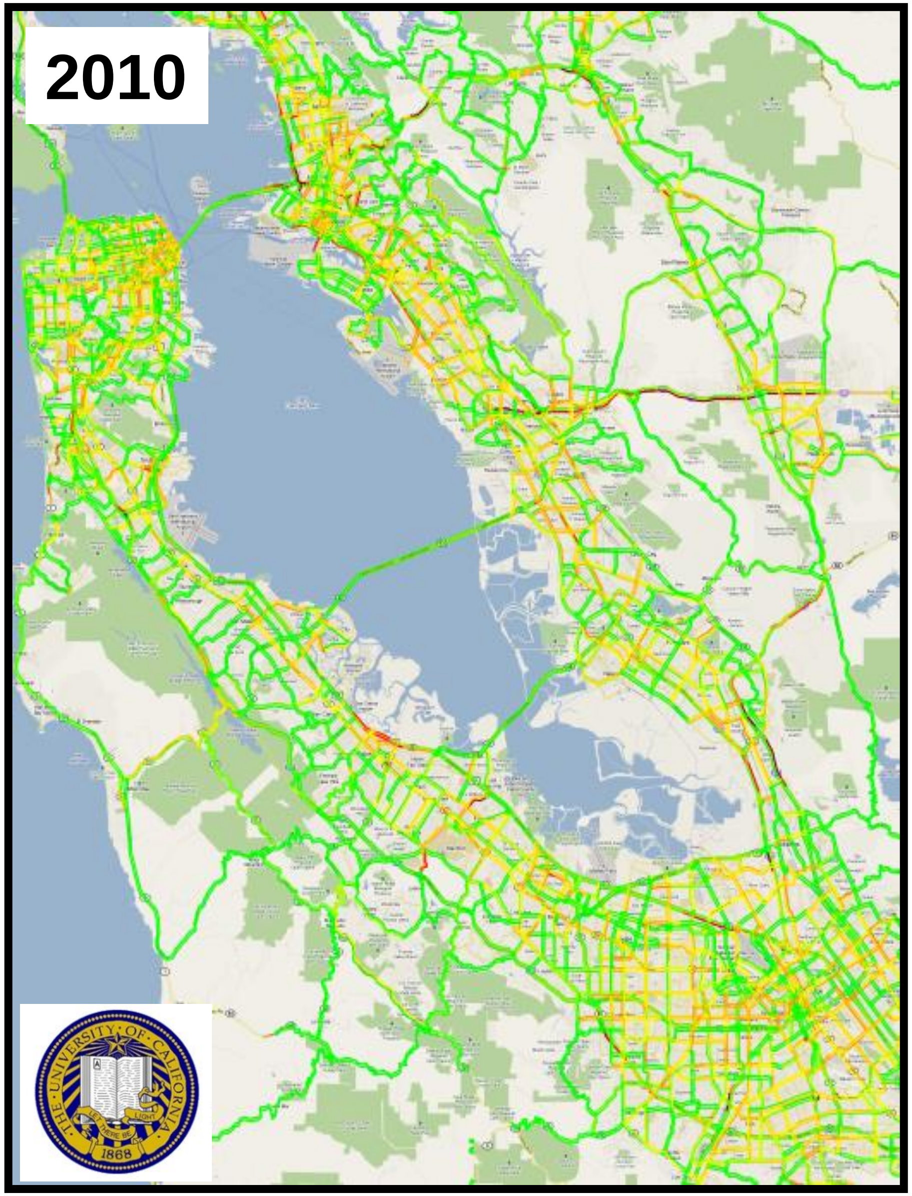
- **Consortium: NSF, US DOT, Caltrans, Nokia, NAVTEQ, + 10 others**
- **2008: 5000 downloads of the FIRST Nokia traffic app worldwide**
- **After a few months: about 60 million data points / day from dozen of sources (smartphones, taxis, fleets, etc.)**





Historical indirect smartphone beneficiary: traffic

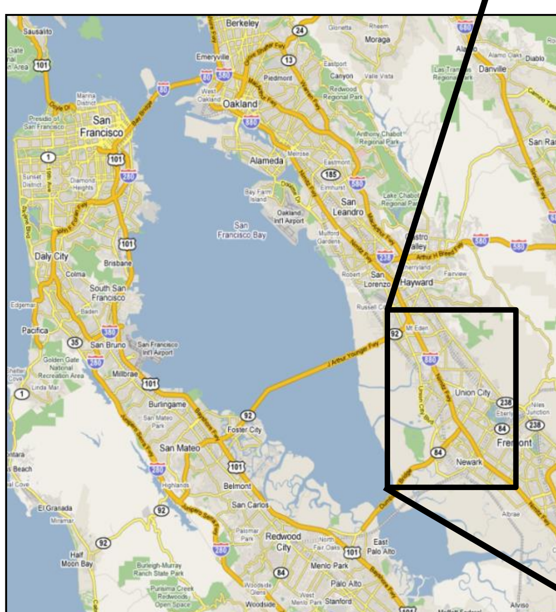


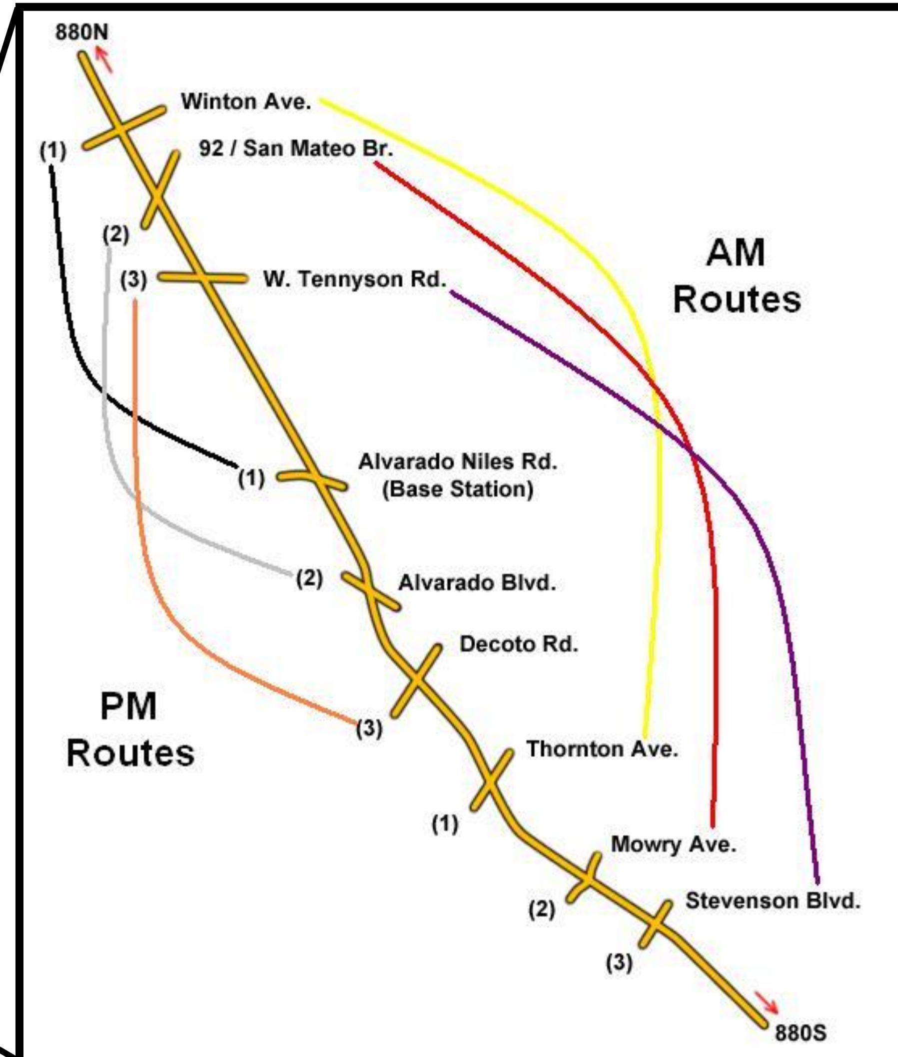


Prototype experiment: *Mobile Century* (2008)

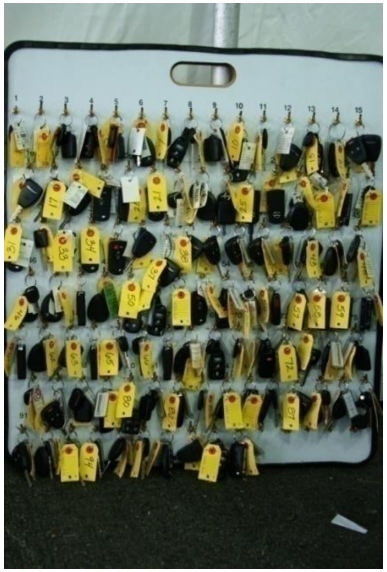
Experimental proof of concept: the *Mobile Century* field test

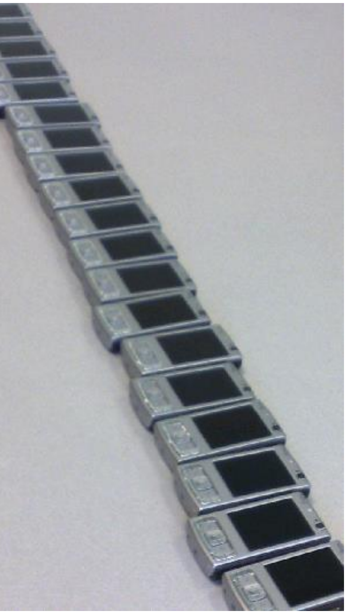
- February 8th 2008
- I80, Union City, CA
- Field test, 100 cars
- 165 Berkeley students drivers
- 10 hours deployment,
- About 10 miles
- 2% - 5% penetration rate

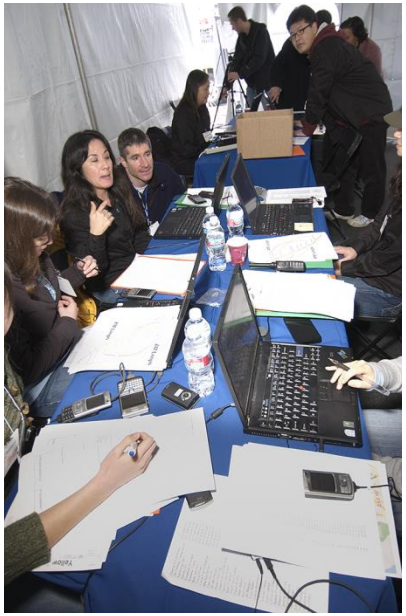




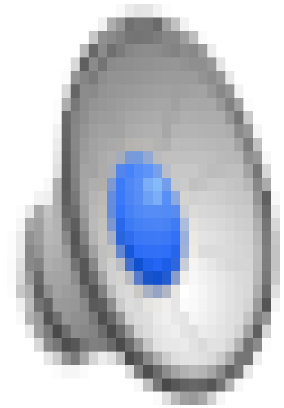
A glimpse of *Mobile Century* (2008)







A glimpse of *Mobile Century* (February 8th, 2008)

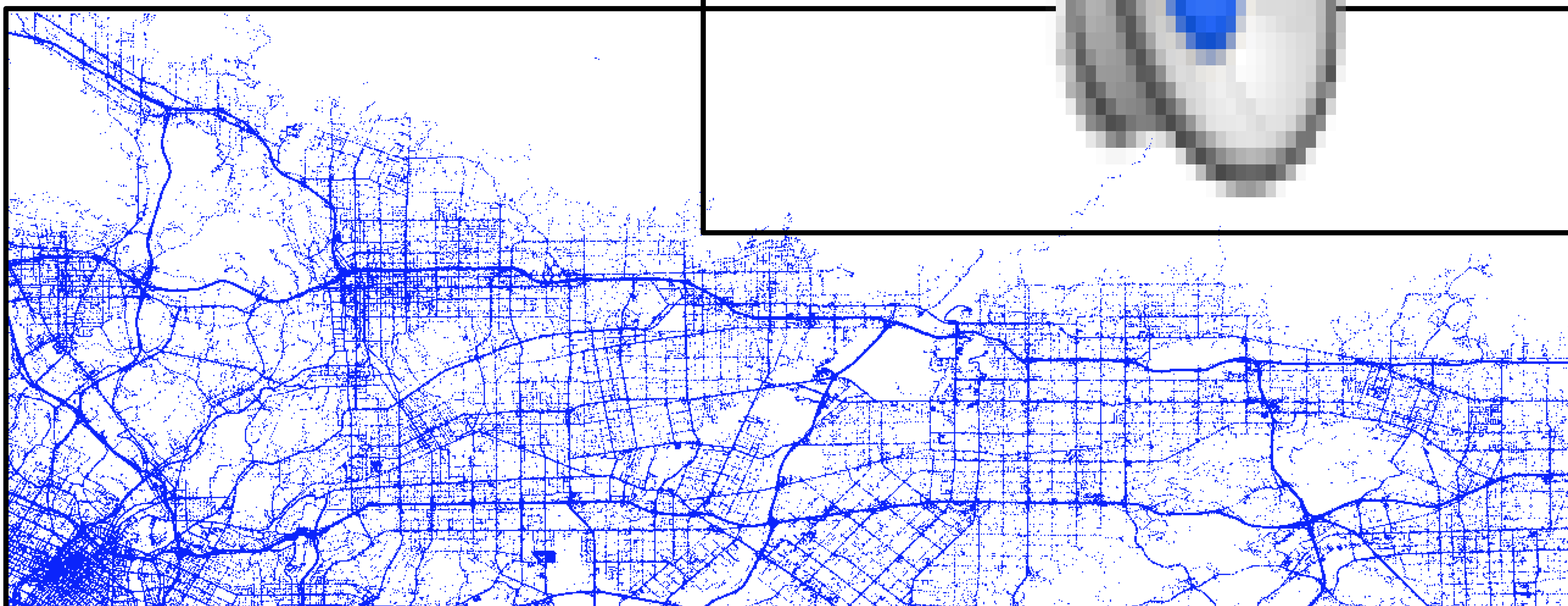


[Herrera et al., TR-C, 2010]

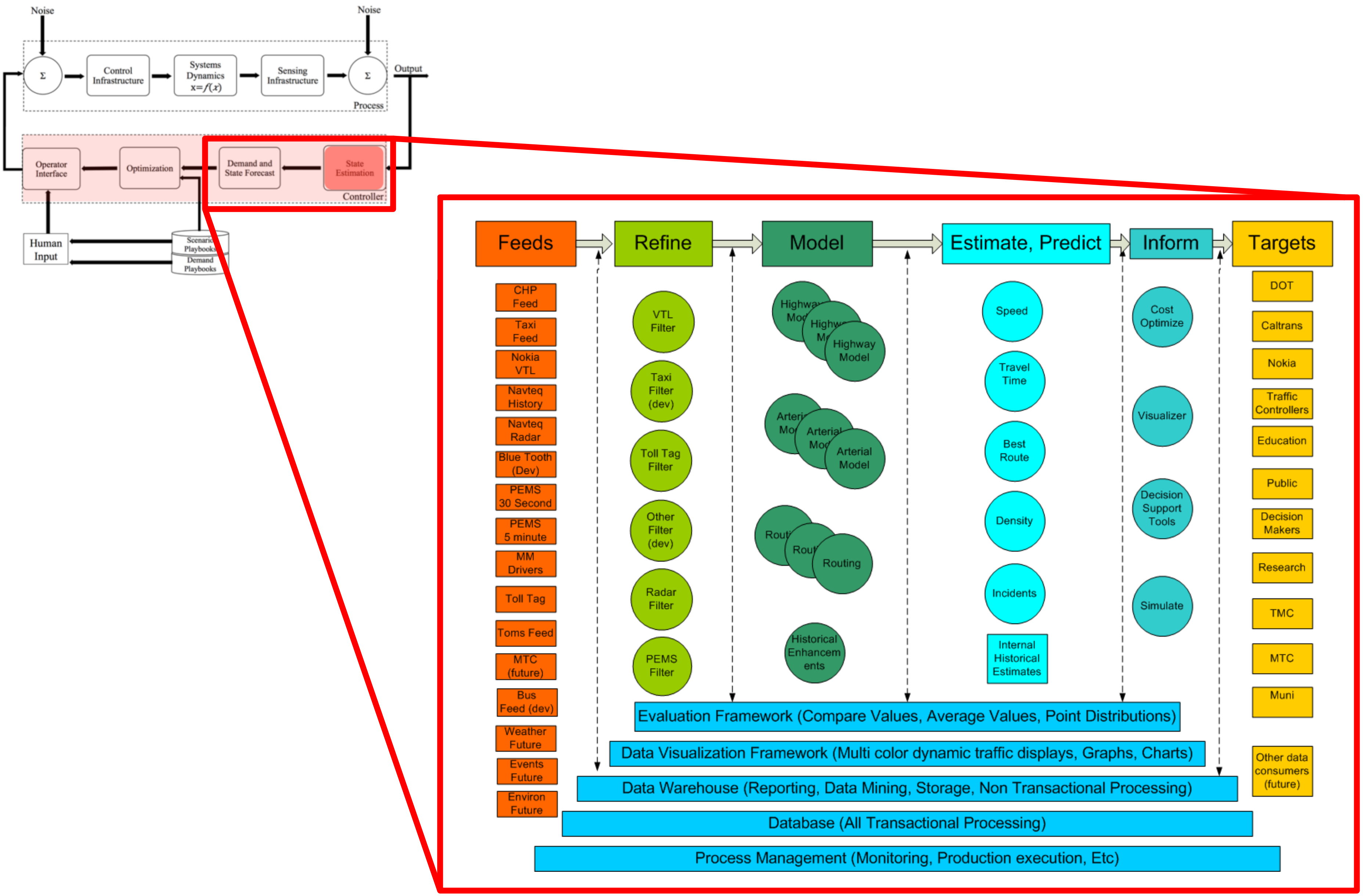
Data sample

500 vehicles sampled at 30' intervals

One day of data, 0.5% penetration



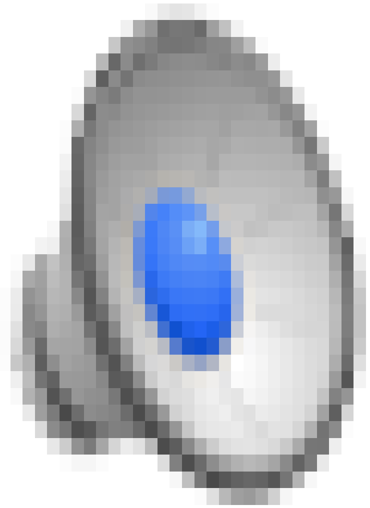
Architecture of *Mobile Millennium*



The early days of *Mobile Millennium*

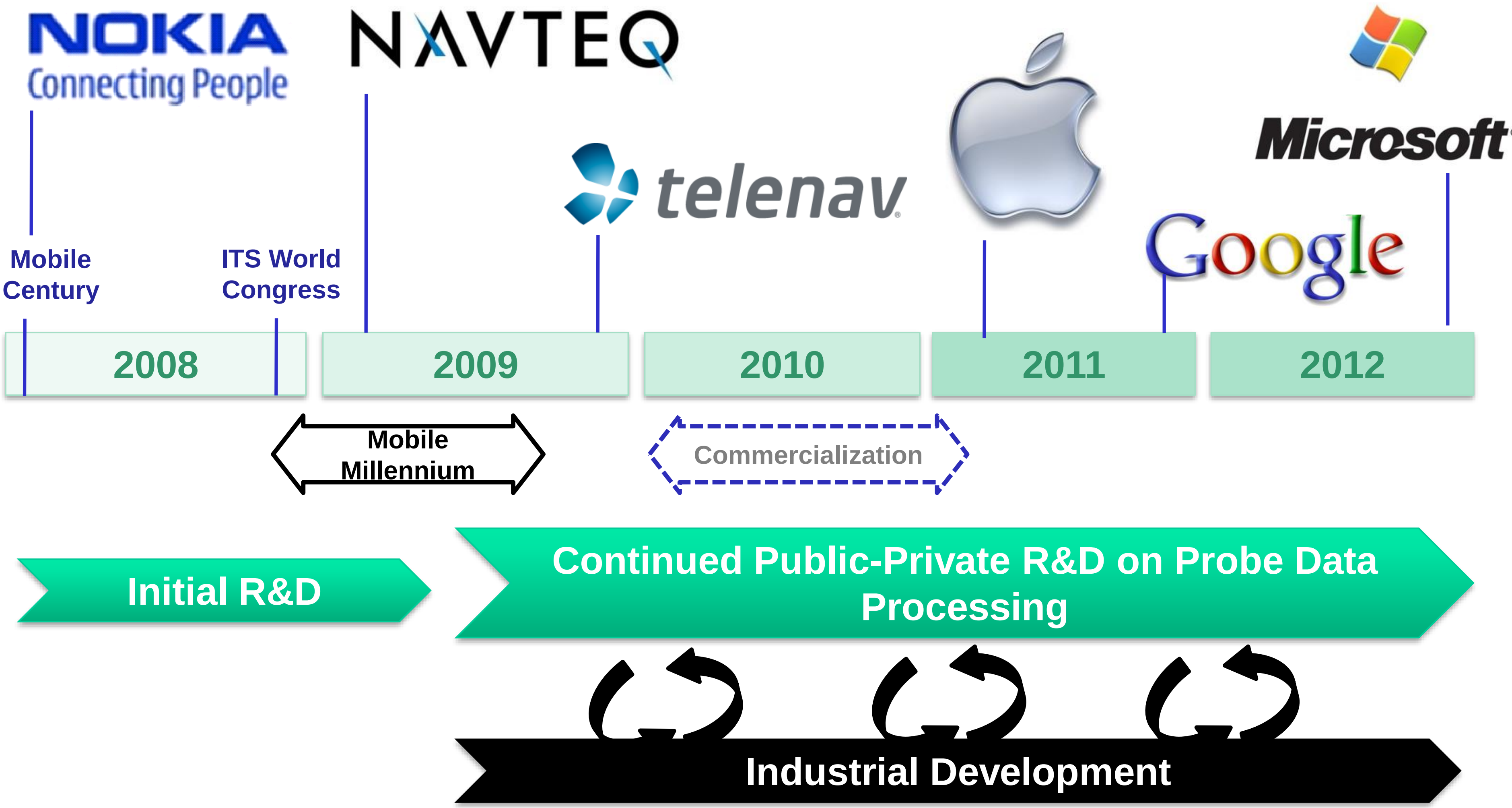
Friday, March 20th, 2009, [accelerated] synchronized movies

- **Acceleration: 1 frame = 30 seconds of physical time**
- **1:30pm (Friday afternoon congestion)**



Google Maps (no probes) **Mobile Millennium (probe based)**

Project and industry timeline



Contributions of the *Mobile Millennium* project

Modeling contributions

- **Flow models for integration of Lagrangian data for highways**
- **Machine learning models for arterial traffic**

Estimation contributions

- **Statistical filtering for discretized PDEs (EnKF, PF, EKF, etc.)**
- **Convex optimization approaches to data assimilation (variational formulations, viability formulations)**

Experimental contributions

- **Building an app and a full backend system (three times…)**
- **Running experiments at scale**
- **Integrating private sector feeds into live system**

Data quality contributions

- **Penetration studies (how much GPS data do we need?)**
- **Procurement for the State of California**

[Belletti et al., Phys. Let. A, 2015, Patire et al. TR-C, 2014, Thai and Bayen, IEEE TAC 2014, Hunter et al., IEEE TASE, 2013]

Outline

1. **General framework for traffic operations**

1. **Inference problems**
    1. **Demand inference**
    2. **Traffic estimation**

2. **Heterogeneous games**
    1. **Heterogeneous game, Nash-Stackelberg solutions**
    2. **Learning dynamics in repeated games**

3. **Other mobile sensor and data and CPS education**

The impact of traffic apps on system dynamics

Fundamental premise of routing services

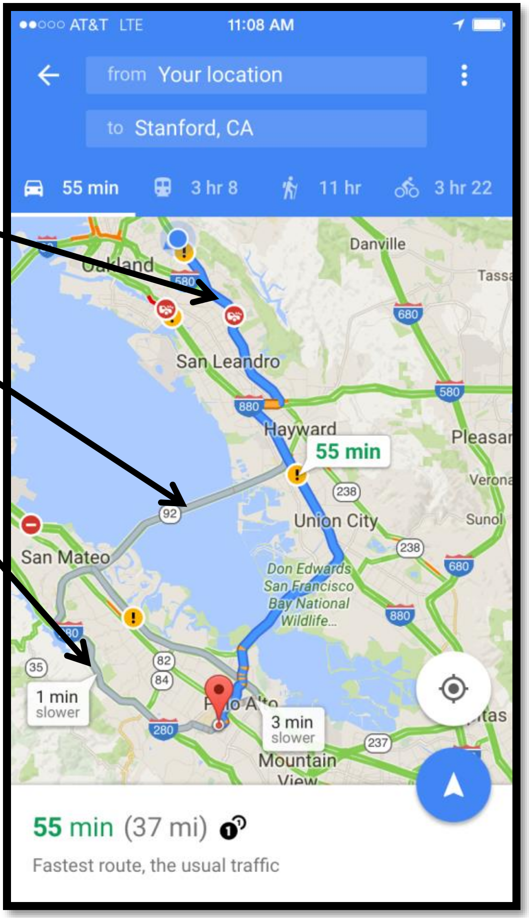
- Each app enabled user receives a [SOTA] shortest path
- Some follow the recommendations

All paths proposed are nearly equal:

- Shortest path (55mins)
- Third shortest path (58 mins)
- Second shortest path (56 mins)

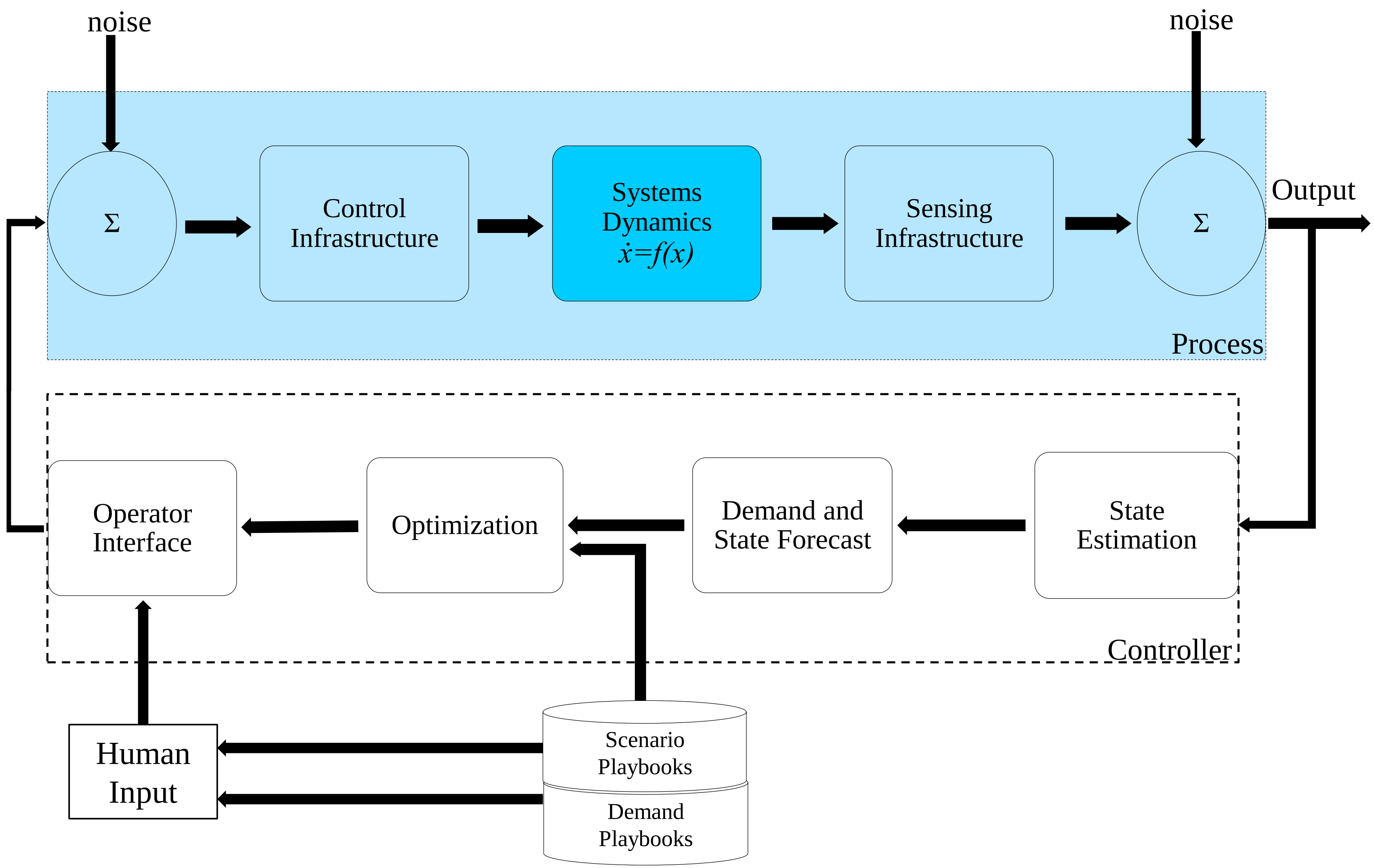
Routing does in general not depend on

- Forecast of the network loading using demand data (incomplete today)
- Forecast of the network using potential impact of routing (i.e. routed users) on the network
- Knowledge of what competitors of the app are doing (in the present case, Apple, INRIX, 511, etc.)

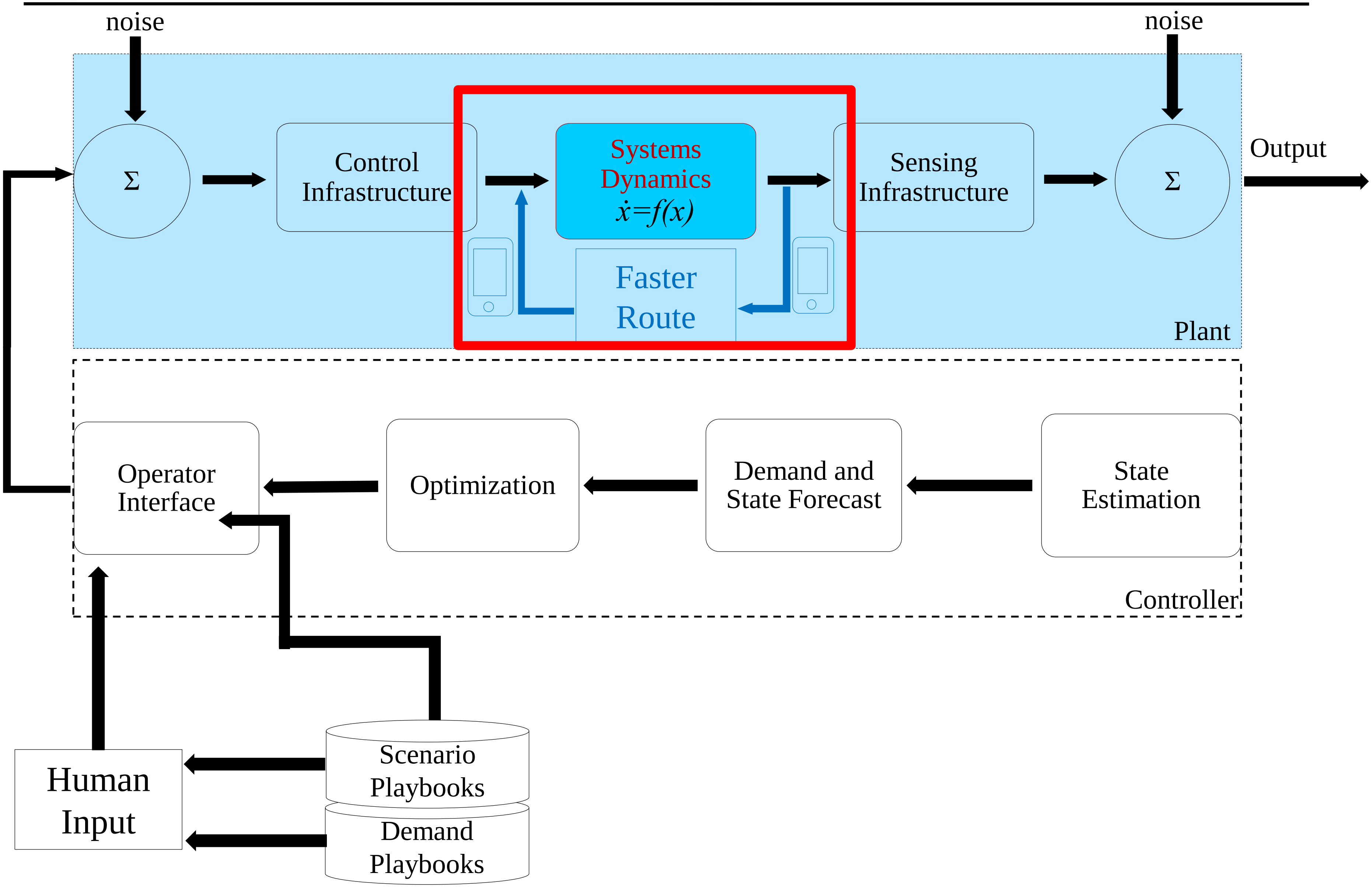


[Samaranayake et al., TR-C, 2012, ALANEX 2014, SIAM MAEE 2014]

Classical operations framework in transportation



There is now an active feedback loop in the plant



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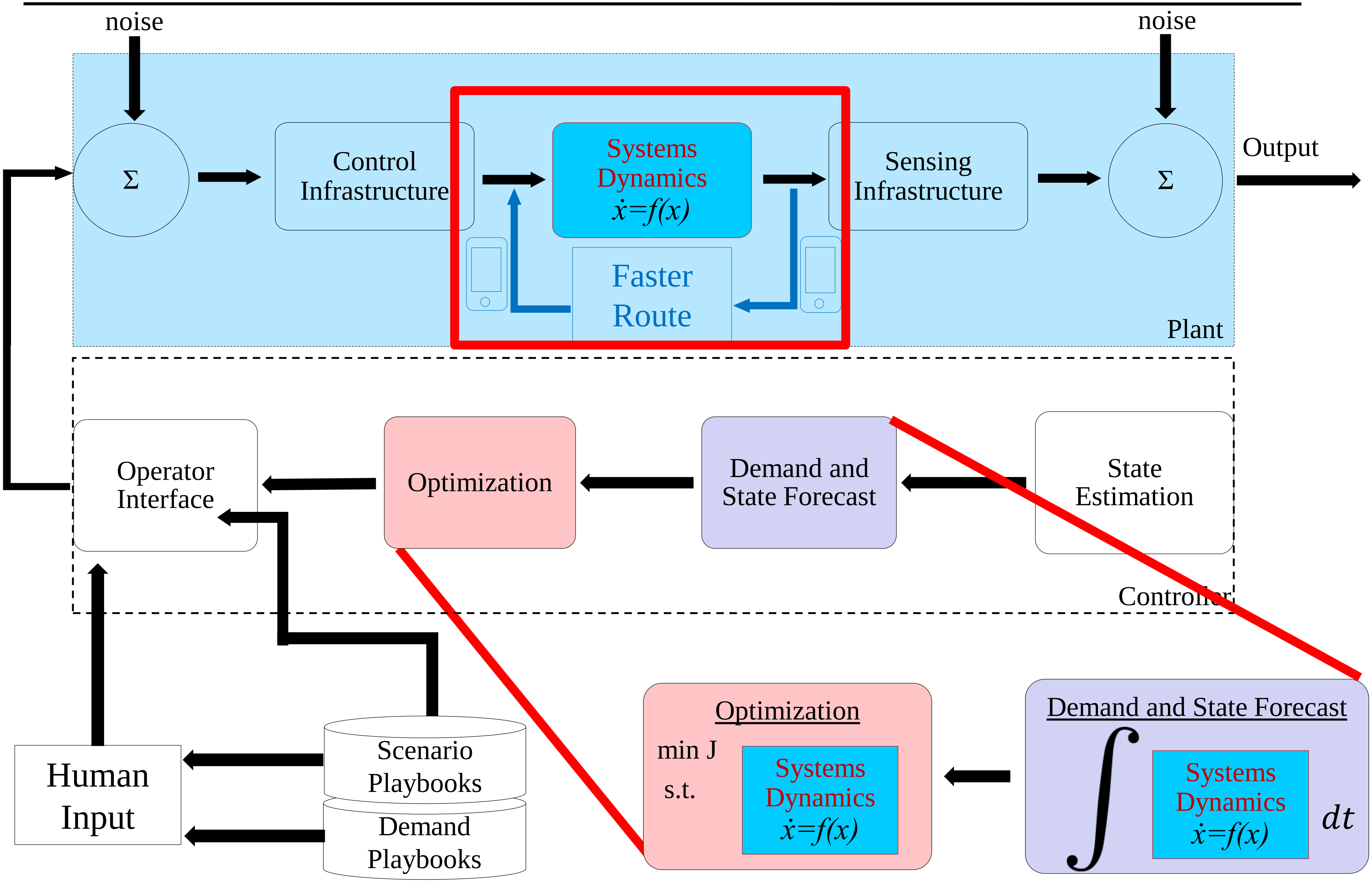
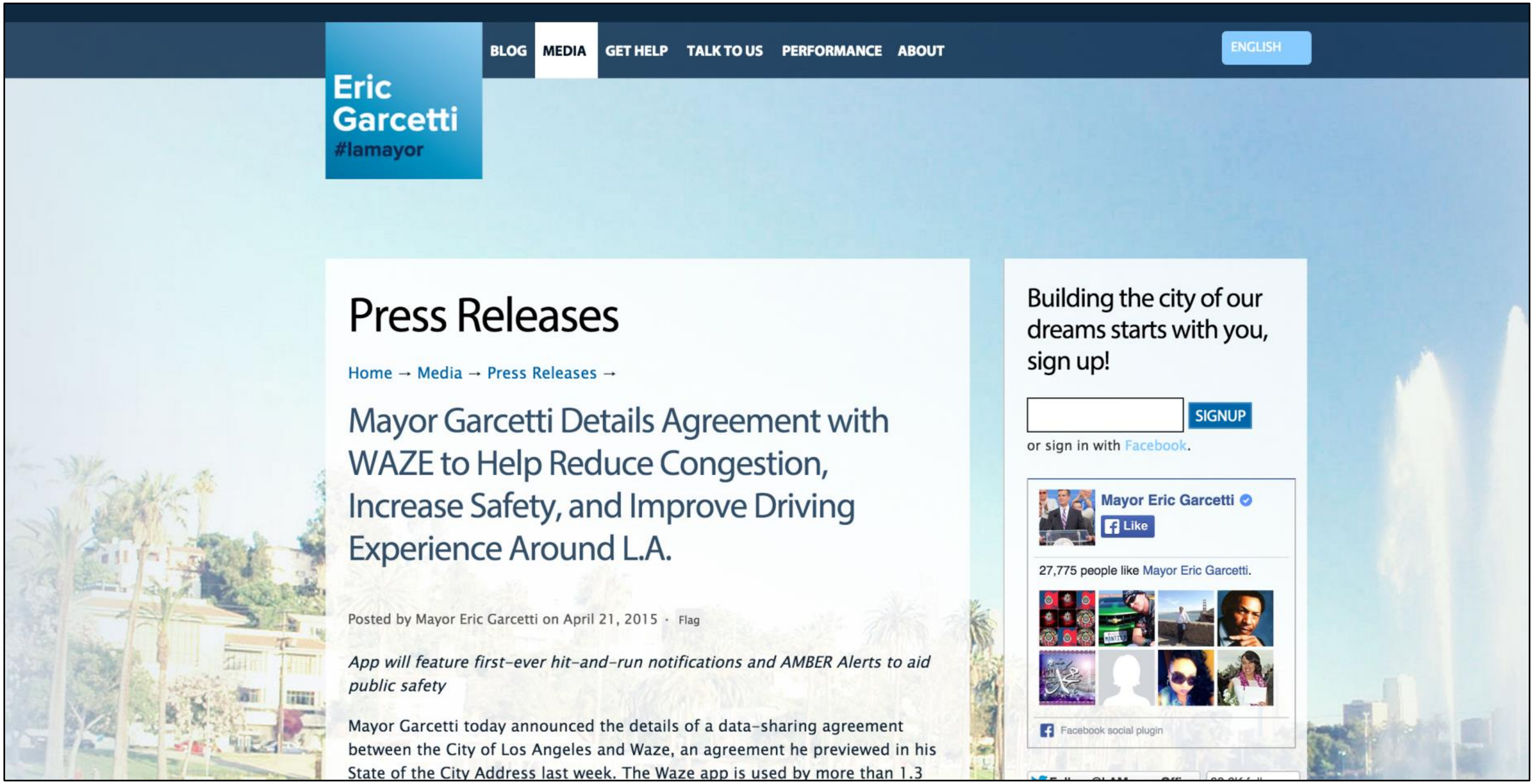


Initially people "thought" app helped



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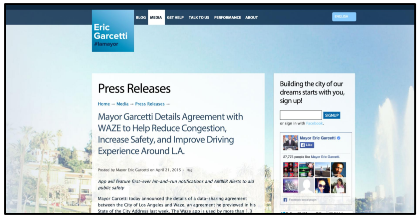

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Los Angeles and Waze Team Up to Combat Traffic Congestion

INSIGHTS | MOBILE DOSE

When Americans think of traffic they think of Los Angeles, even if they’ve never visited. So it makes sense that the LA mayor’s office has announced that the city is partnering with traffic app Waze to help combat the congestion. The deal allows data to be shared between the two parties—the city will alert Waze about hazards, construction and crashes while the app will give the city a wealth of data to analyze how traffic moves. Ideally this will allow for changes that will improve commutes.

Until more and more people started using it

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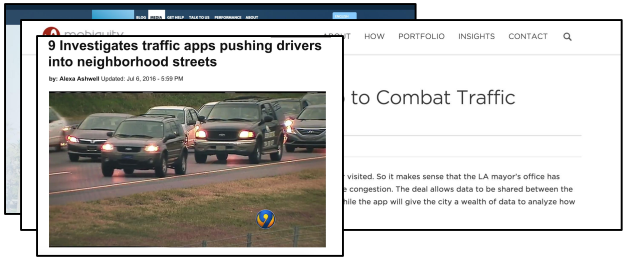
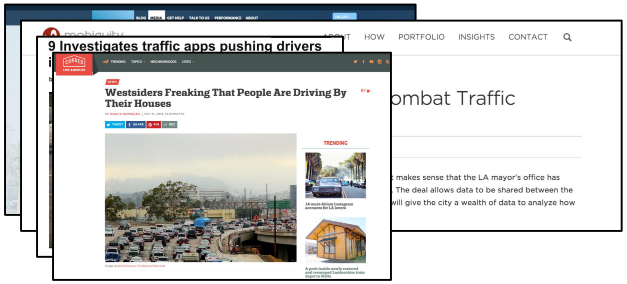
9 Investigates traffic apps pushing drivers into neighborhood streets

by: Alexa Ashwell Updated: Jul 6, 2016 - 5:59 PM

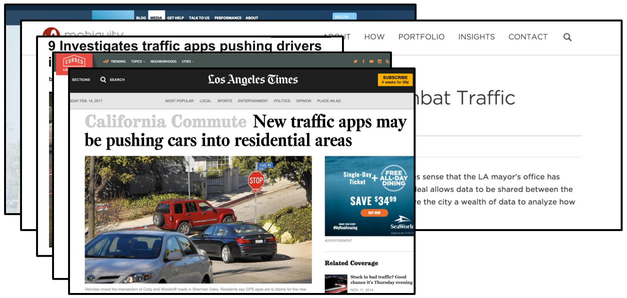
Until more and more people started using it

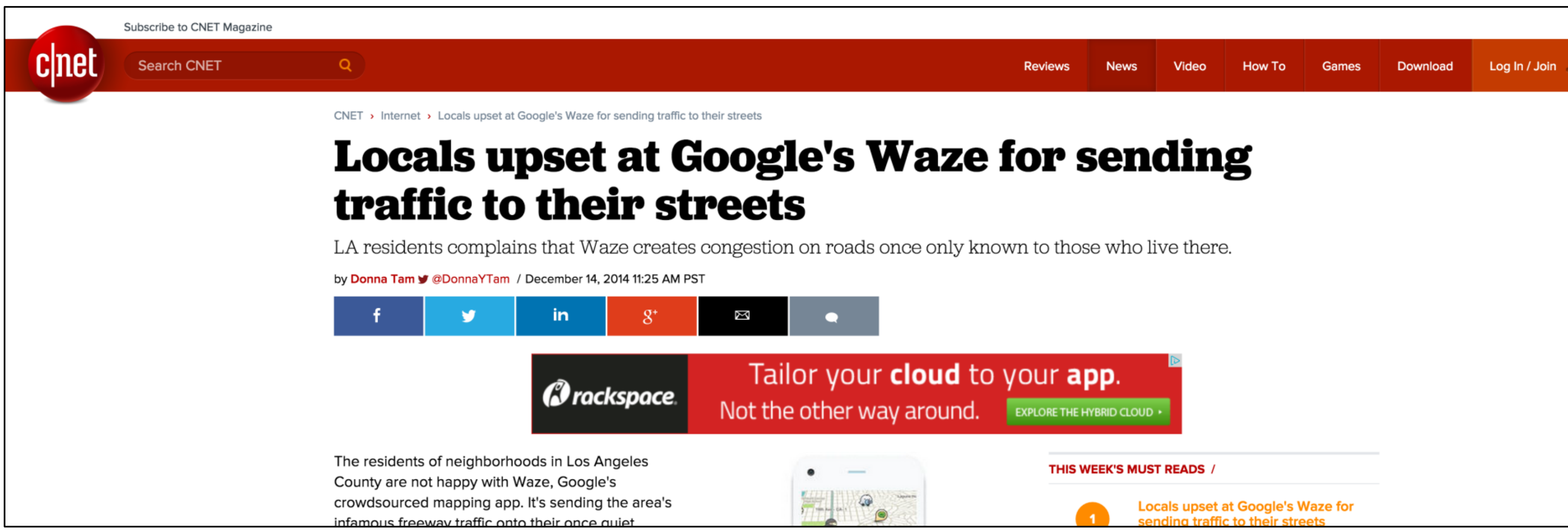


Until more and more people started using it



Specific apps are identified as responsible





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Locals upset at Google's Waze for sending traffic to their streets

LA residents complains that Waze creates congestion on roads once only known to those who live there.

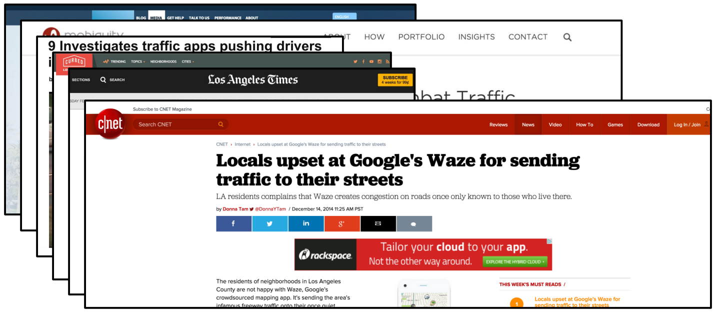
by Donna Tam @DonnaYTam / December 14, 2014 11:25 AM PST

The residents of neighborhoods in Los Angeles County are not happy with Waze, Google's crowdsourced mapping app. It's sending the area's infamous freeway traffic onto their once quiet

THIS WEEK'S MUST READS /

1 Locals upset at Google's Waze for sending traffic to their streets

Specific apps are identified as responsible



Get the Waze out of here! Echo Park residents blame apps for sending traffic to steep streets

October 24, 2016 32 Comments ECHO PARK

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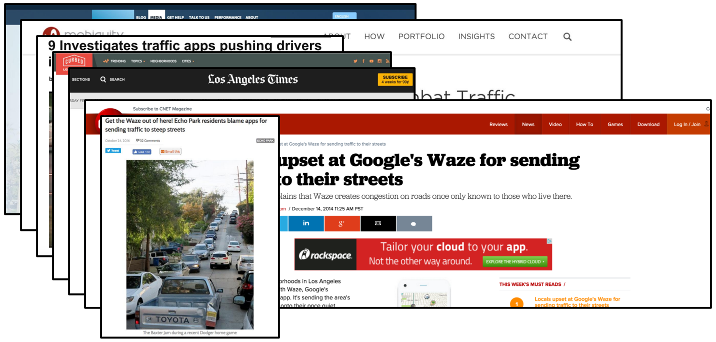
The Baxter Jam during a recent Dodger home game

Specific apps are identified as responsible



Readers React **How an app destroyed their streets: Readers count the Waze**

Vehicles crowd the intersection of Cody Road and Woodcliff Road in Sherman Oaks on Jan. 5. Residents say the worsening traffic on side streets is partially to blame on Waze. (Los Angeles Times)

Related Coverage

Time to rein in California's traffic ticket surcharges

MAY 1, 2015

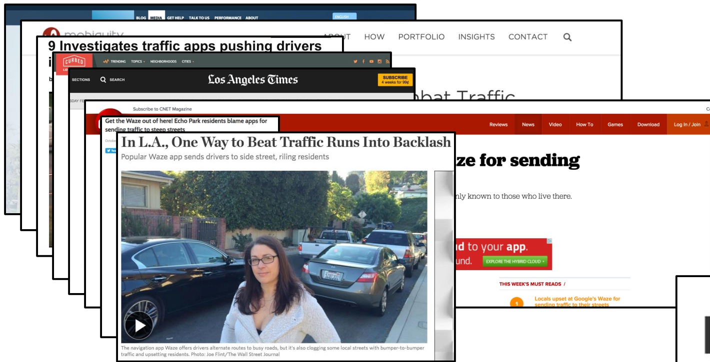
Specific apps are identified as responsible

In L.A., One Way to Beat Traffic Runs Into Backlash

Popular Waze app sends drivers to side street, riling residents

The navigation app Waze offers drivers alternate routes to busy roads, but it's also clogging some local streets with bumper-to-bumper traffic and upsetting residents. Photo: Joe Flint/The Wall Street Journal

Specific apps are identified as responsible



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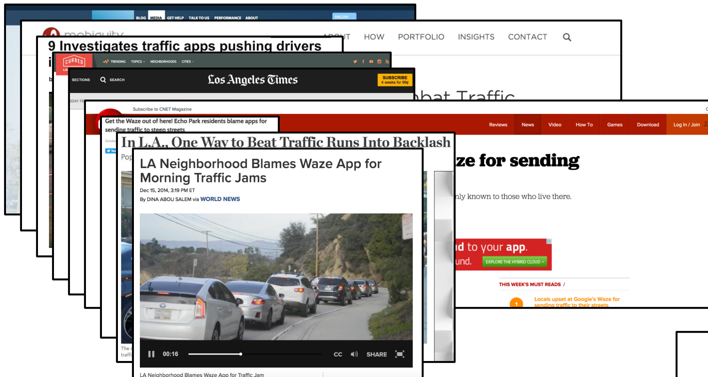


LA Traffic Is Getting Worse And People Are Blaming The Shortcut App Waze

AP

JOHN ROGERS, Associated Press

Dec. 14, 2014, 12:59 PM 13,855

FACEBOOK LINKEDIN TWITTER

LOS ANGELES (AP) — When the people whose houses hug the narrow warren of streets paralleling the busiest urban freeway in America began to see bumper-to-bumper traffic crawling by their homes a year or so ago, they were baffled.

Specific apps are identified as responsible

CNBC HOME U.S. NEWS MARKETS INVESTING TECH MAKE IT VIDEO

LA residents complain about 'Waze Craze'

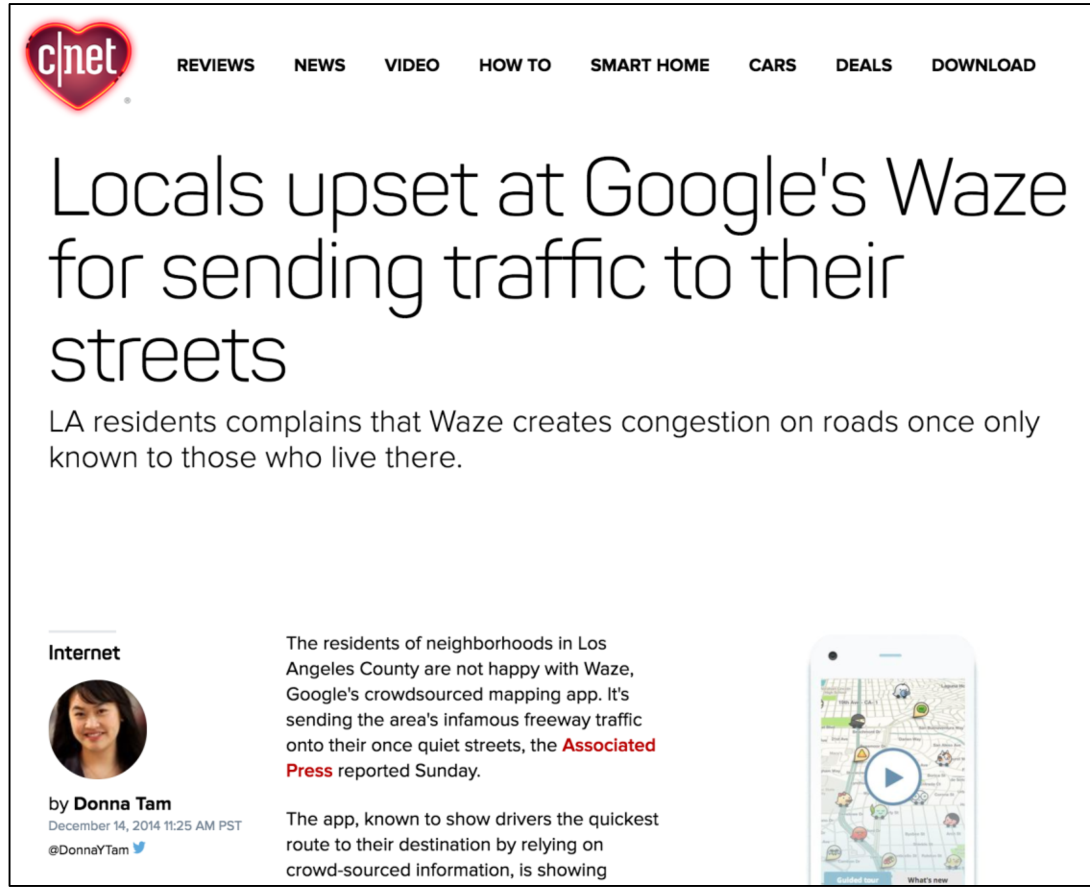
Jane Wells | @janewells

Thursday, 11 Dec 2014 | 2:54 PM ET

CNBC

Drivers in Los Angeles pride themselves on their ability to strategize the daily commute. Every day presents a new challenge: Find the best shortcut, the secret alternative route, to shave off precious minutes from a cruel trek. "Saturday Night Live's" "The Californians" is played for laughs, but it rings true.

Specific apps are identified as responsible



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Locals upset at Google's Waze for sending traffic to their streets

LA residents complains that Waze creates congestion on roads once only known to those who live there.

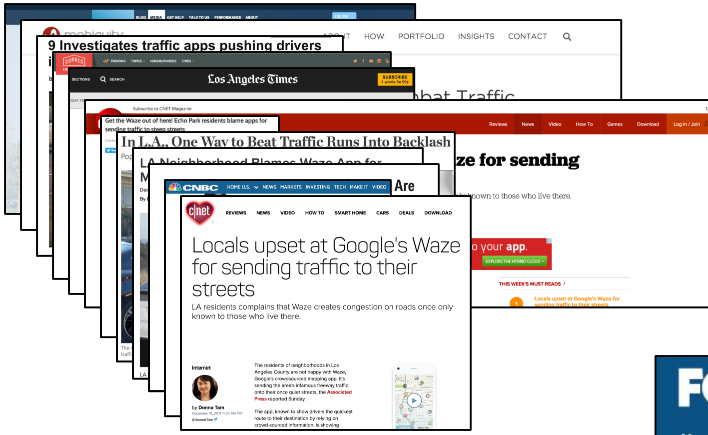
Internet

by **Donna Tam**
December 14, 2014 11:25 AM PST
@DonnaYTam

The residents of neighborhoods in Los Angeles County are not happy with Waze, Google's crowdsourced mapping app. It's sending the area's infamous freeway traffic onto their once quiet streets, the Associated Press reported Sunday.

The app, known to show drivers the quickest route to their destination by relying on crowd-sourced information, is showing

Specific apps are identified as responsible



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AUTO TECH

Popular smartphone app causes traffic jam uproar in California neighborhoods

Published December 15, 2014 · Associated Press

Dec. 9, 2014: Early morning rush hour traffic winds it's way along a narrow street in Sherman Oaks section of Los Angeles. When the people whose houses hug the narrow warren of streets paralleling the busiest urban freeway in America began to see bumper-to-bumper traffic rushing by their homes a year or so ago they were baffled. When word spread that the explosively popular new smartphone app Waze was sending many of those cars through their neighborhood in a quest to shave five minutes off a daily rush-hour commute, they were angry and ready to fight back.

More from Fox News

Topless mystery woman the most distracting driver ever?

Is Elizabeth Warren in political trouble in Massachusetts?

Specific apps are identified as responsible


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Shortcut-finding app Waze creating residential traffic headaches

The outcry echoes 25-year-old protests against the app's book equivalent during the L.A. Olympics

By Kim Brunhuber, CBC News Posted: Feb 29, 2016 5:00 PM ET | Last Updated: Feb 29, 2016 5:00 PM ET

A commuter uses the Waze app to escape traffic in Vancouver. (Doug Trent/CBC)

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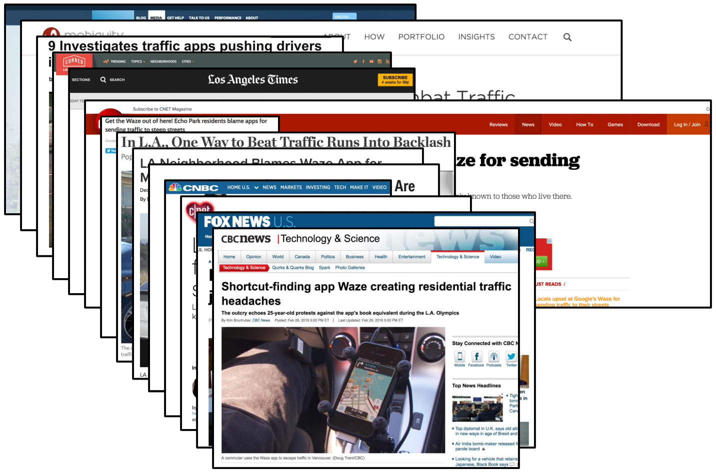

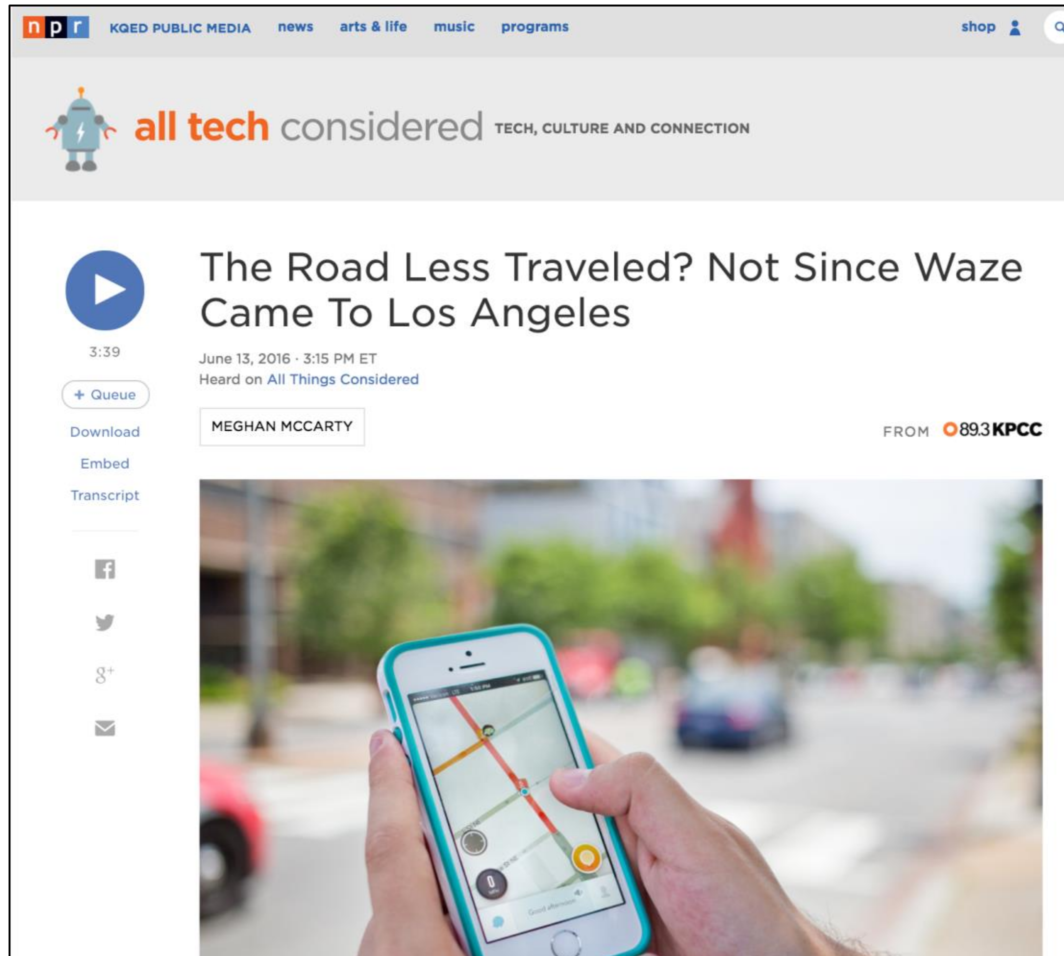

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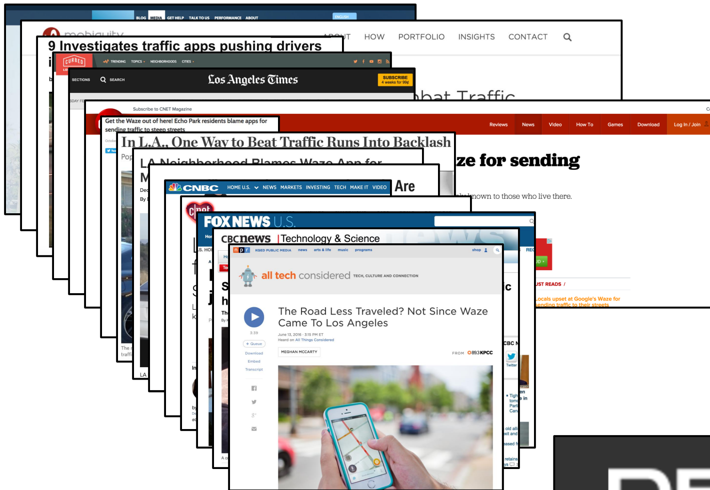

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TECH

Waze App Angers California Residents Due To Increased Traffic In Neighborhoods

Inigo Monzon , Design & Trend
Dec, 16, 2014, 09:47 PM

SHARES TWEET

TAGS : WAZE APP , TRAFFIC , TECHNOLOGY

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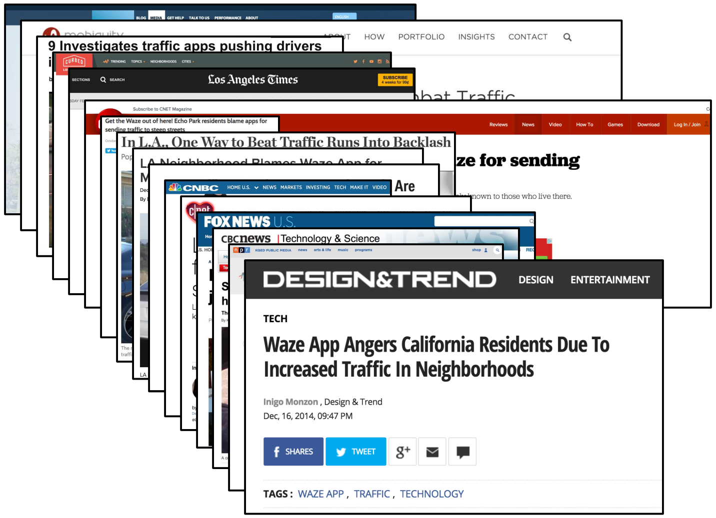

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Posted by Dyana Bagby on September 2, 2016.

Reporter analysis: Waze directions send traffic through quiet streets

A traffic-navigation app called Waze is taking over the roadways and drawing controversy for encouraging cut-through traffic in neighborhoods locally and nationwide.

A recent experiment with Waze found the app indeed directs drivers through quiet neighborhood streets in Buckhead, Dunwoody and Sandy Springs. But, ironically, Waze would not send drivers into a Brookhaven neighborhood that has been a hot spot of protests about app-driven traffic.

With the motto “Outsmarting traffic, together,” Waze uses a combination of GPS tracking and local roadway information crowdsourced from its own users. It both directs drivers to their destinations by the fastest route and allows them to report where heavy traffic is, where road work is happening, and even where police have set up roadblocks.

Waze advertises 50 million users worldwide. It has gained popularity from drivers, and

Recent Posts

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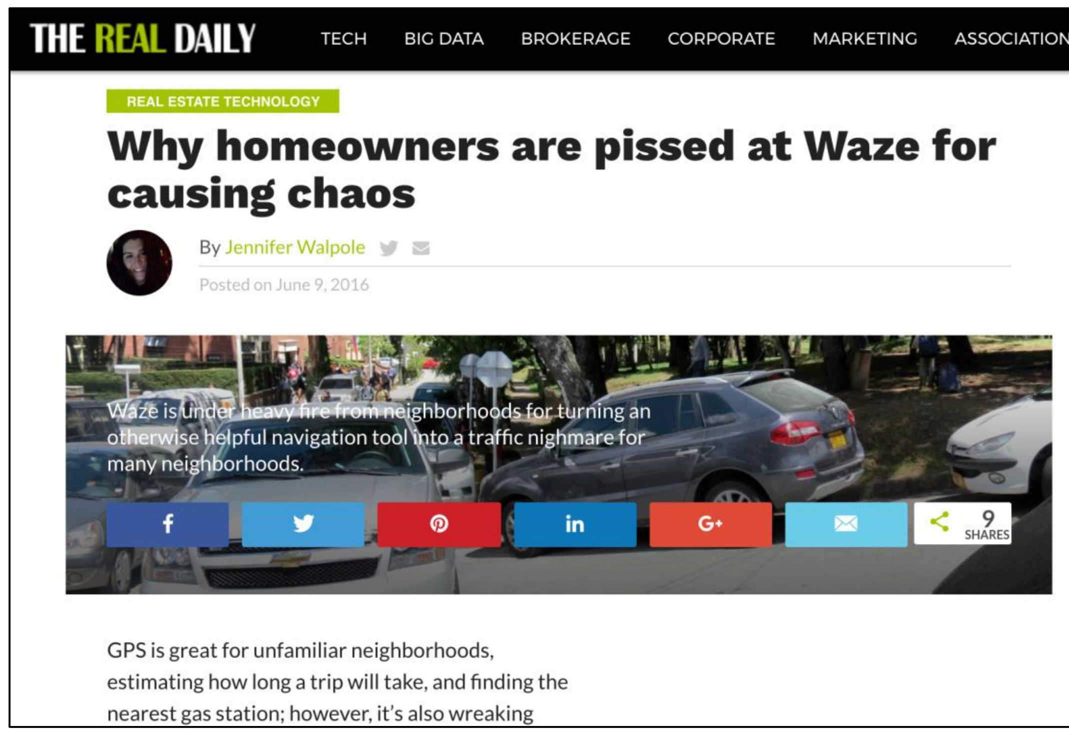
- DeKalb Commissioners to meet in Dunwoody Feb. 21
- 7 more candidates join crowded Congressional race
- Sandy Springs, Cobb Chambers form stadium traffic task force
- Brookhaven receives $5.7m loan to purchase PDK Airport land

Community Links

- City of Atlanta
- City of Brookhaven
- City of Brookhaven – Calendar of Events
- City of Dunwoody
- City of Dunwoody – Calendar of Events
- City of Sandy Springs
- City of Sandy Springs – Community Calendar

Specific apps are identified as responsible



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REAL ESTATE TECHNOLOGY

Why homeowners are pissed at Waze for causing chaos

By Jennifer Walpole

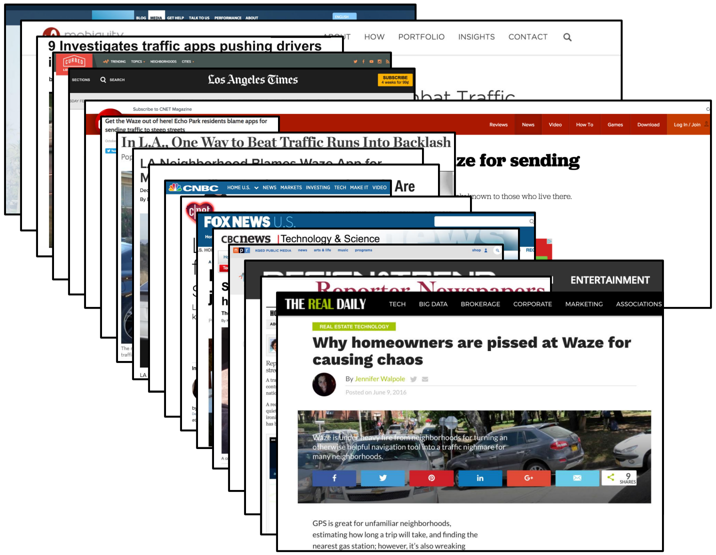
Posted on June 9, 2016

Waze is under heavy fire from neighborhoods for turning an otherwise helpful navigation tool into a traffic nighmare for many neighborhoods.

9 SHARES

GPS is great for unfamiliar neighborhoods, estimating how long a trip will take, and finding the nearest gas station; however, it's also wreaking

Citizens start to resist



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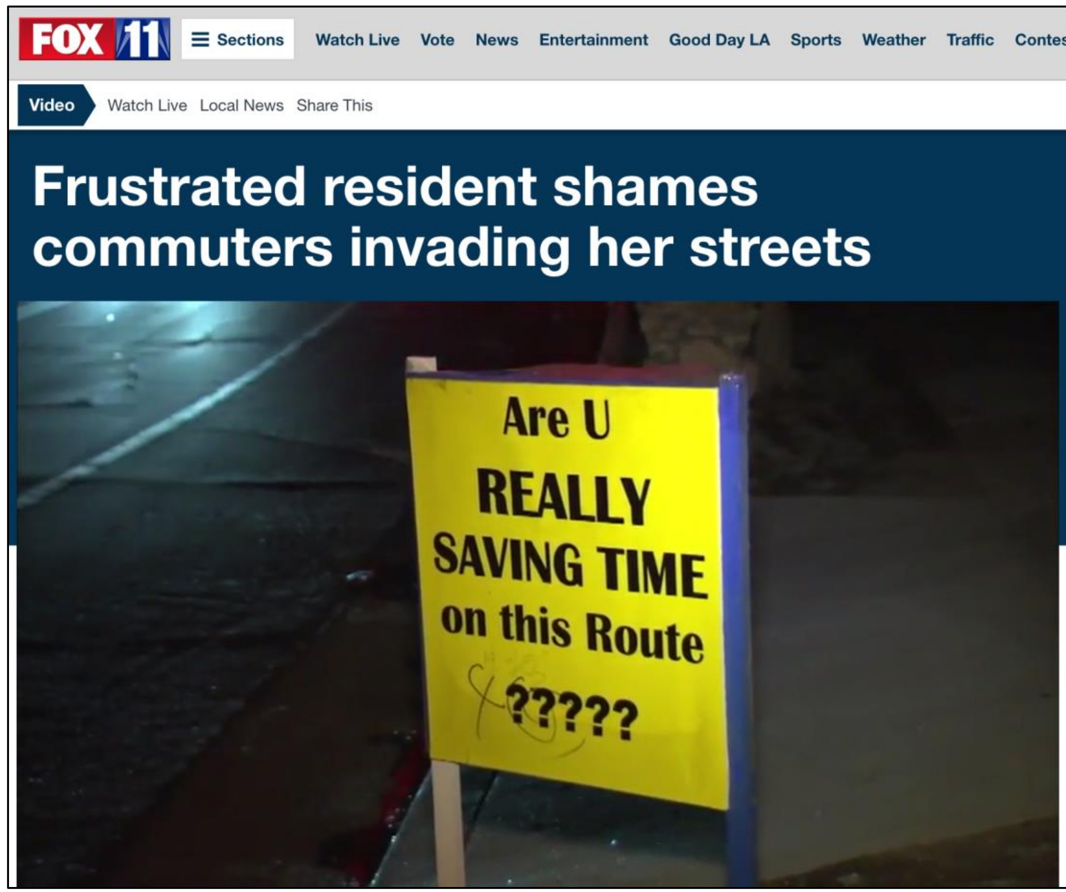
Ashcroft Residents Work to Stop 'Waze Craze' Traffic

Fri, May 01, 2015 By Staff 20 Comments

Joel Becker, right, and Joel Ring, ask traffic to slow on Ashcroft Avenue.

Citizens start to resist



Citizens start to resist

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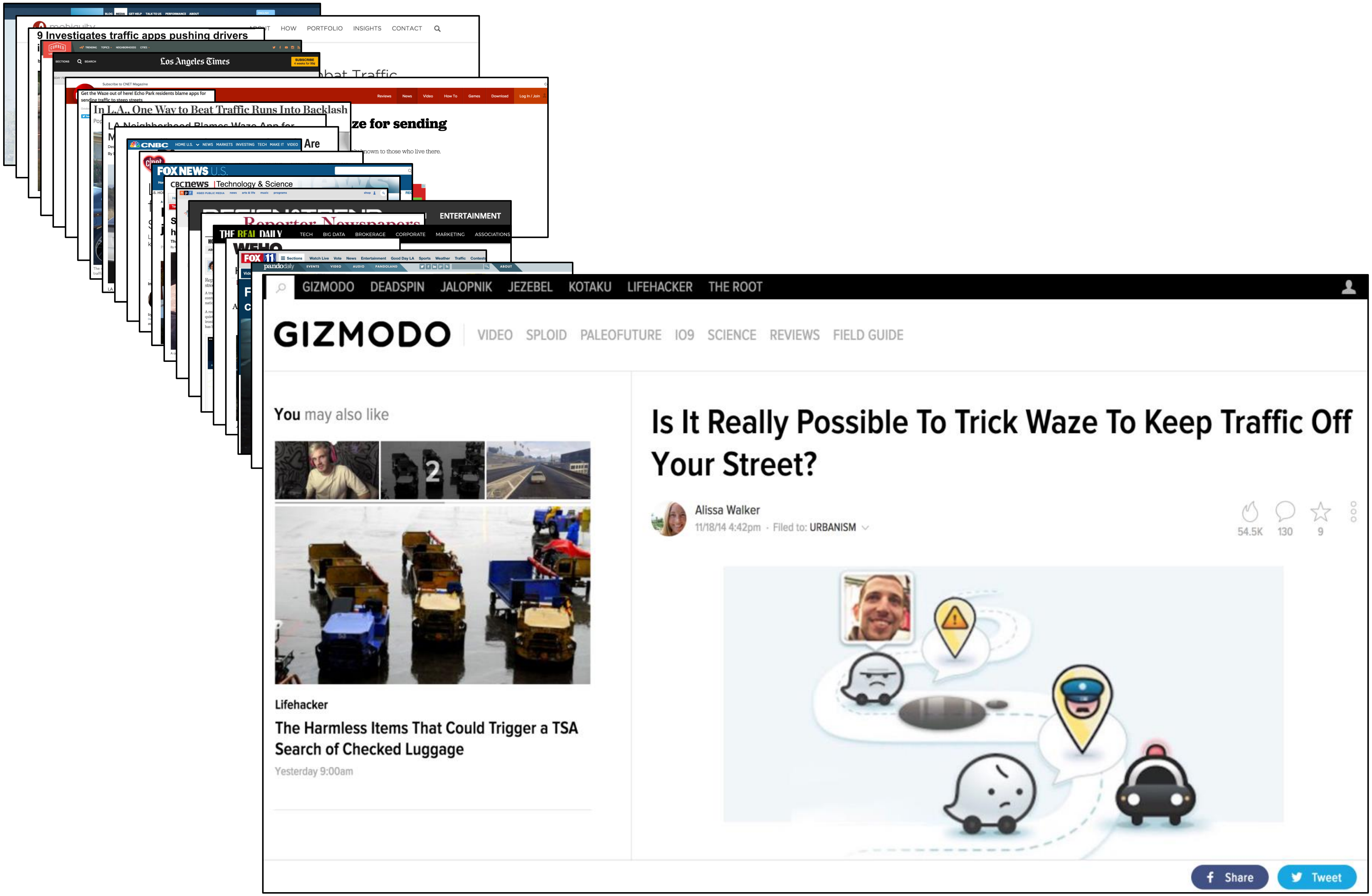

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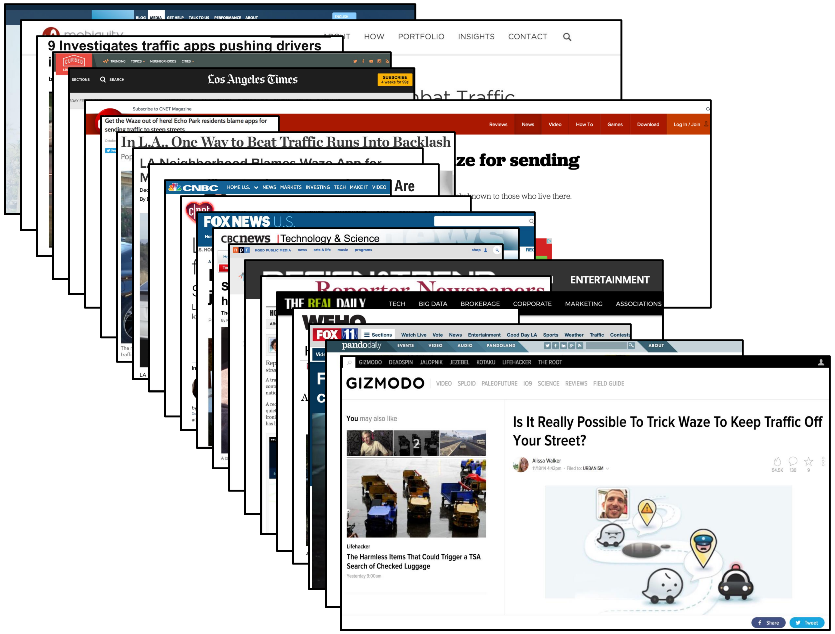

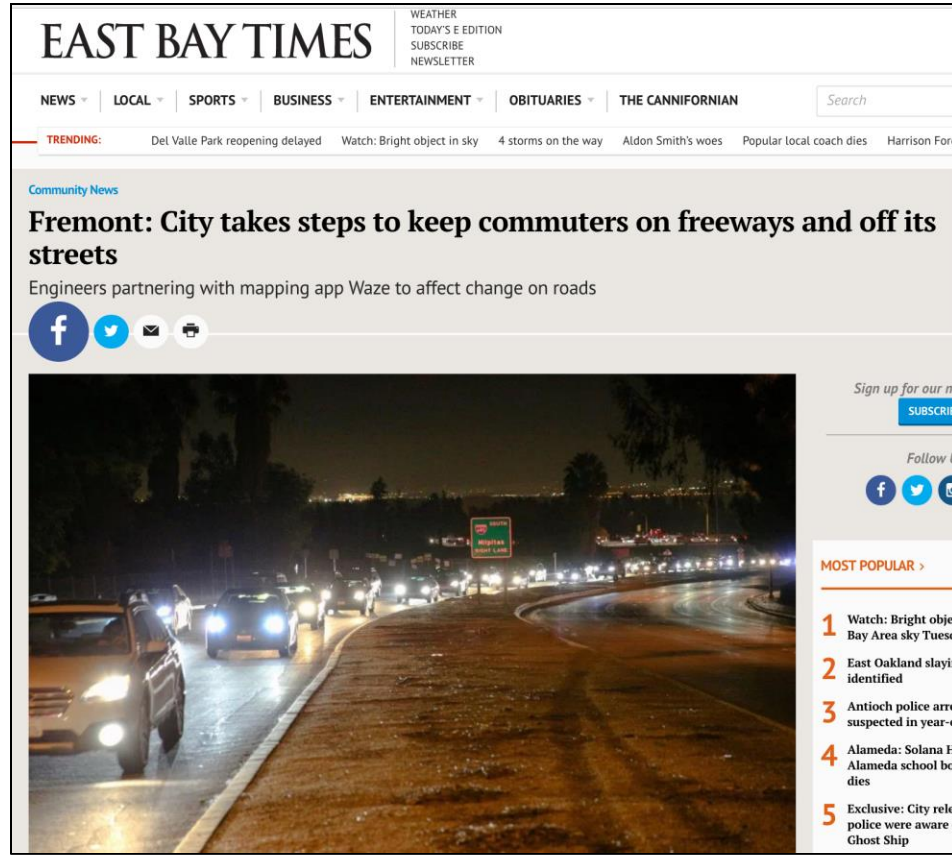
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Neighborhoods and cities start to resist

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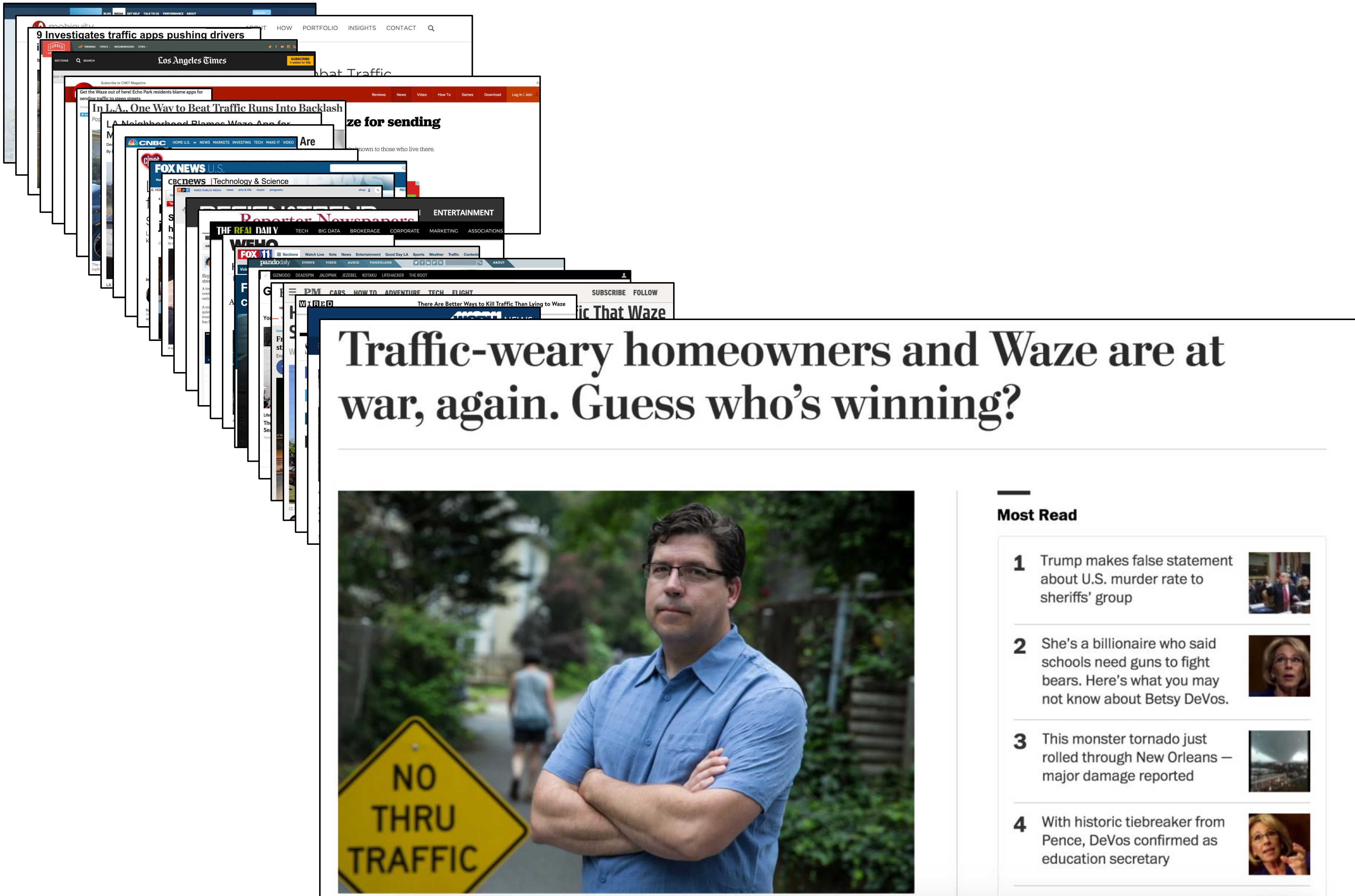

Traffic-weary homeowners and Waze are at war, again. Guess who's winning?

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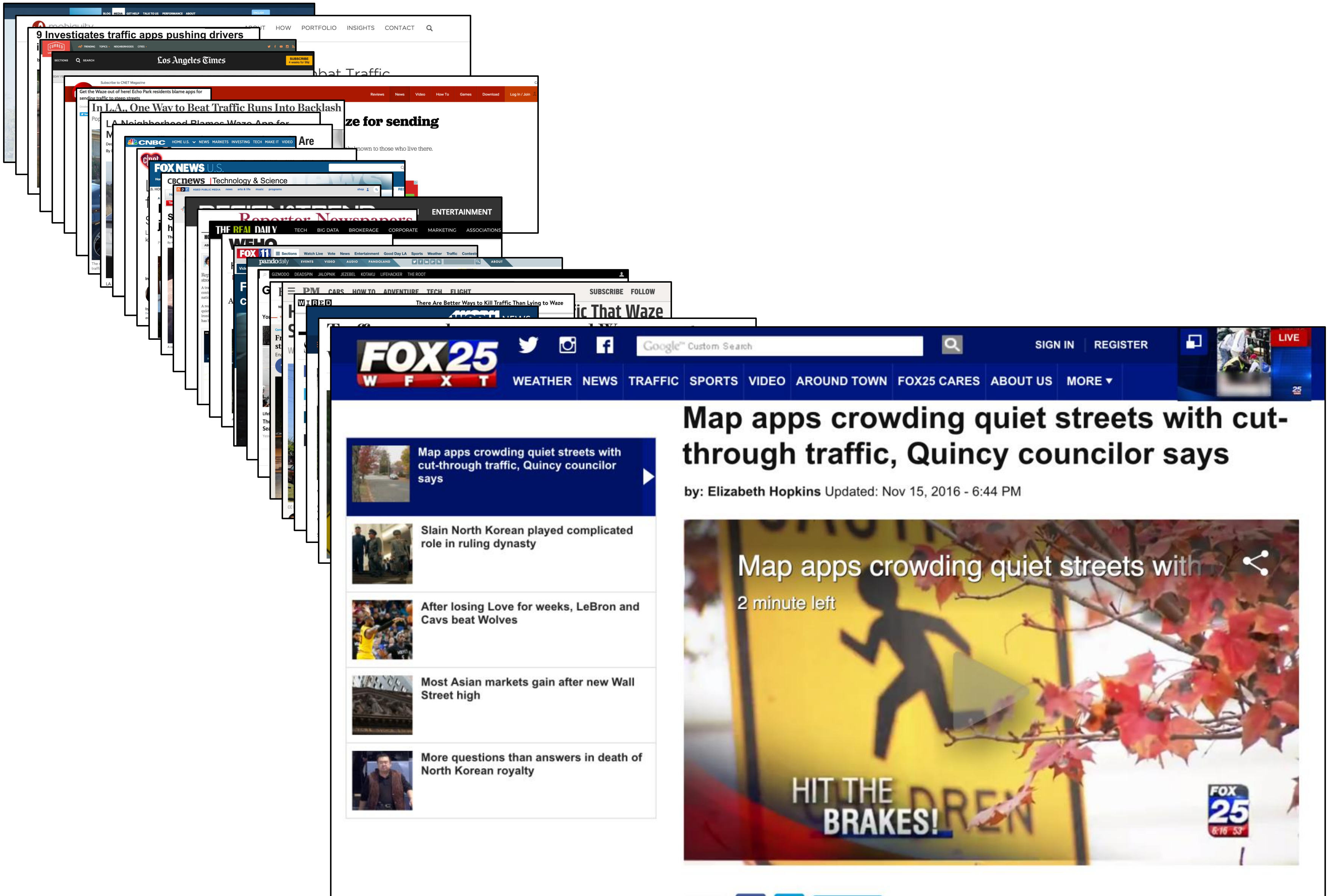

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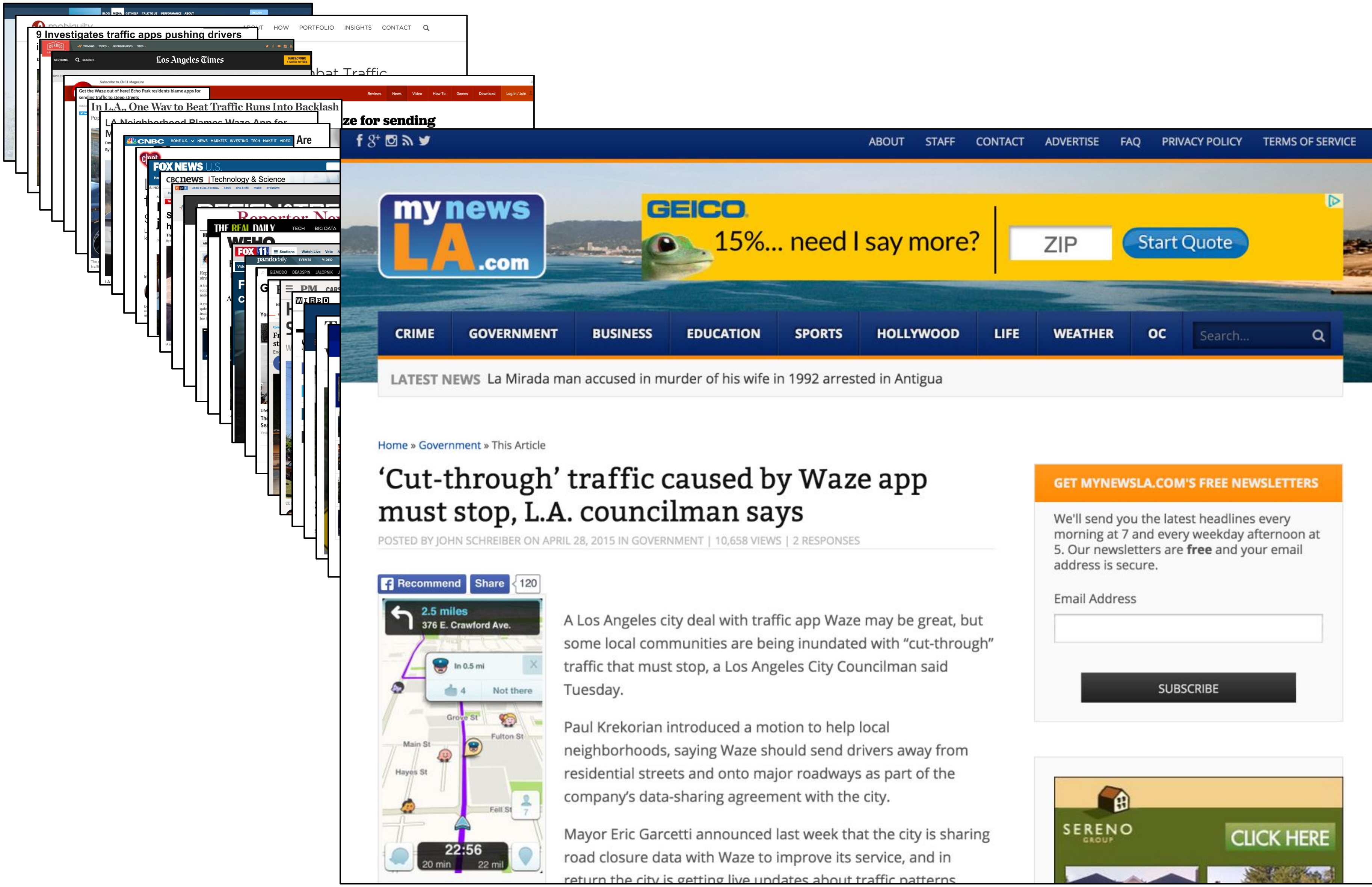

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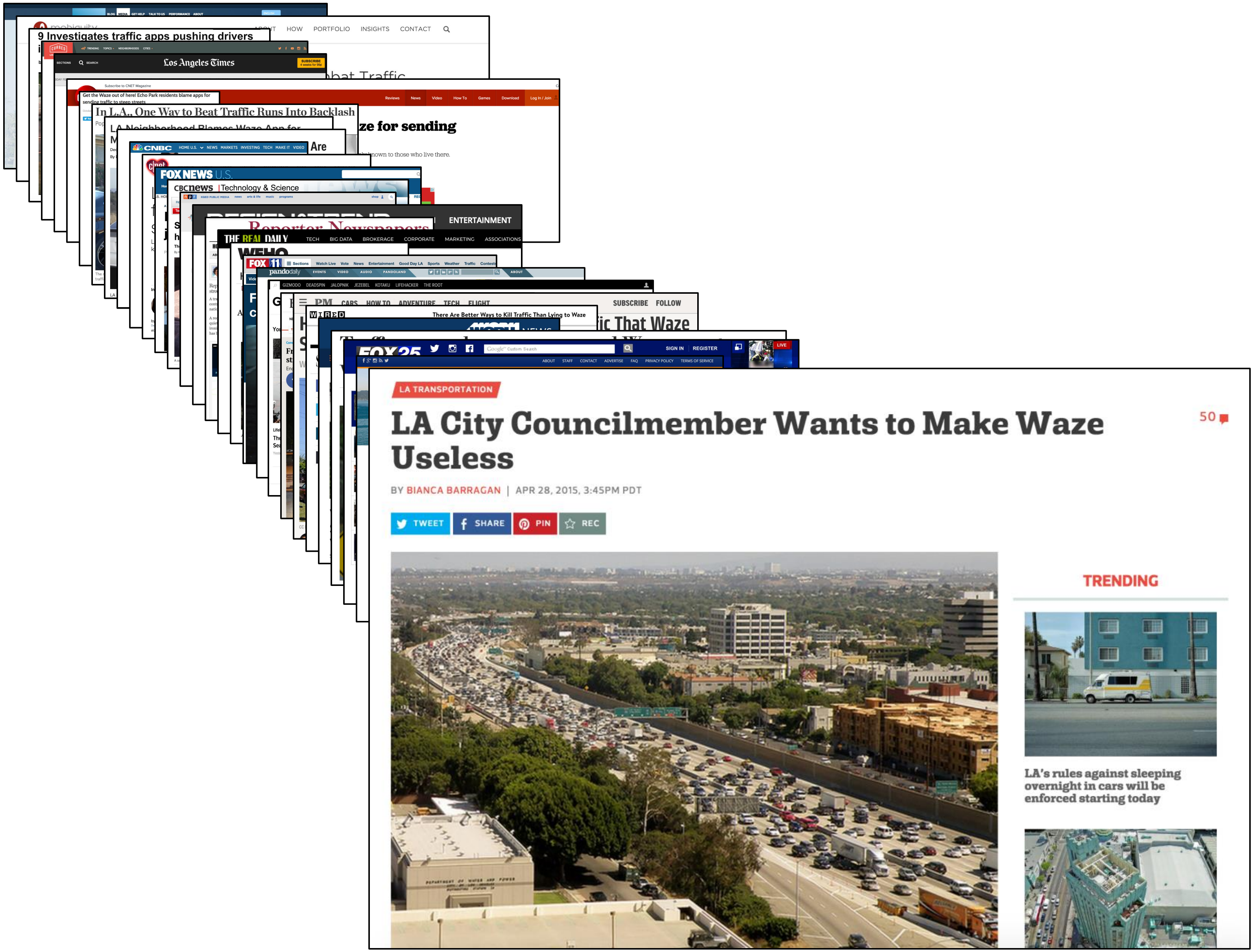
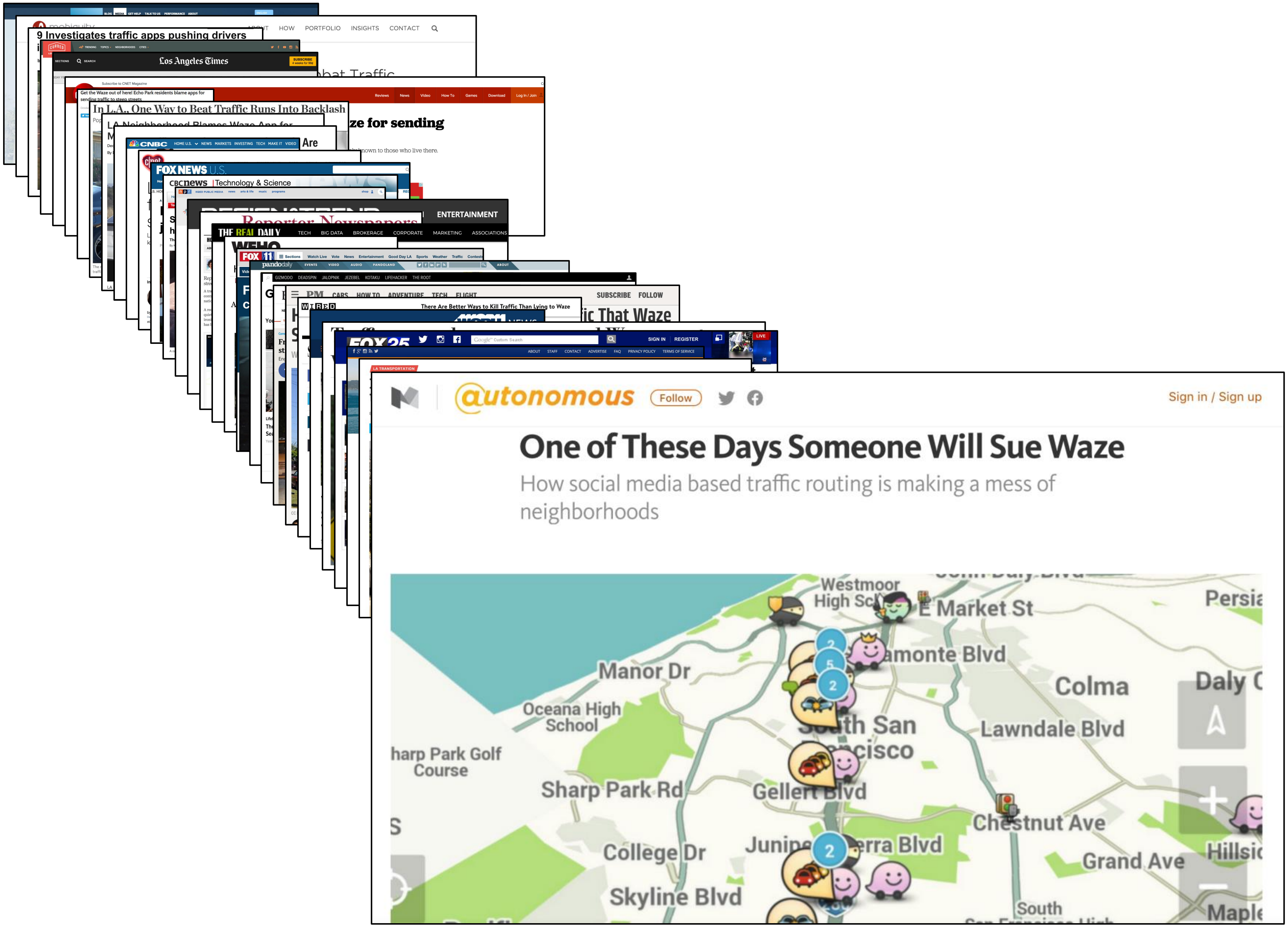


People predict lawsuits



Lawsuits happen

But few people are asking the right question

The Mercury News

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TRENDING: OMG, you forgot?! Michael Flynn Grammy snubs Aldon Smith Westminster dog show California's best lagers

Opinion > Commentary

Is Google's Waze app making traffic worse?

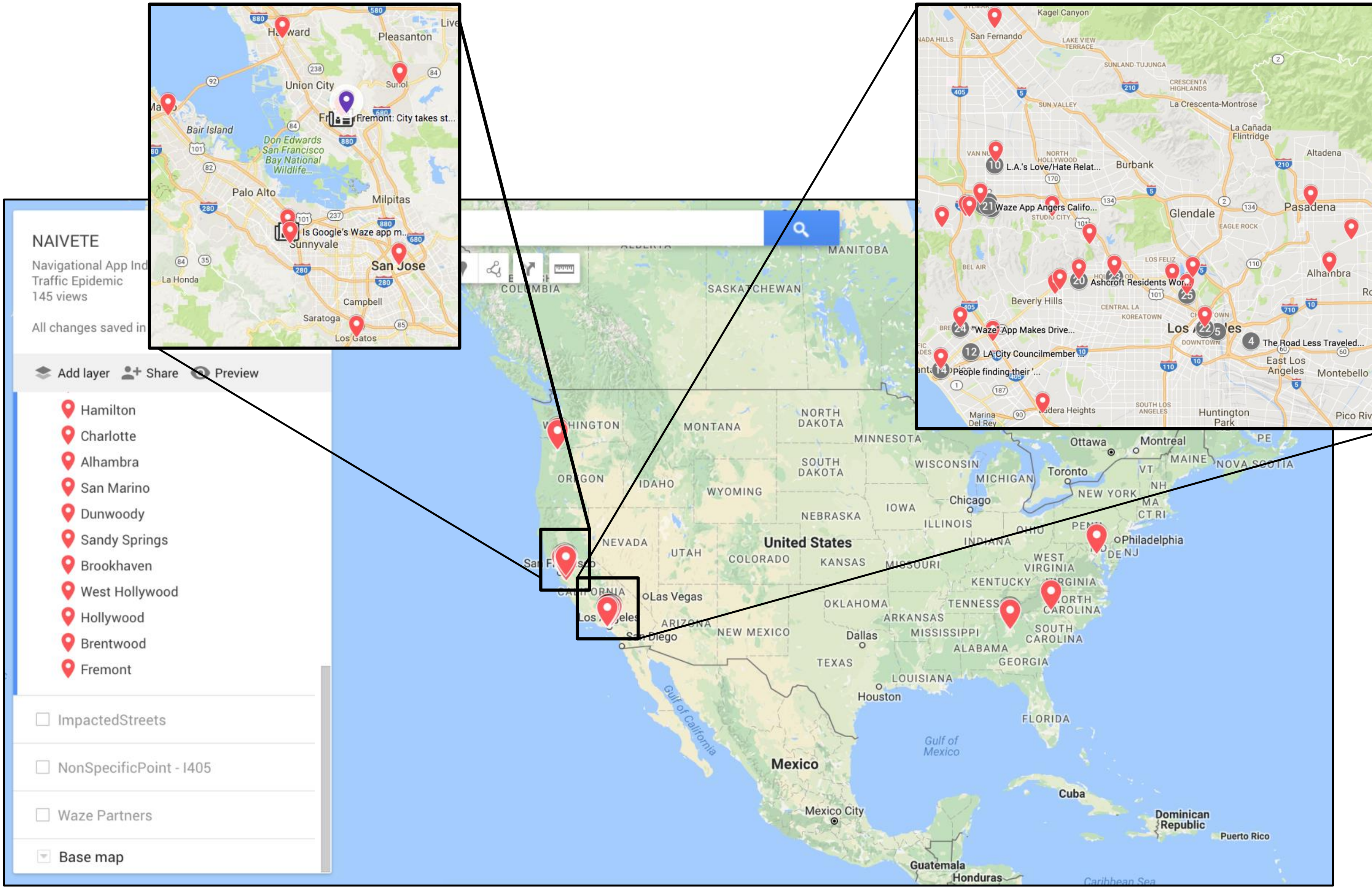
By GARY RICHARDS | grichards@bayareanewsgroup.com |
PUBLISHED: January 21, 2016 at 6:58 am | UPDATED: August 11, 2016 at 11:04 pm

I love Waze. Absolutely love it.

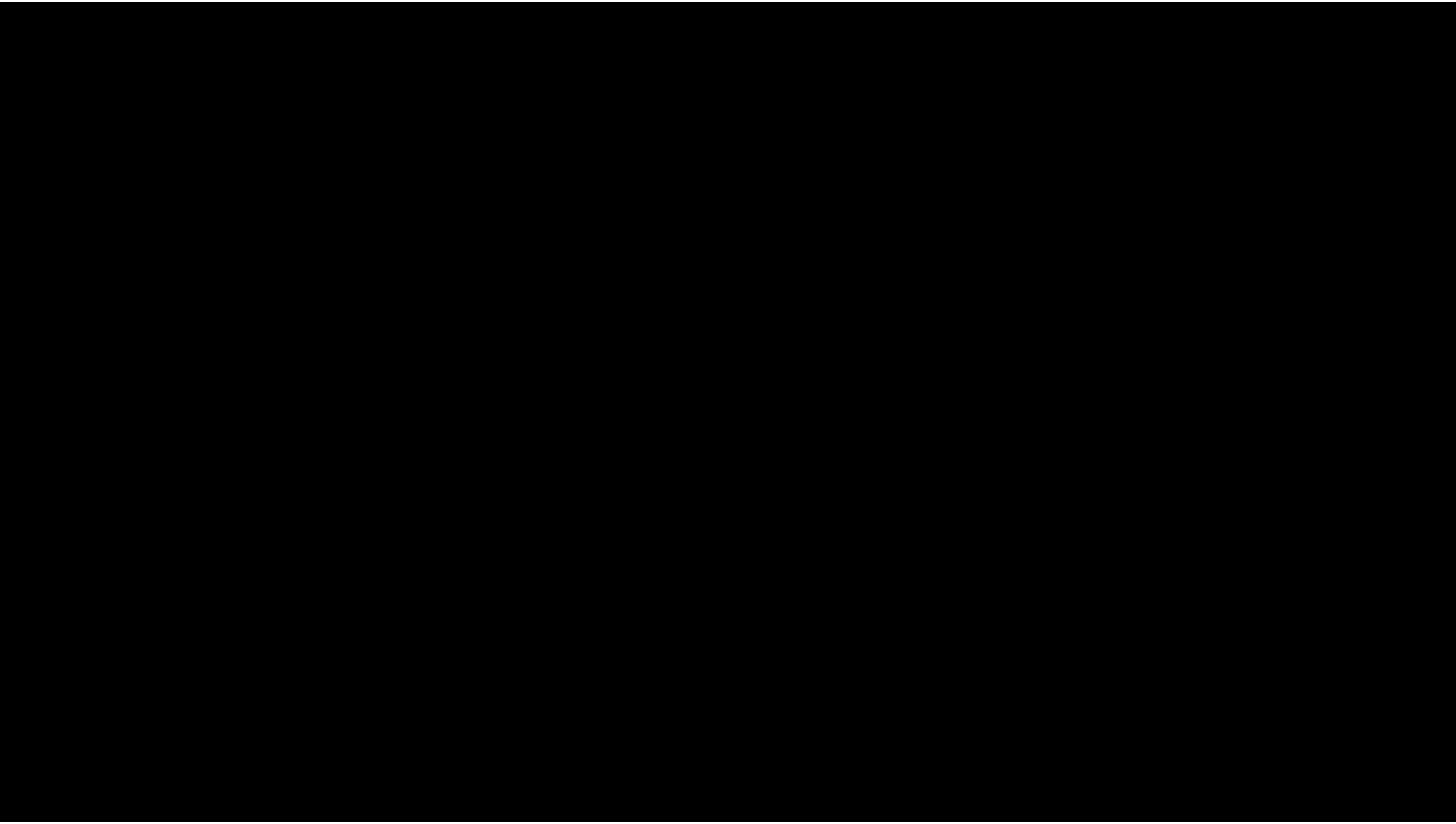
It's a great traffic app for alerting drivers to everything from a major crash ahead to a guy fixing a flat on the shoulder to road crews trimming weeds. Just punch a destination into your smartphone and a Siri-like voice tells you how to skirt traffic slowdowns.

But not everyone shares my passion.

Extent of the problem in the US



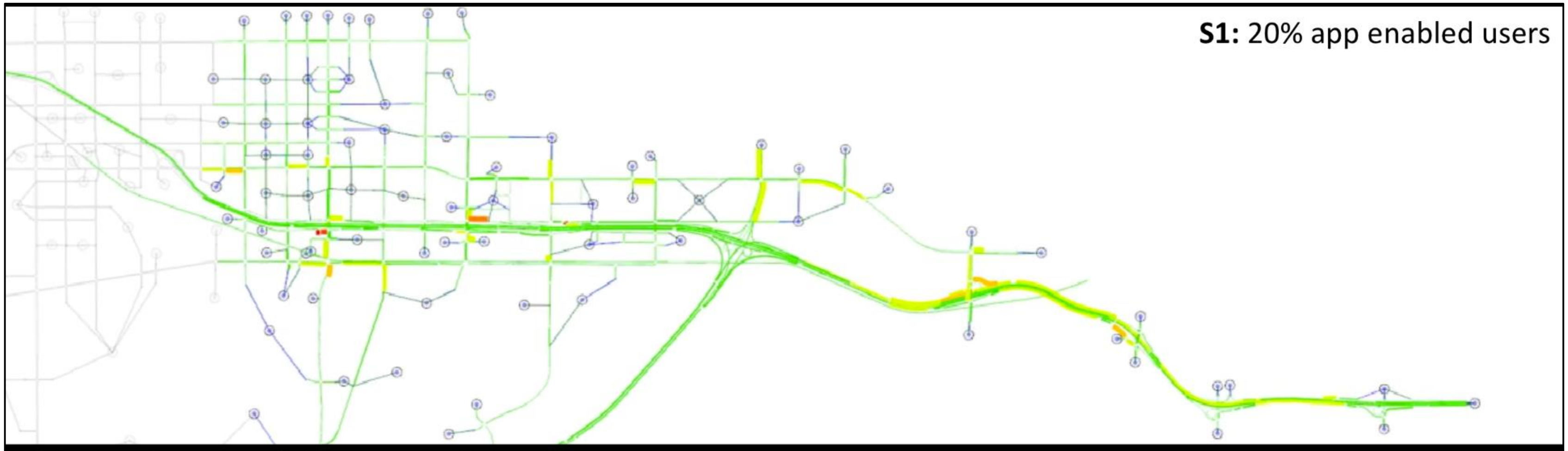
Experimental validation

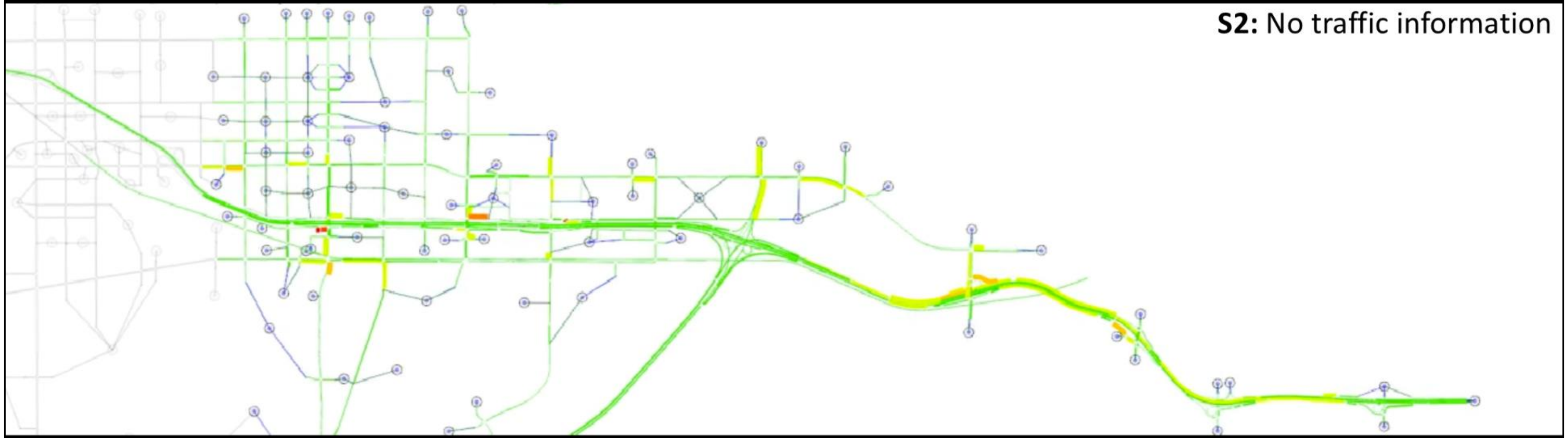


A sequence of events well studied in isolation

2-hour scenario, simulated from 7am to 9m

- **Coupling hard to model**
- **Coupling dependent on information patterns**





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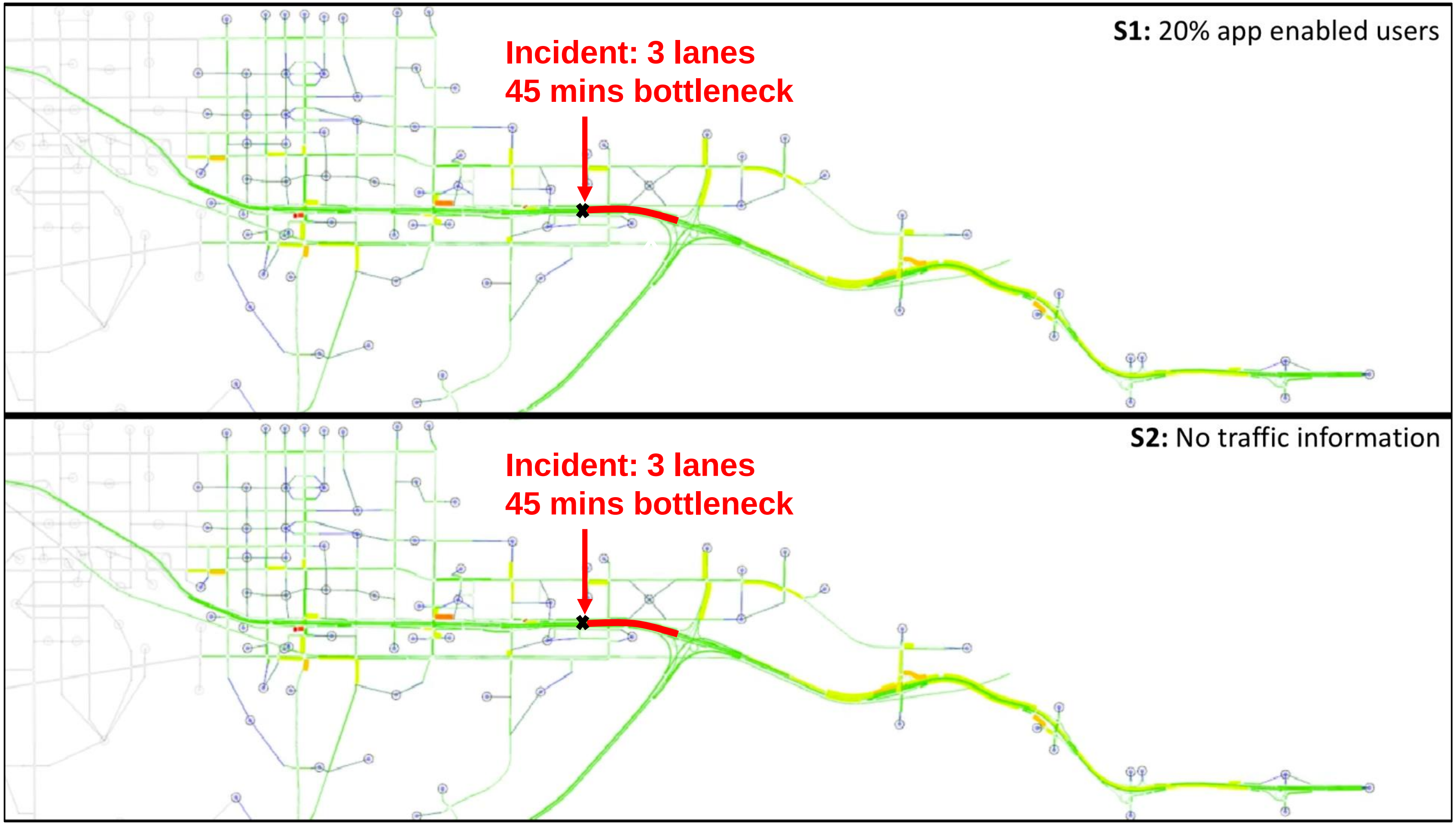

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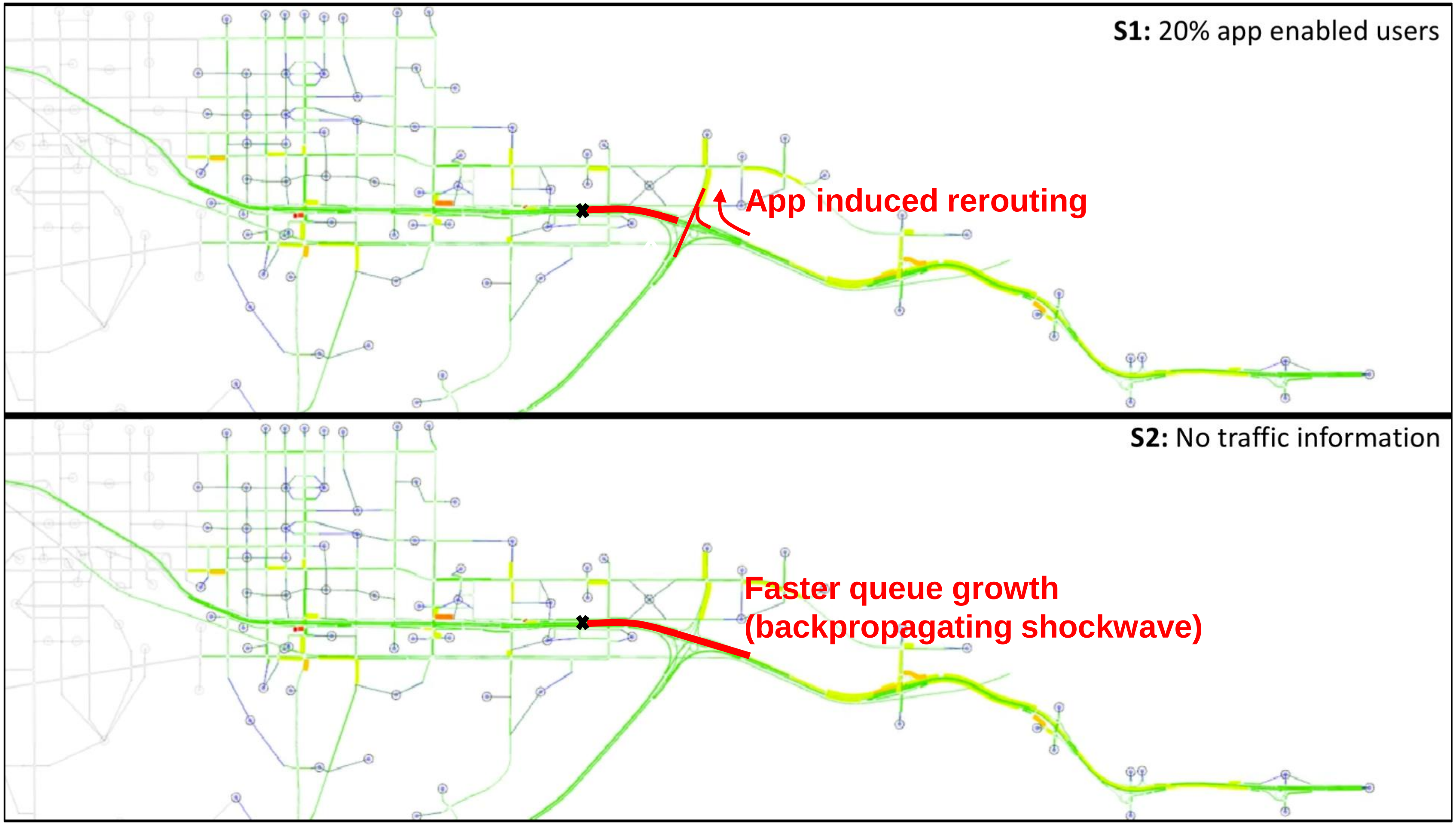

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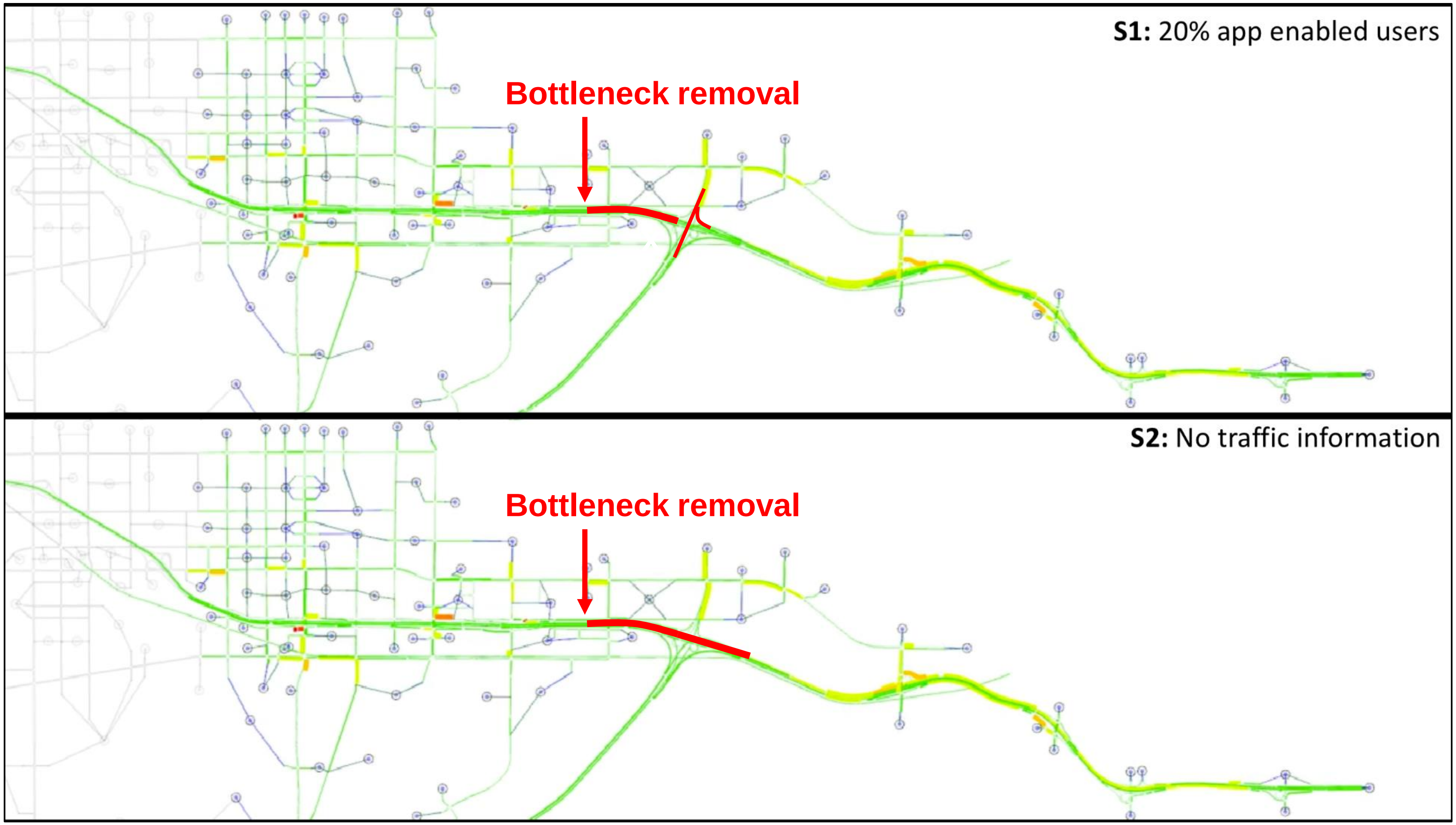

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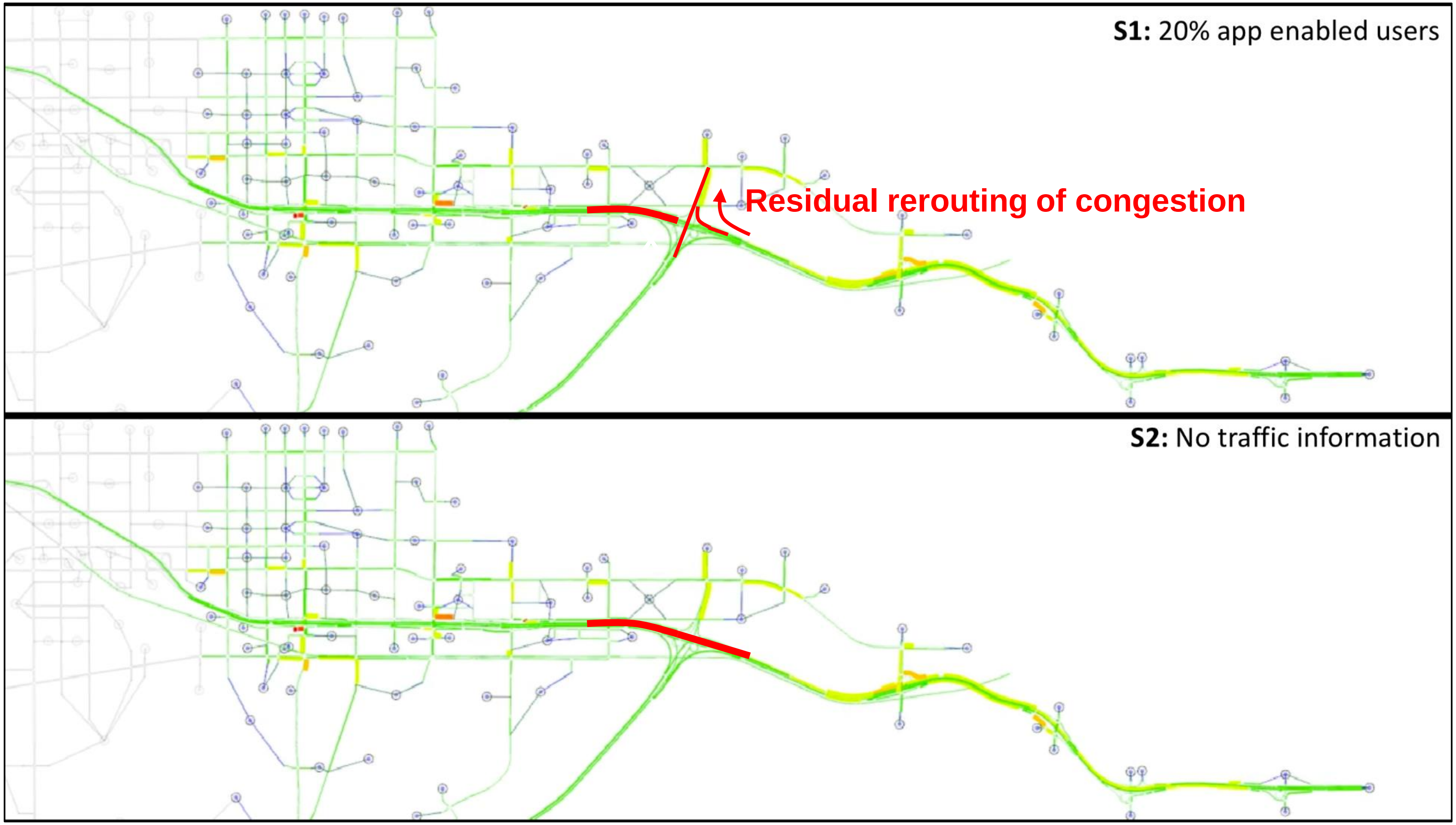

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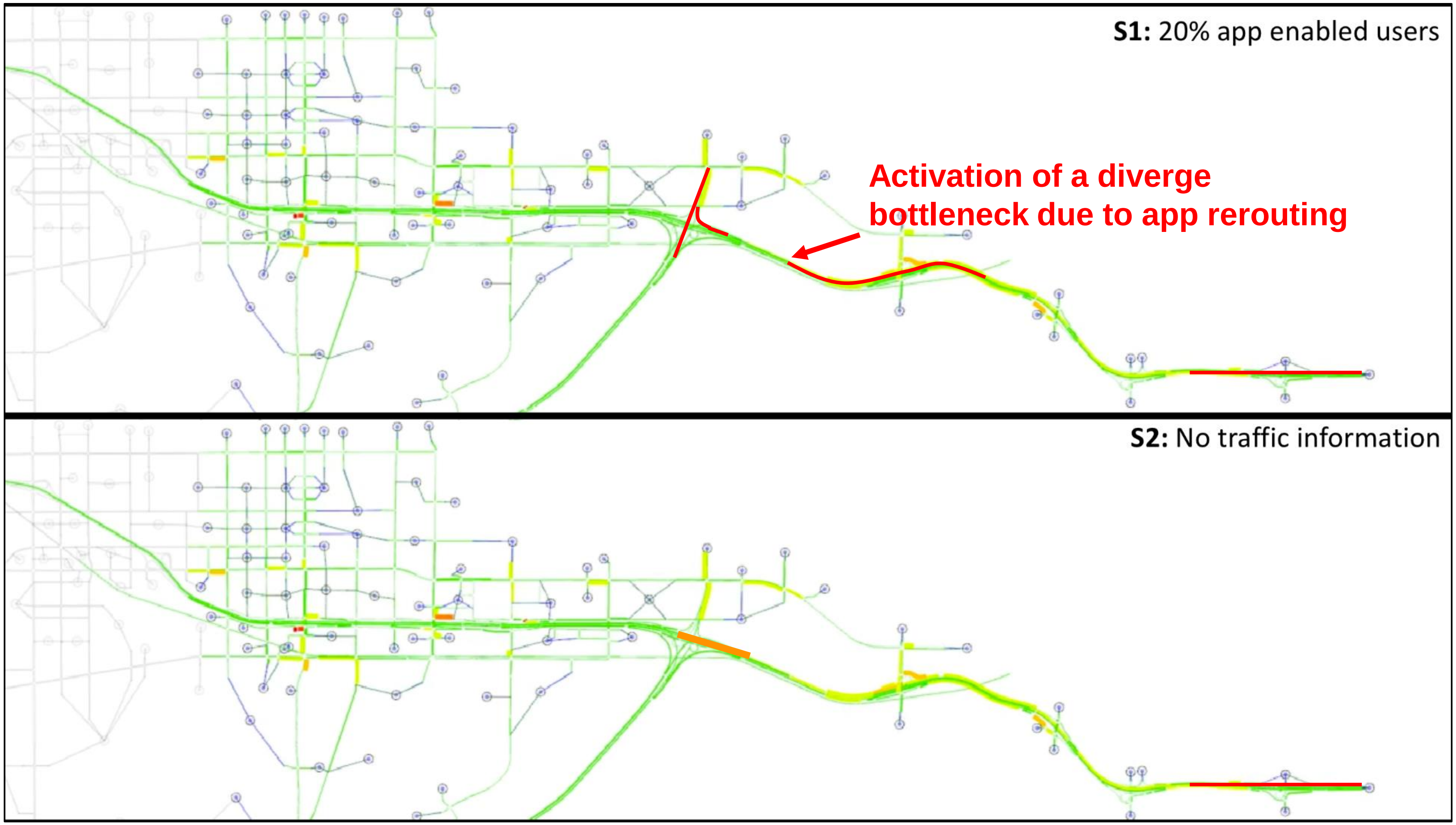

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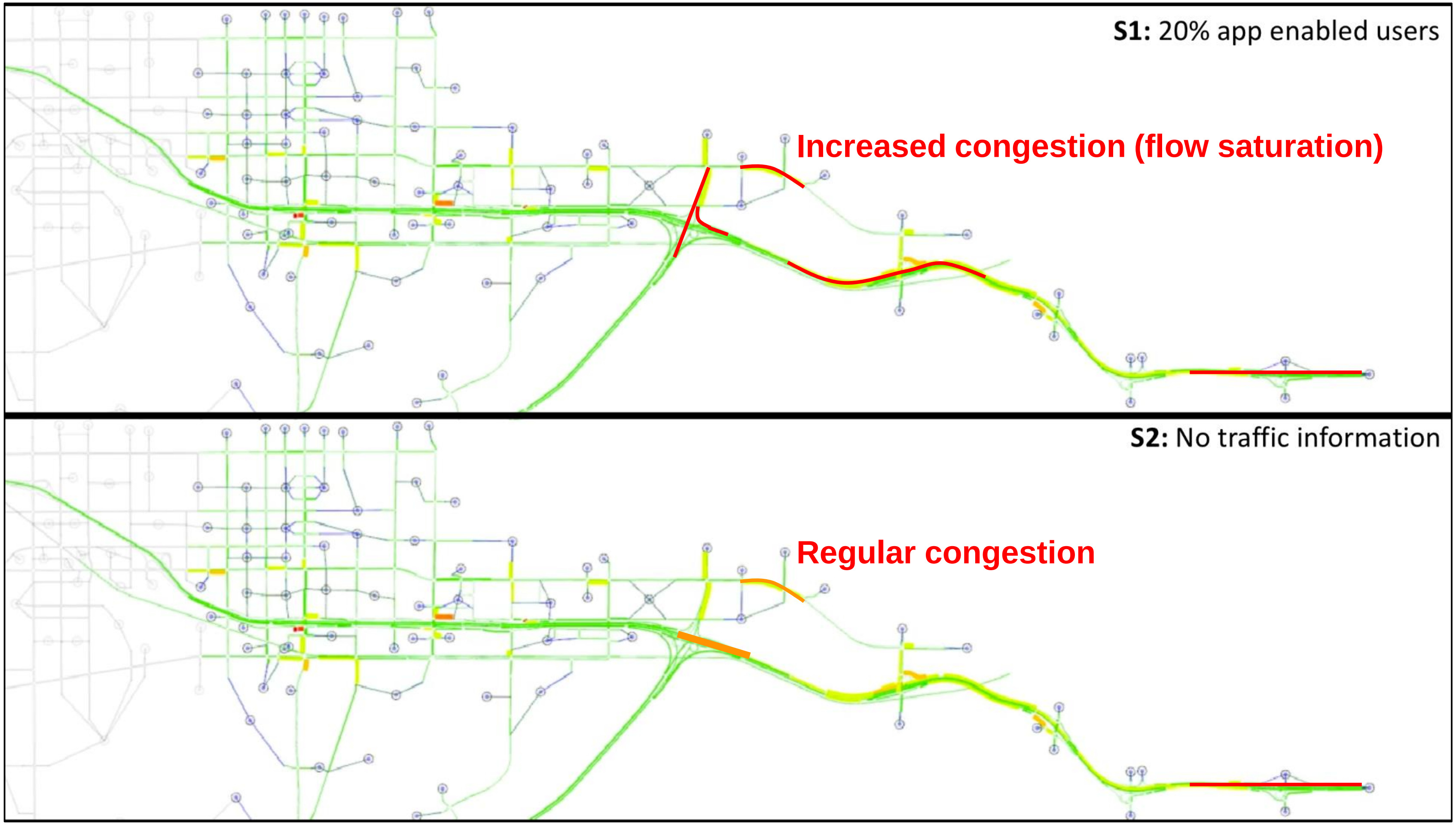

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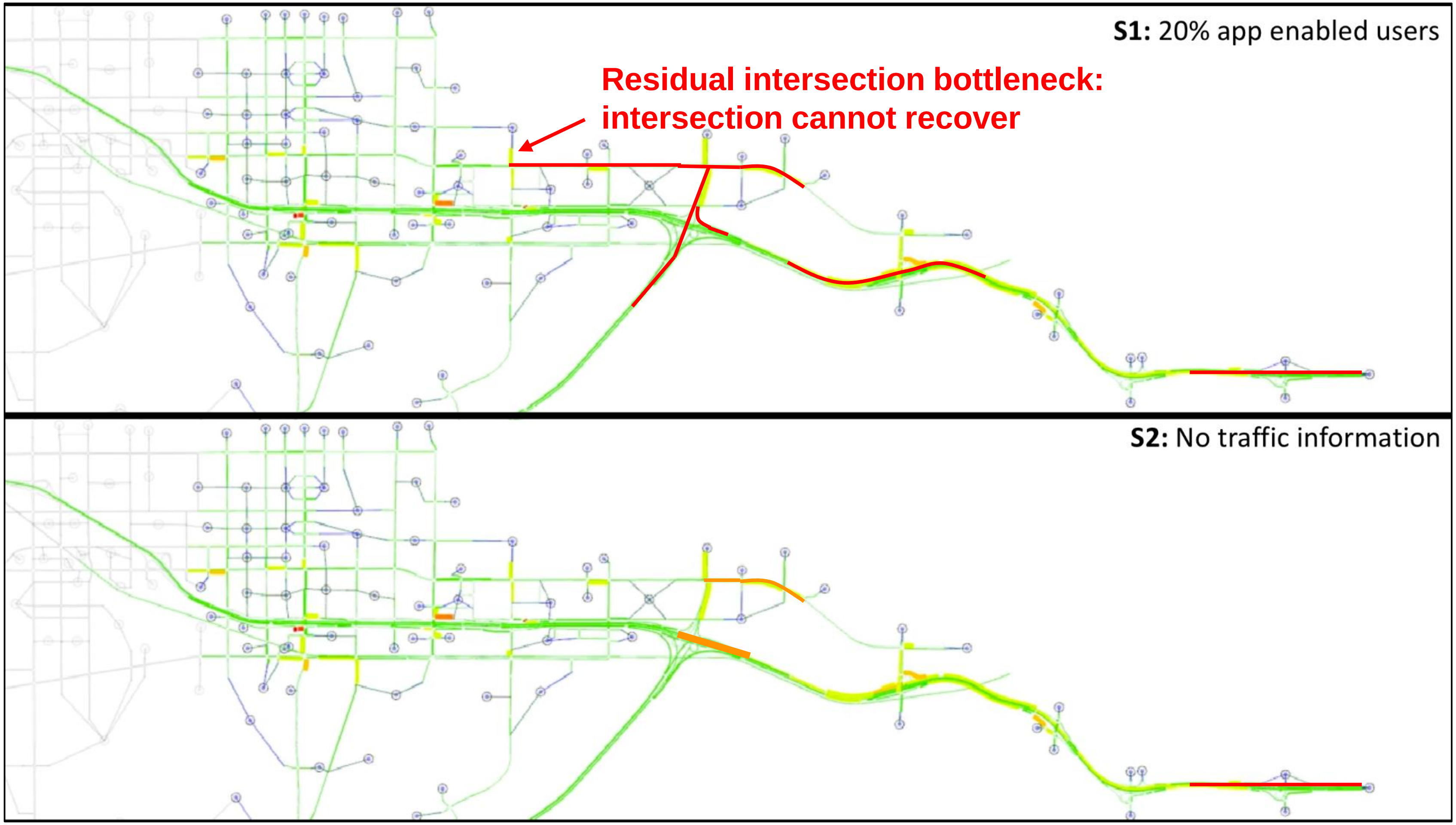

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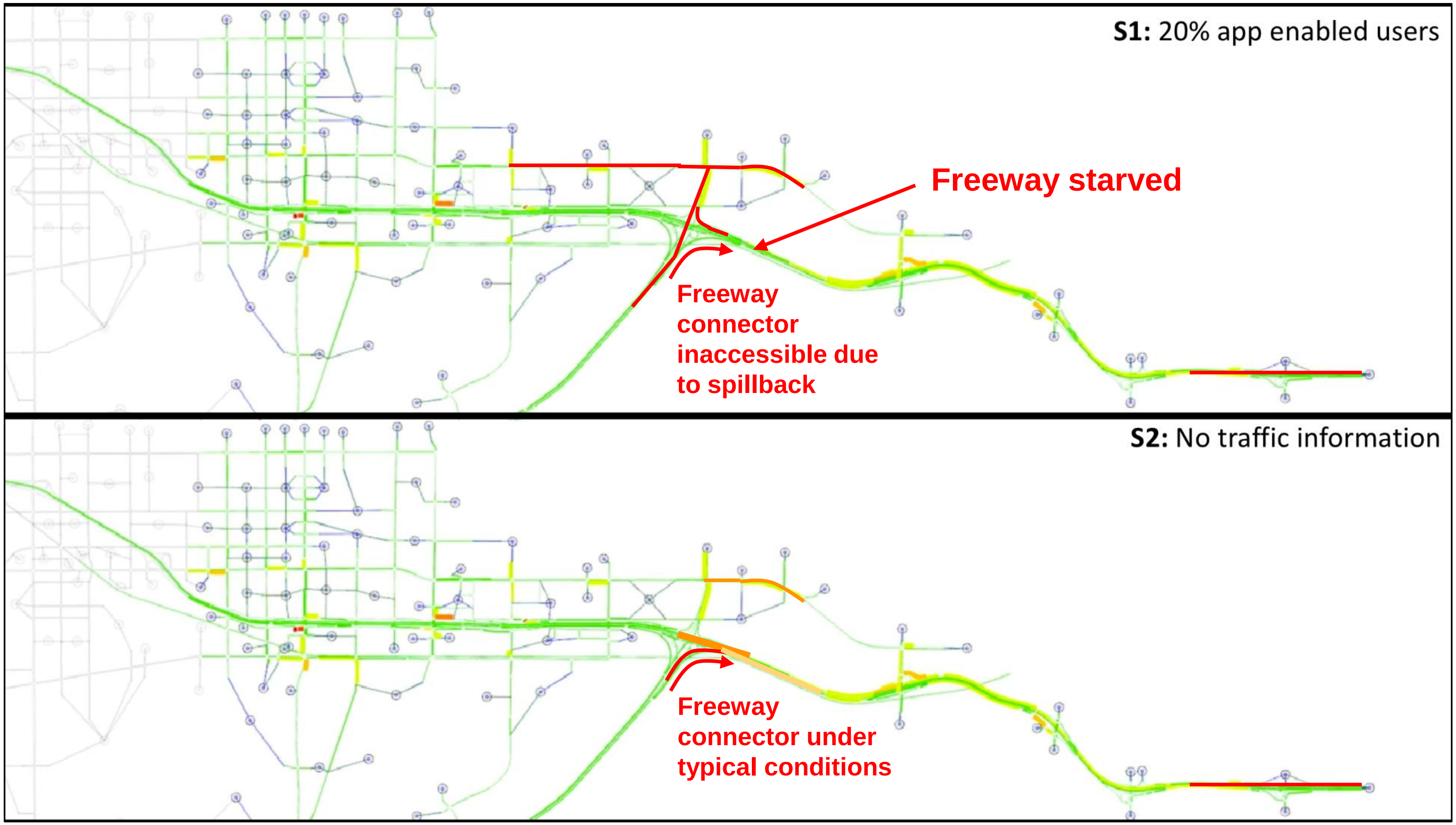

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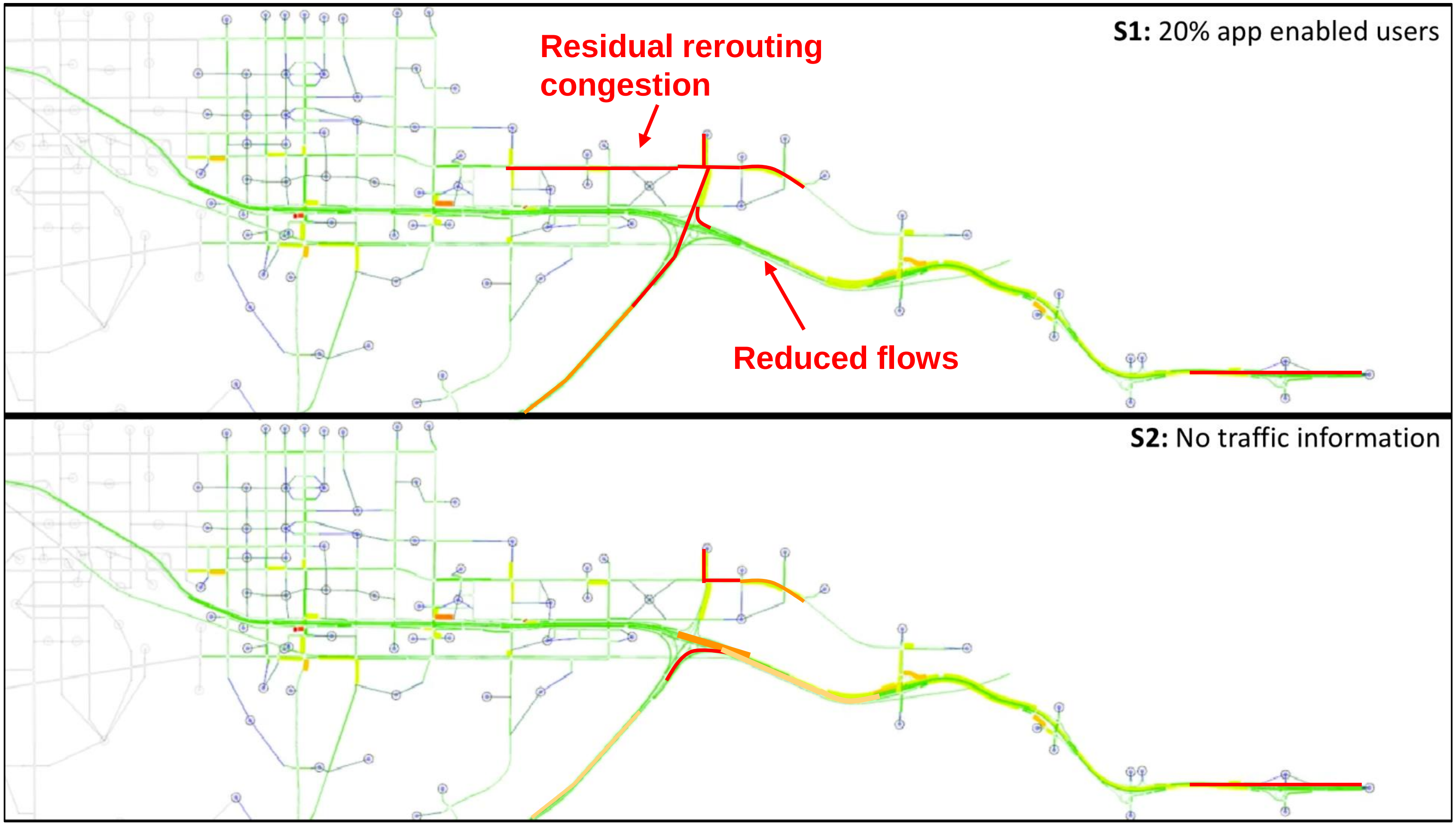


Outline

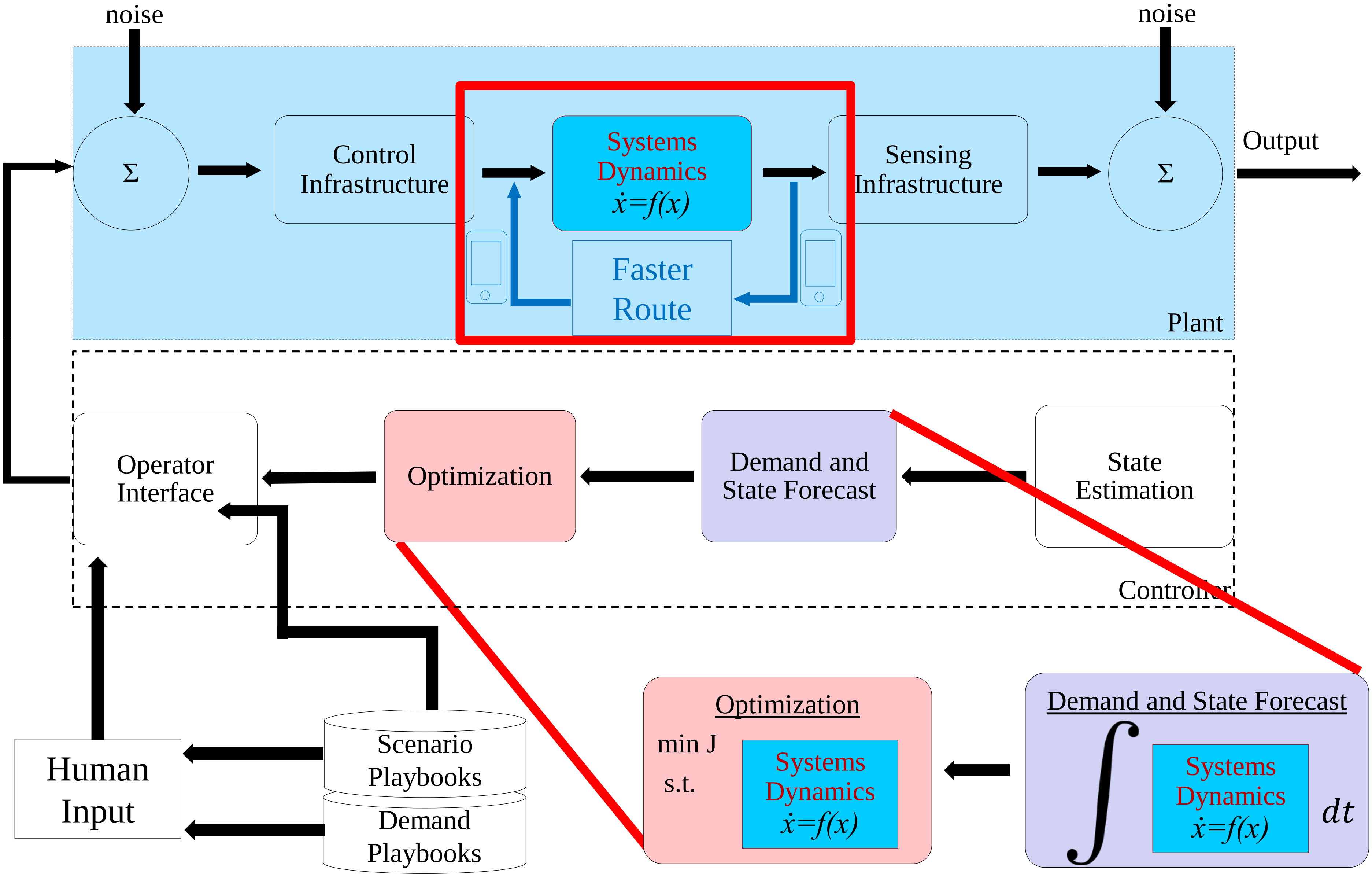
1. **General framework for traffic operations**

1. **Inference problems**
    1. **Demand inference**
    2. **Traffic estimation**

2. **Heterogeneous games**
    1. **Heterogeneous game, Nash-Stackelberg solutions**
    2. **Learning dynamics in repeated games**

3. **Other mobile sensor and data and CPS education**

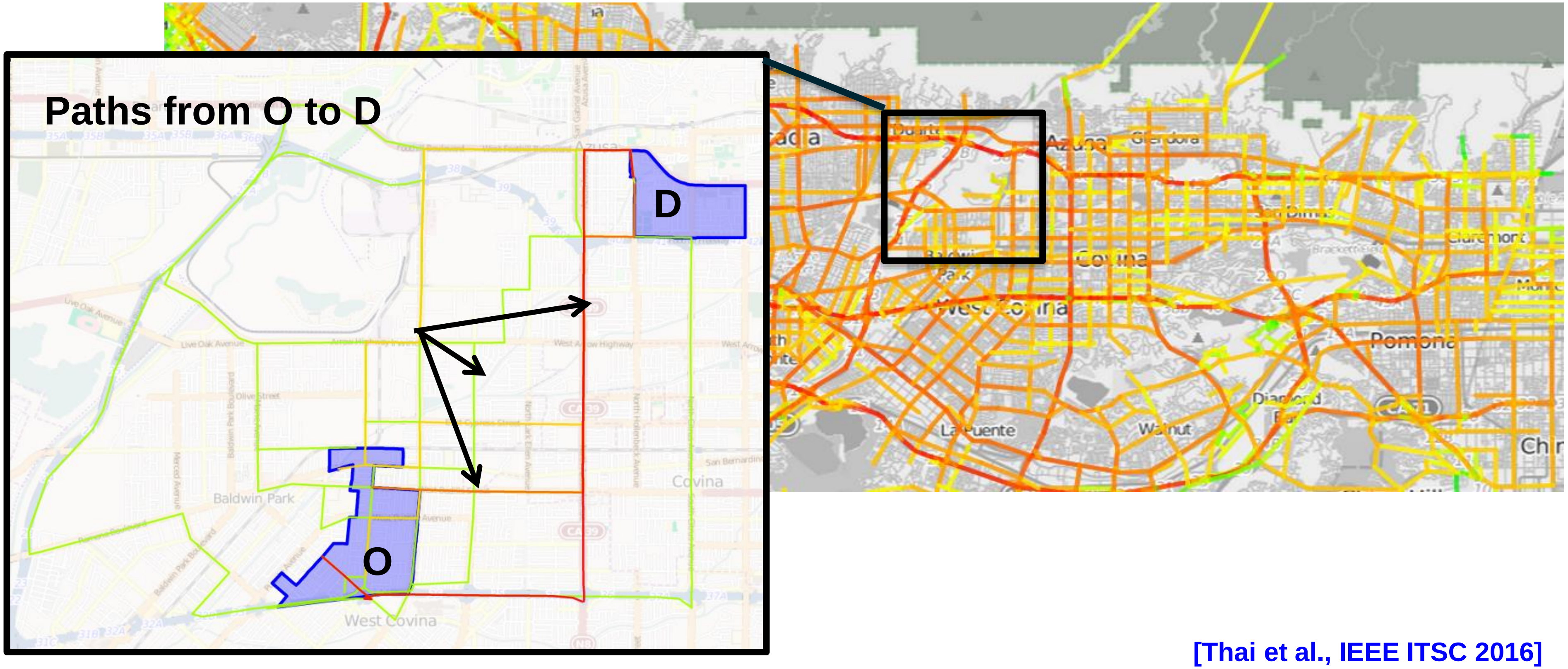
Steady state modeling of “feedback in plant”



Static problem statement

Using a classical Nash (user equilibrium) framework

- **Model two classes of users:**
  - **App-enabled users, who have access to traffic information and follow shortest path**
  - **Non-app enabled users, who keep choosing high-capacity roads over low capacity roads**



[Thai et al., IEEE ITSC 2016]

Static results

Disjoint union of high and low capacity edges: $\mathcal{E} = \mathcal{E}^{\text{hi}} \sqcup \mathcal{E}^{\text{lo}}$

Non-app enabled users cost of $e \in \mathcal{E}^{\text{lo}}$: $c_e^{\text{nae}}(x_e) = C\, t_e(x_e)$, $C \gg 1$

Non-app enabled user cost path p: $\ell_p^{\text{nae}} = \sum_{e \in p^{\text{hi}}} t_e(x_e) + C \sum_{e \in p^{\text{lo}}} t_e(x_e)$

App enabled cost of path p: $\ell_p^{\text{ae}} = \sum_{e \in p} t_e(x_e)$

Nash equilibrium path flows $(f^{\text{nae}}, f^{\text{ae}}) \in \Delta^{\mathcal{P}^{\text{nae}}} \times \Delta^{\mathcal{P}^{\text{ae}}}$ satisfy

$$\begin{bmatrix} \ell^{\text{nae}} \\ \ell^{\text{ae}} \end{bmatrix}^T \begin{bmatrix} g^{\text{nae}} - f^{\text{nae}} \\ g^{\text{ae}} - f^{\text{ae}} \end{bmatrix} \geq 0 \quad \forall\, (g^{\text{nae}}, g^{\text{ae}}) \in \Delta^{\mathcal{P}^{\text{nae}}} \times \Delta^{\mathcal{P}^{\text{ae}}}$$

Property: Convergence guarantee on the heterogeneous game

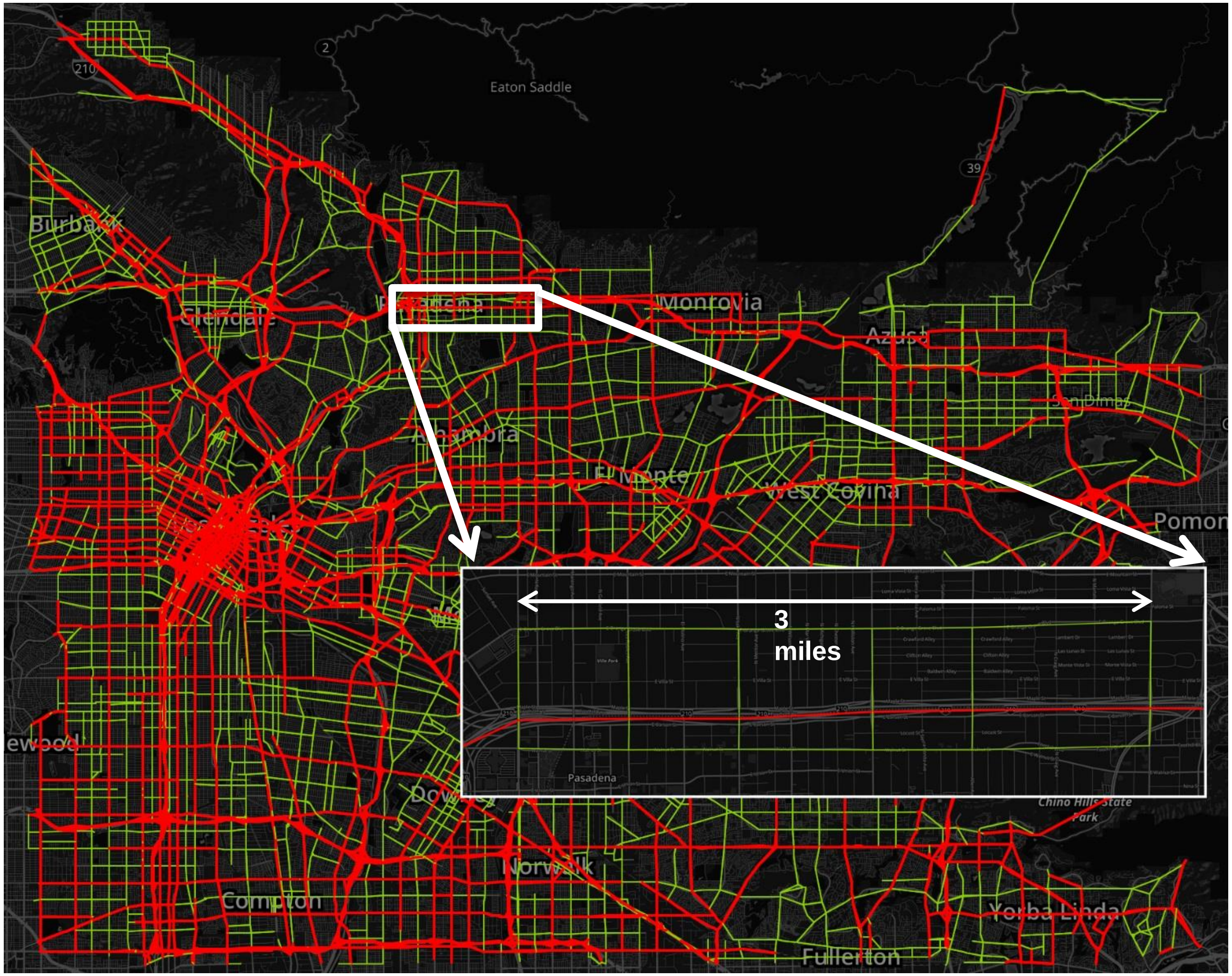
Since the heterogeneous game is a variational inequality problem, Frank-Wolfe (a.k.a. conditional gradient) algorithm gives iterates that converge to the Nash equilibrium.

[Thai et al., IEEE ITSC 2016]

Application for 3 miles in Pasadena

Impact of increased app use for through traffic

- **Immediate massive reroute through Pasadena (2 reroutes)**
- **Travel time in Pasadena increases by 17%**



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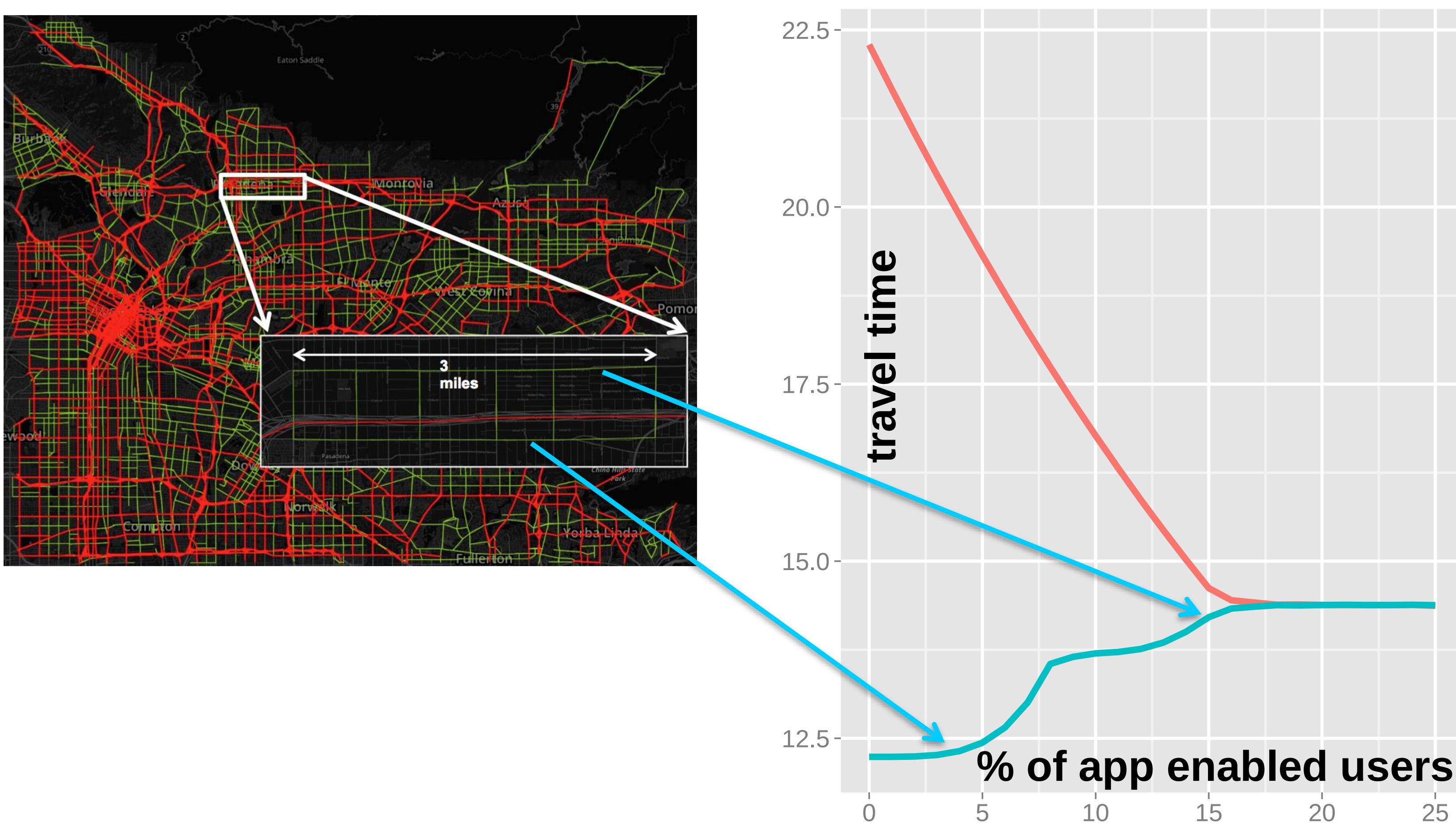

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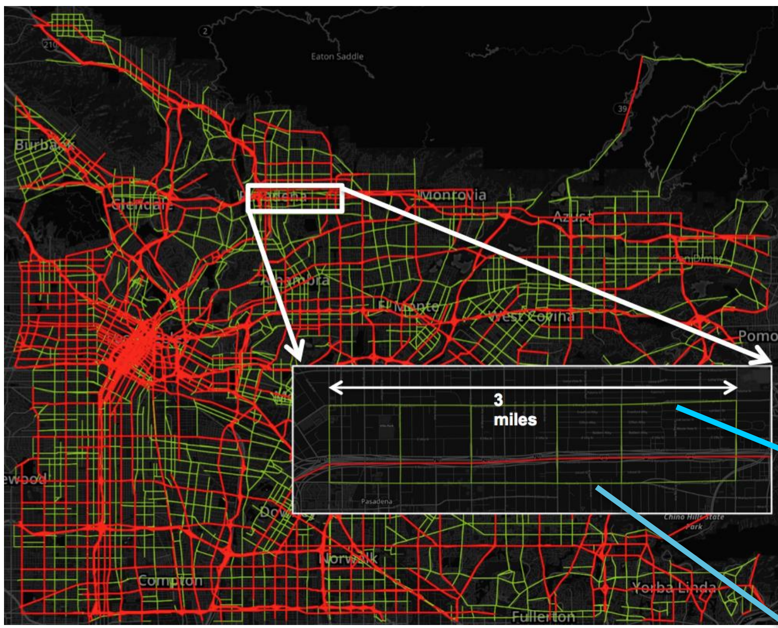
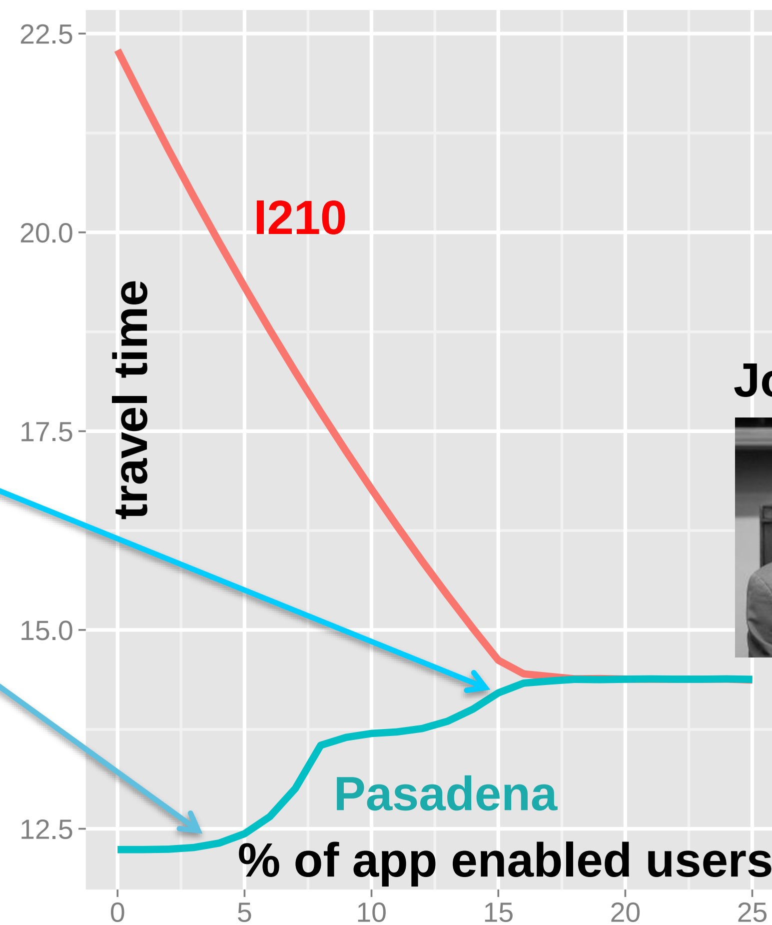




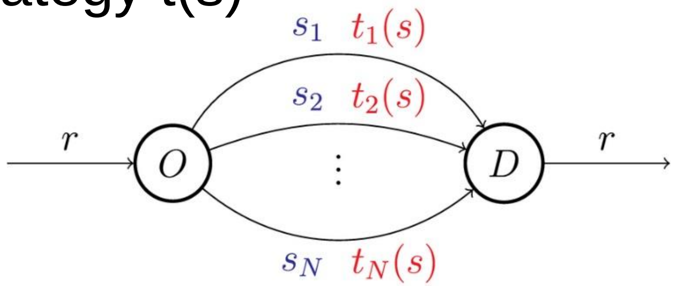
John Nash

[Thai et al., IEEE ITSC 2016]

Control: Nash-Stackelberg games (parallel networks)

First, leader routes compliant drivers αr : strategy $s \in \mathbb{R}_+^N$

Second, followers (non-compliant drivers $(1-\alpha)r$) choose their routes selfishly: strategy t(s)



Leader seeks to minimize system-wide cost: $\min C(s+t(s))$

Optimal Stackelberg strategies $\arg\min_s C(s+t(s))$ are NP-hard to compute (in the size N) for monotonically increasing latency.

Theorem

For set valued latency functions obtained by inversion of the Hamiltonian $q(\cdot)$ of the Hamilton-Jacobi equation, optimal Stackelberg strategies can be computed in $O(N^2)$, by use of a *non-compliant first* strategy.

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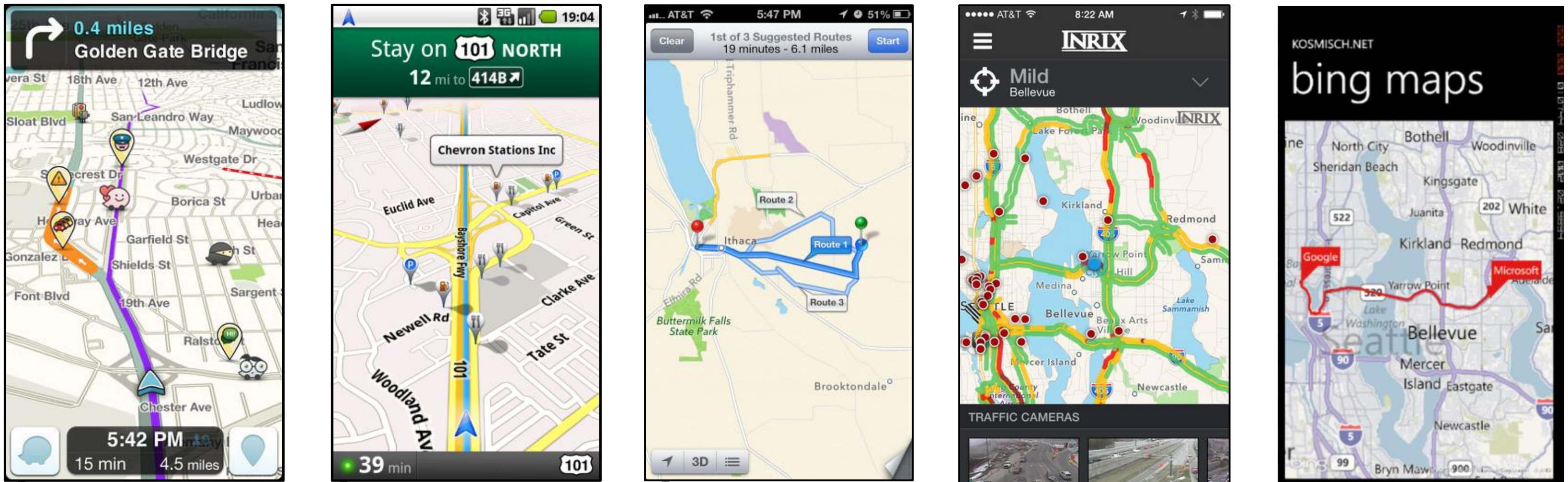
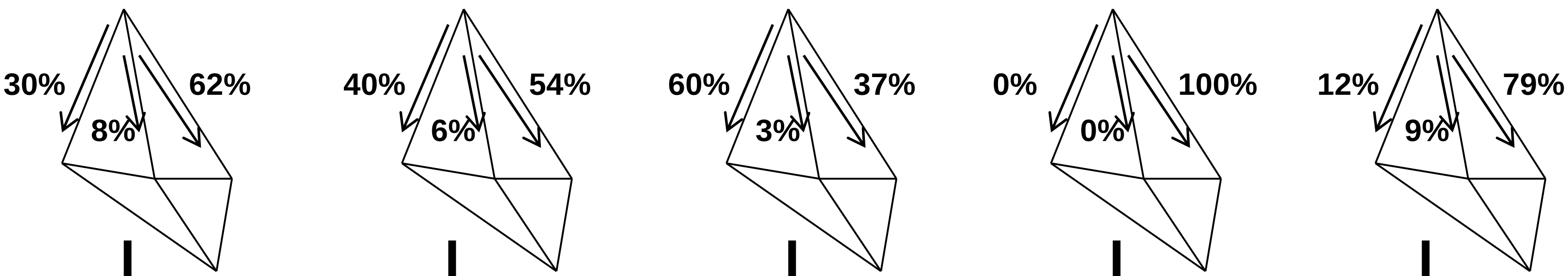
Multi-player situation

Waze Google Apple INRIX Bing (Microsoft)

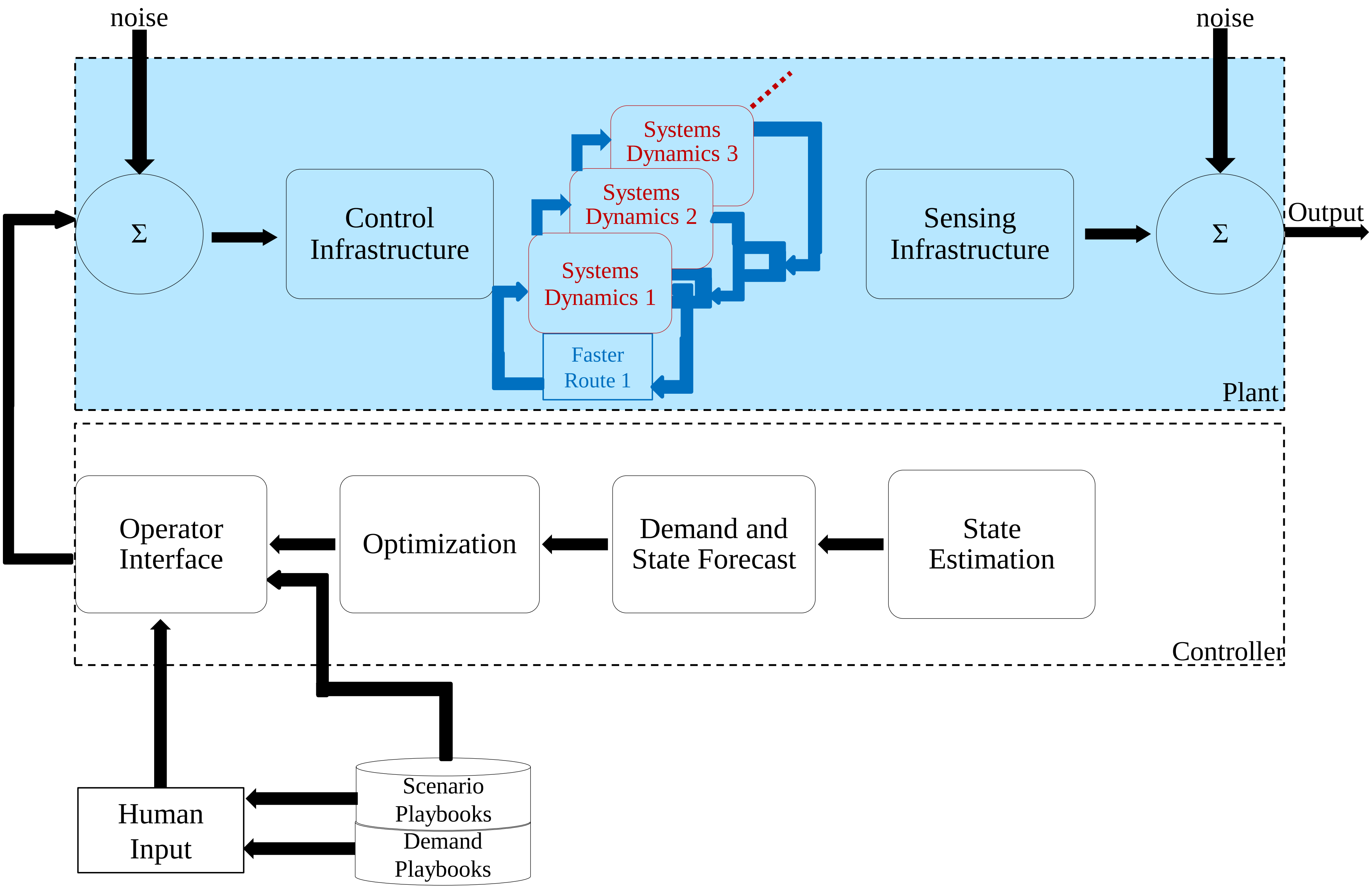


$p_{\text{Waze}} \sim x^{(t)}_{\text{Waze}}$ $p_{\text{Google}} \sim x^{(t)}_{\text{Google}}$ $p_{\text{Apple}} \sim x^{(t)}_{\text{Apple}}$ $p_{\text{INRIX}} \sim x^{(t)}_{\text{INRIX}}$ $p_{\text{Bing}} \sim x^{(t)}_{\text{Bing}}$



All users of each company "equal" by standards of the company i.e. same (shortest) travel time according to the company, "Nash-sampling".

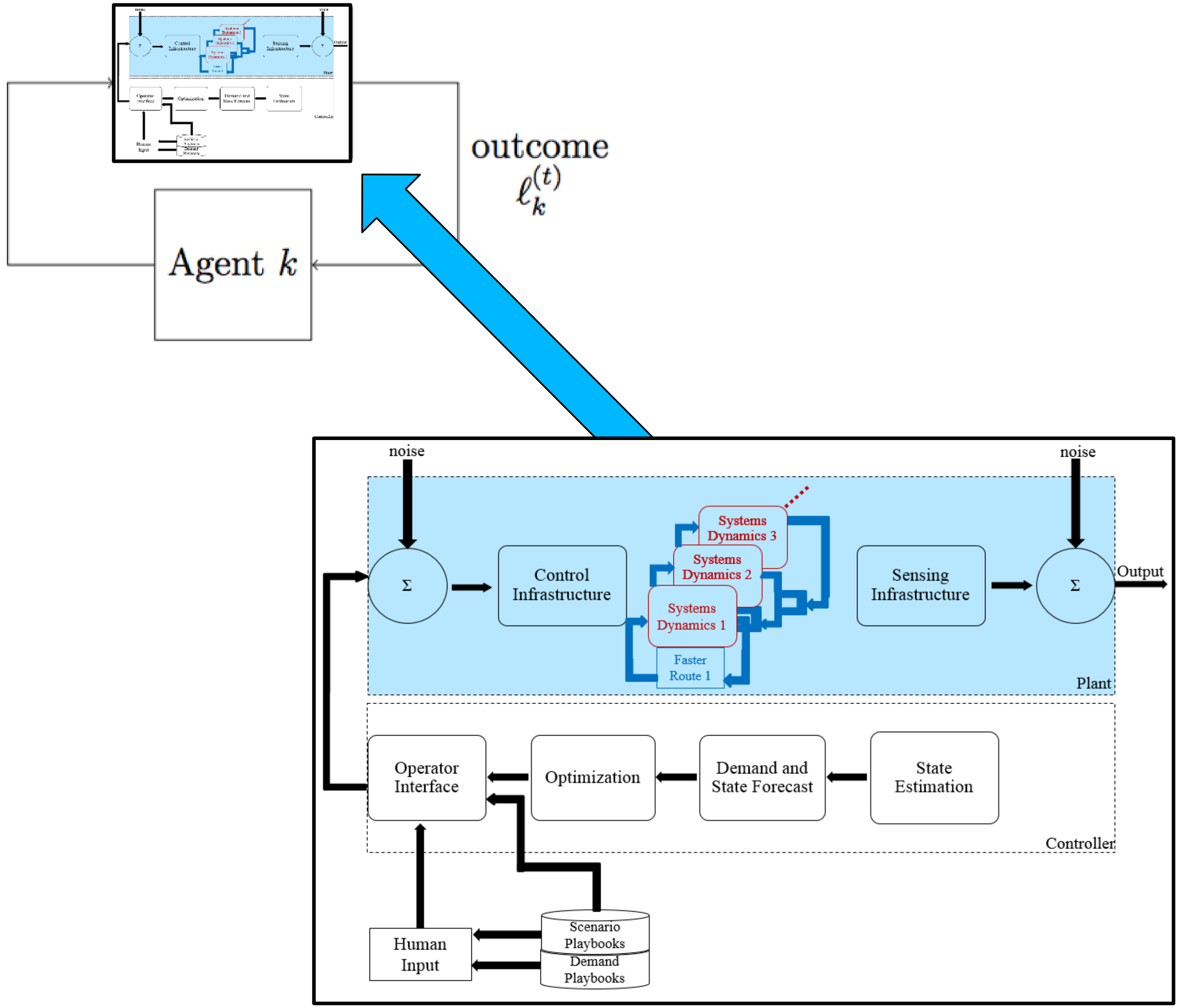
Heterogeneous populations



Dynamics learning

learning algorithm

$$x_k^{(t+1)} = u\left(x_k^{(t)}, \ell_k^{(t)}\right)$$



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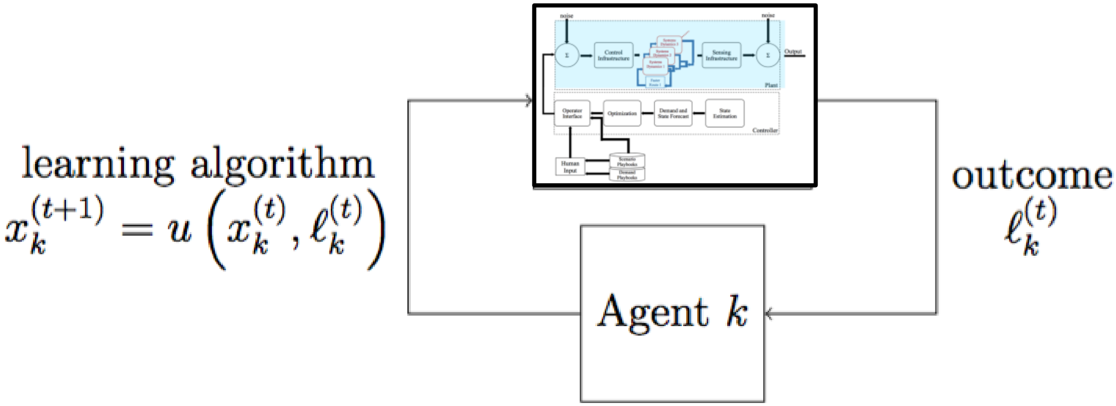


Online Learning Model

1: **for** $t \in \mathbb{N}$ **do**
2: Play $p \sim x_k^{(t)}$
3: Discover $\ell_k^{(t)}$
4: Update $x_k^{(t+1)} = u_k\left(x_k^{(t)}, \ell_k^{(t)}\right)$
5: **end for**

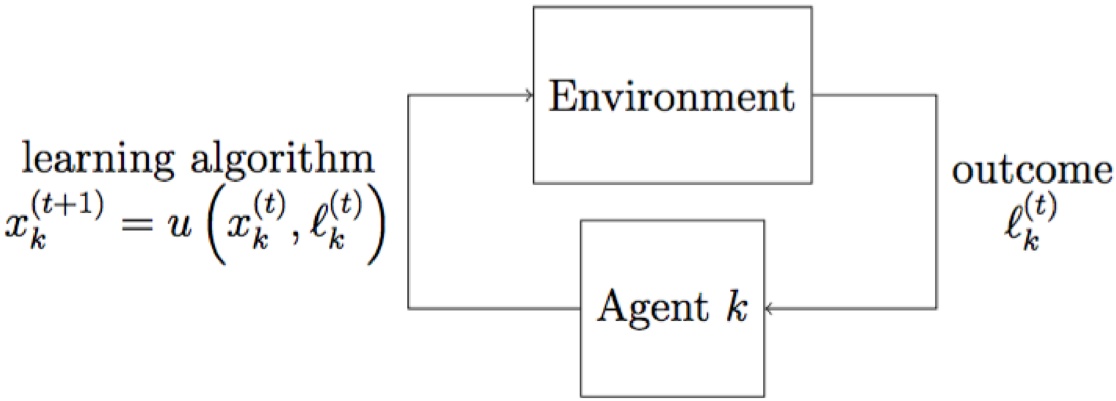
As more historical data, routing systems (companies) learn and evolve

- **These "learning" algorithms are unknown outside the companies**
- **Companies have non-cooperative strategies among each other**
- **This is in addition to providing selfish routing to their users**

Distributed learning dynamics in routing games

- **Each player routes population *k* according to distribution** $p \sim x_k^{(t)}$ **(corresponding to one OD pair)**
- **At each iteration, the population *k* discovers their outcome** $\ell_k^{(t)}$
- **The routing of population *k* at the next step is subsequently updated according to the following law** $x_k^{(t+1)} = u_k\left(x_k^{(t)}, \ell_k^{(t)}\right)$

Dynamics learning



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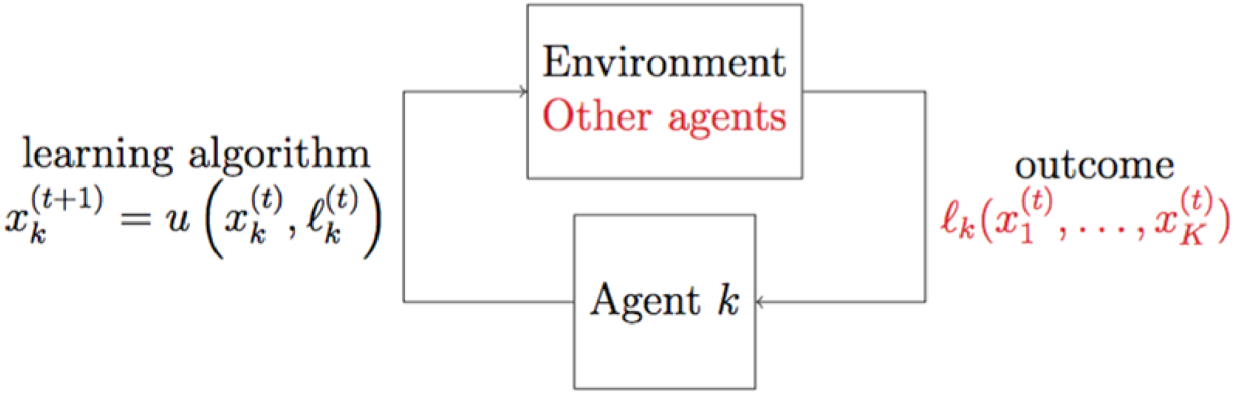

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Dynamics learning

Non equilibrium situations

- **Equilibria: good description of system efficiency at steady-sate.**
- **But systems rarely operate at equilibrium, hence**
  - **Prescriptive models: How do we drive system to eq.?**
  - **Descriptive models: How would players behave in the game?**

Goals of the work

- **Define algorithm classes for which we can prove convergence**
- **Robustness to stochastic perturbations.**
- **Heterogeneous learning: different agents use different algorithms**
- **Convergence rates.**

Related work

- **Discrete time: Hannan consistency (Hannan 1957), Hedge algorithm for two-player games (Freund 1999), regret based algorithms: (Hart 2001), online learning in games (Cesa 2006)**
- **Continuous time: Potential games under dynamics with positive correlation condition (Sandholm 2009), replicator dynamics in evolutionary game theory (Weibull 1997), no-regret dynamics for two player games (Hart 2001)**

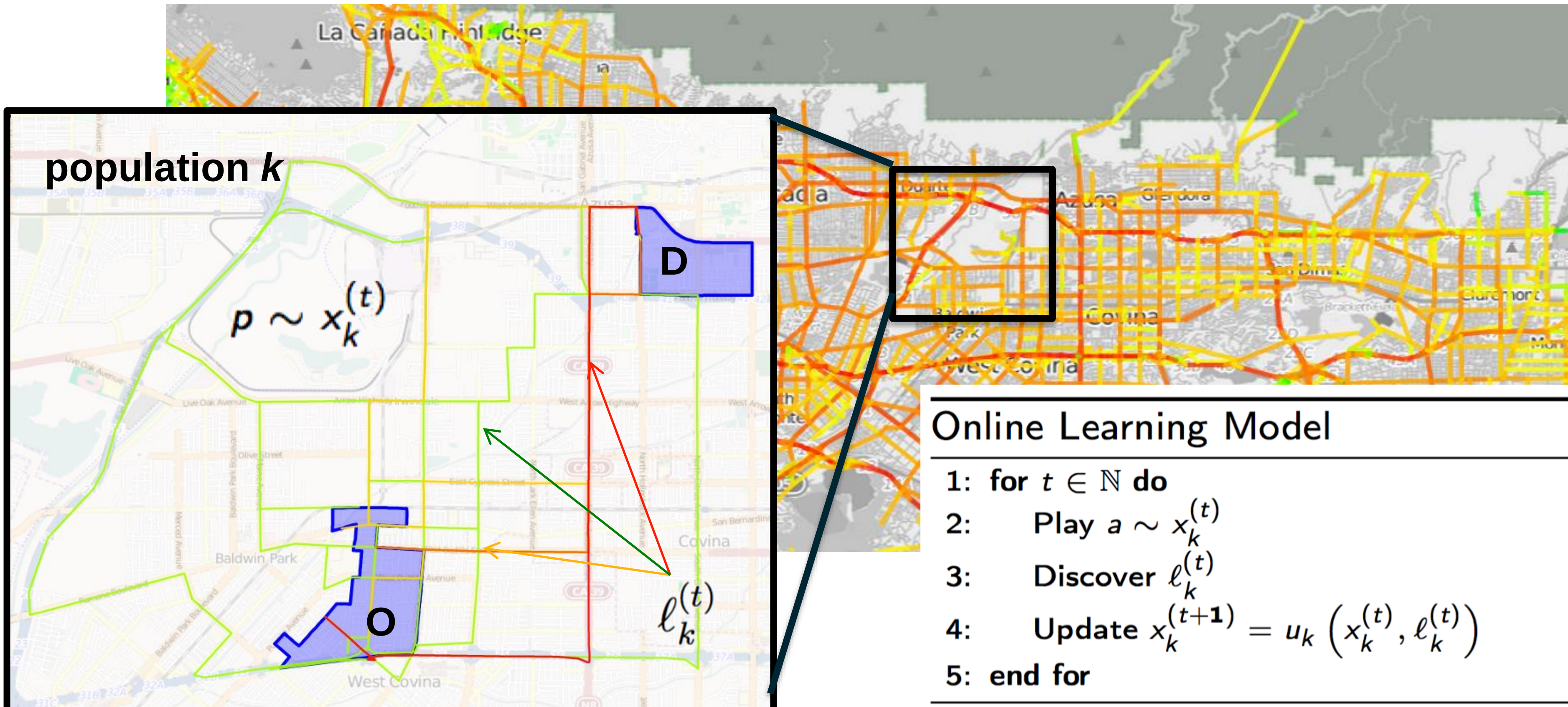
Problem formulation

Main problem

Define class of algorithms $\mathcal{C}$ such that

$$u_k \in \mathcal{C}\ \forall k \Rightarrow x^{(t)} \rightarrow \mathcal{X}^\star$$

Important question: what is $\mathcal{X}^\star$?



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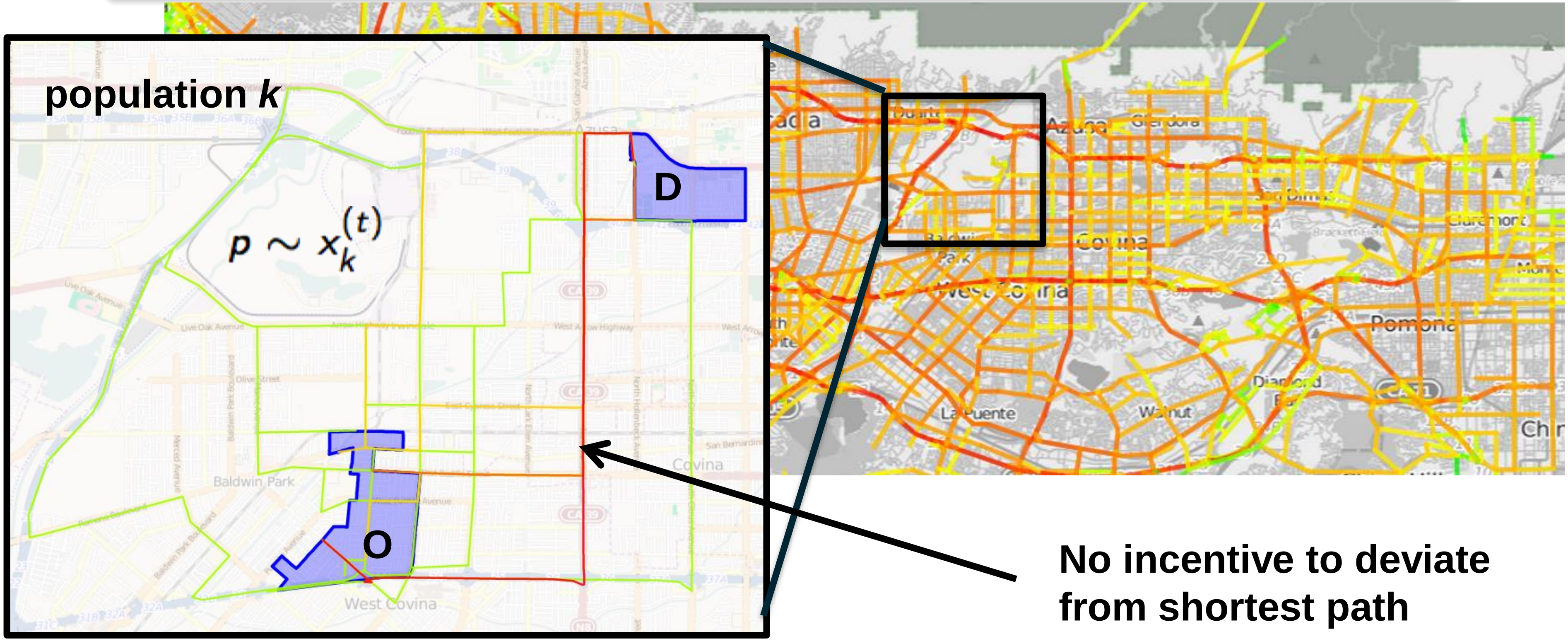
Nash equilibrium

Write
$x = (x_1, \dots, x_K) \in \Delta^{\mathcal{A}_1} \times \dots \times \Delta^{\mathcal{A}_K}$
$\ell(x) = (\ell_1(x), \dots, \ell_K(x))$

Nash equilibria $\mathcal{X}^\star$

$x^\star$ is a Nash equilibrium if for all k, paths in the support of $x_k^\star$ have minimal loss.

$$\forall x, \ \langle \ell(x^\star), x - x^\star \rangle \geq 0$$



Model 1: regret analysis

Interpretation of the regret and the convergence

- **Cumulative regret models the comparison of playing over time the best strategy possible (without changing it), and comparing it to the strategy obtained by the game.**
- **In the case of sublinear regret, the game converges on average towards a Nash equilibrium**
- **Good for optimization purposes**
- **Bad for operational purposes (no guarantee on what the outcome of the game is)**

Cumulative regret

$$R_k^{(t)} = \sup_{x_k \in \Delta^{\mathcal{A}_k}} \sum_{\tau \leq t} \left\langle x_k^{(t)} - x_k, \ell_k(x^{(t)}) \right\rangle$$

"Online" optimality condition. Sublinear if $\limsup_t \frac{R_k^{(t)}}{t} \leq 0$.

Convergence of averages

$$\left[\forall k, R_k^{(t)} \text{ is sublinear}\right] \Rightarrow \bar{x}^{(t)} \to \mathcal{X}^\star$$

$\bar{x}^{(t)} = \frac{1}{t} \sum_{\tau=1}^{t} x^{(\tau)}$.

Model 2: stochastic approximation

Idea:

- **View the learning dynamics as a discretization of an ODE**
- **Study the convergence of the ODE**
- **Relate the convergence of the discrete algorithm to the convergence of the ODE**

In Hedge $x_a^{(t+1)} \propto x_a^{(t)} e^{-\eta_t \ell_a^{(t)}}$, take $\eta_t \to 0$.

Replicator equation [25]

$$\forall a \in \mathcal{A}_k, \frac{dx_a}{dt} = x_a \left(\langle \ell(x), x \rangle - \ell_a(x) \right)$$

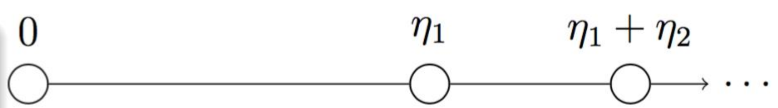


Figure: Underlying continuous time

Definitions:

- η_t **Discretization (in time)**
- x_a **Distribution of flow along one arc**
- $\mathcal{A}_k$ **Set of arcs for population *k***

Weibull, *Evolutionary Game Theory*, 1997

AREP: approximate replicator dynamics

Idea:

- **View the learning dynamics as a discretization of an ODE**
- **Study the convergence of the ODE**
- **Relate the convergence of the discrete algorithm to the convergence of the ODE**

In Hedge $x_a^{(t+1)} \propto x_a^{(t)} e^{-\eta_t \ell_a^{(t)}}$, take $\eta_t \to 0$.

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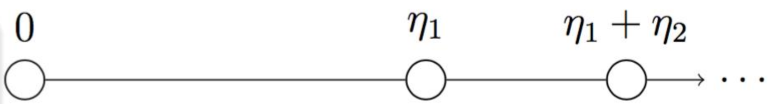


Figure: Underlying continuous time

Discretization of the continuous-time replicator dynamics

$$\frac{x_a^{(t+1)} - x_a^{(t)}}{\eta_t} = x_a^{(t)} \left(\left\langle \ell(x^{(t)}), x^{(t)} \right\rangle - \ell_a(x^{(t)}) \right) + U_a^{(t+1)}$$

Benaim, *Dynamics of stochastic approximation algorithms,* 1999

AREP: approximate replicator dynamics

Idea:

- **View the learning dynamics as a discretization of an ODE**
- **Study the convergence of the ODE**
- **Relate the convergence of the discrete algorithm to the convergence of the ODE**

$$\frac{dx_a}{dt} = x_a \left(\langle \ell(x), x \rangle - \ell_a(x) \right)$$

Discretization of the continuous-time replicator dynamics

$$\frac{x_a^{(t+1)} - x_a^{(t)}}{\eta_t} = x_a^{(t)} \left(\left\langle \ell(x^{(t)}), x^{(t)} \right\rangle - \ell_a(x^{(t)}) \right) + U_a^{(t+1)}$$

- η_t discretization time steps.
- $(U^{(t)})_{t \geq 1}$ perturbations that satisfy for all $T > 0$,
  $\lim_{\tau_1 \to \infty} \max_{\tau_2 : \sum_{t=\tau_1}^{\tau_2} \eta_t < T} \left\| \sum_{t=\tau_1}^{\tau_2} \eta_t U^{(t+1)} \right\| = 0$

(a sufficient condition is that $\exists q \geq 2$: $\sup_\tau \mathbb{E} \| U^{(\tau)} \|^q < \infty$ and $\sum_\tau \eta_\tau^{1+\frac{q}{2}} < \infty$)

AREP: approximate replicator dynamics

Idea:

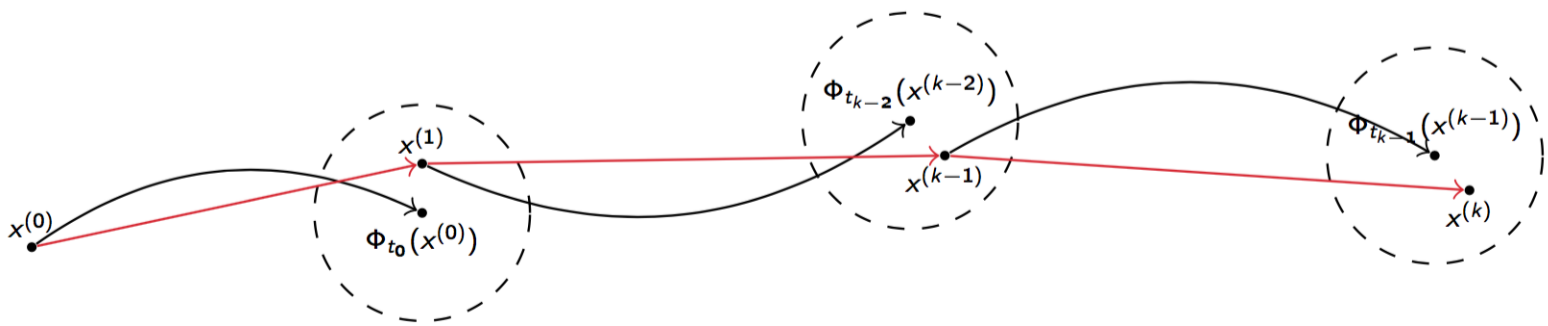
- **View the learning dynamics as a discretization of an ODE**
- **Study the convergence of the ODE**
- **Relate the convergence of the discrete algorithm to the convergence of the ODE, but no convergence rates**

Theorem [13]

In convex potential games, under AREP updates, if $\eta_t \downarrow 0$ and $\sum \eta_t = \infty$, then

$$x^{(t)} \to \mathcal{X}^\star \text{ a.s.}$$

- Affine interpolation of $x^{(t)}$ is an asymptotic pseudo trajectory of ODE.



- Use f as a Lyapunov function.

AREP: approximate replicator dynamics

Idea:

- **View the learning dynamics as a discretization of an ODE**
- **Study the convergence of the ODE**
- **Relate the convergence of the discrete algorithm to the convergence of the ODE, but no convergence rates**

Theorem [13]

In convex potential games, under AREP updates, if $\eta_t \downarrow 0$ and $\sum \eta_t = \infty$, then

$$x^{(t)} \to \mathcal{X}^\star \text{ a.s.}$$

Model 3: convex optimization

Idea:

- **View the learning dynamics as a distributed algorithm to minimize the function *f*.**
- **Allows us to analyze convergence rates.**

Here:

- **Class of distributed optimization methods: stochastic mirror descent**

minimize	$f(x)$	convex function
subject to	$x \in \mathcal{X} \subset \mathbb{R}^d$	convex, compact set

Bregman Divergence

Strongly convex function ψ

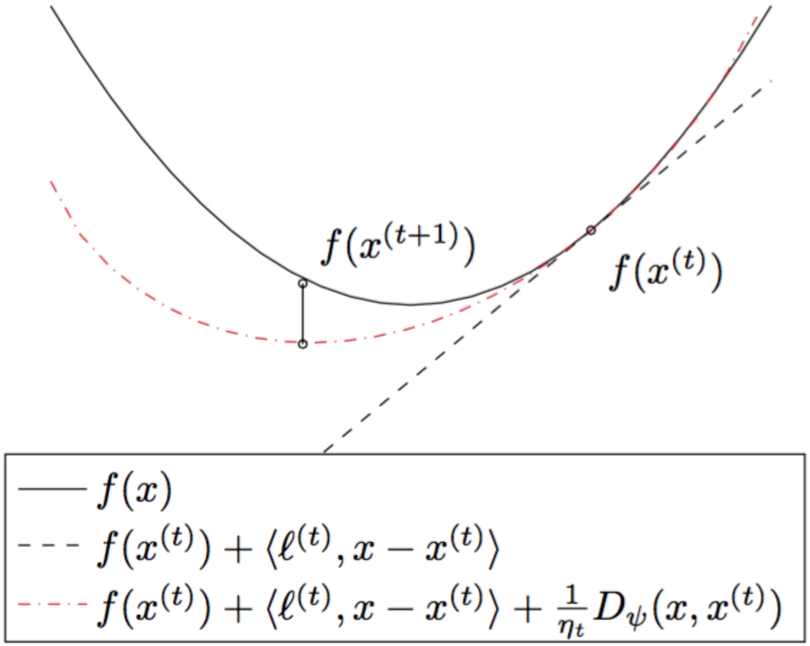
$$D_\psi(x, y) = \psi(x) - \psi(y) - \langle \nabla\psi(y), x - y \rangle$$

Approach 3: convex optimization

minimize	$f(x)$	convex function
subject to	$x \in \mathcal{X} \subset \mathbb{R}^d$	convex, compact set

Algorithm 2 MD Method with learning rates (η_t)

for $t \in \mathbb{N}$ **do**
  observe $\ell^{(t)} \in \partial f(x^{(t)})$
  $x^{(t+1)} = \arg\min_{x \in \mathcal{X}} \langle \ell^{(t)}, x \rangle + \frac{1}{\eta_t} D_\psi(x, x^{(t)})$
end for



- η_t: learning rate
- D_ψ: ▸ Bregman divergence

Bregman Divergence

Strongly convex function ψ

$$D_\psi(x, y) = \psi(x) - \psi(y) - \langle \nabla \psi(y), x - y \rangle$$

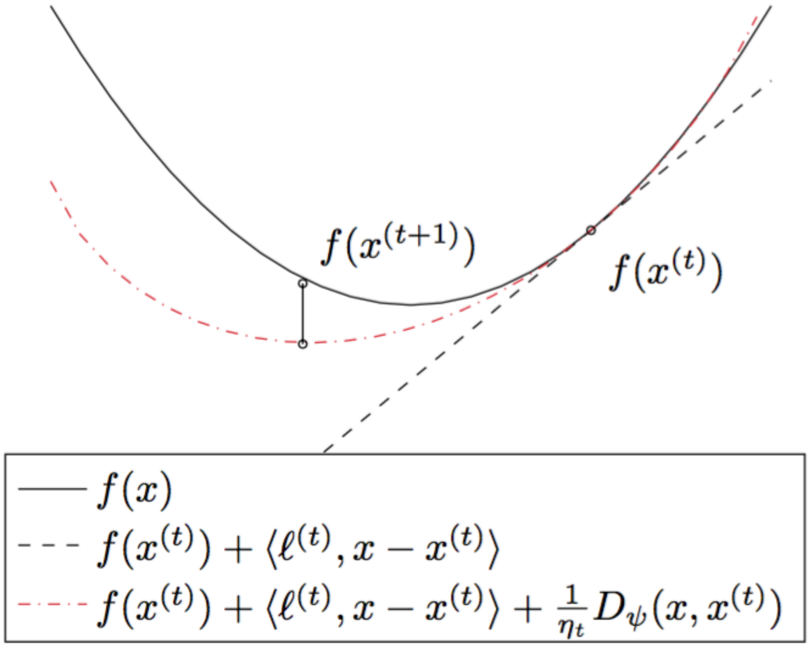
Approach 3: convex optimization

minimize	$f(x)$	convex function
subject to	$x \in \mathcal{X} \subset \mathbb{R}^d$	convex, compact set

Algorithm 2 MD Method with learning rates (η_t)

1: **for** $t \in \mathbb{N}$ **do**
2: observe $\ell_k^{(t)} \in \partial_k f(x^{(t)})$
3: $x_k^{(t+1)} = \arg\min_{x \in \mathcal{X}_k} \left\langle \ell_k^{(t)}, x \right\rangle + \frac{1}{\eta_t^k} D_{\psi_k}(x, x_k^{(t)})$
4: **end for**

- η_t: learning rate
- D_ψ: ▸ Bregman divergence



Bregman Divergence

Strongly convex function ψ

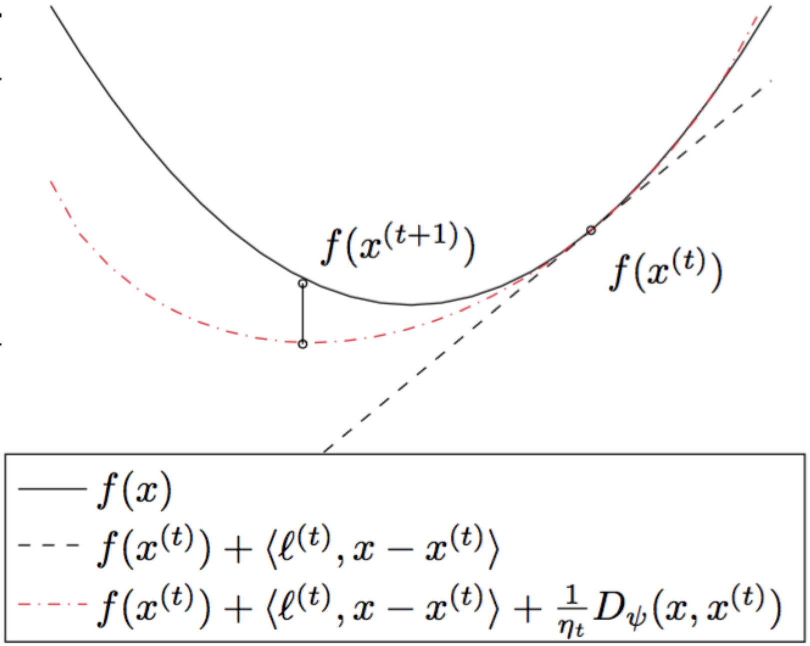
$$D_\psi(x, y) = \psi(x) - \psi(y) - \langle \nabla \psi(y), x - y \rangle$$

Approach 3: convex optimization

minimize	$f(x)$	convex function
subject to	$x \in \mathcal{X} \subset \mathbb{R}^d$	convex, compact set

Algorithm 2 SMD Method with learning rates (η_t)

1: **for** $t \in \mathbb{N}$ **do**
2: observe $\hat{\ell}_k^{(t)}$ with $\mathbb{E}\left[\hat{\ell}_k^{(t)} | \mathcal{F}_{t-1}\right] \in \partial_k f(x^{(t)})$
3: $x_k^{(t+1)} = \arg\min_{x \in \mathcal{X}_k} \left\langle \hat{\ell}_k^{(t)}, x \right\rangle + \frac{1}{\eta_t^k} D_{\psi_k}(x, x_k^{(t)})$
4: **end for**



- η_t: learning rate
- D_ψ: ▸ Bregman divergence

Bregman Divergence

Strongly convex function ψ

$$D_\psi(x, y) = \psi(x) - \psi(y) - \langle \nabla\psi(y), x - y \rangle$$

Convergence

- To show convergence $\mathbb{E}\left[f(x^{(t)})\right] \to f^\star$, generalize the technique of Shamir et al. [22].

Convergence of Distributed Stochastic Mirror Descent

For $\eta_t^k = \frac{\theta_k}{t^{\alpha_k}}$, $\alpha_k \in (0,1)$,

$$\mathbb{E}\left[f(x^{(t)})\right] - f^\star = \mathcal{O}\left(\sum_k \frac{\log t}{t^{\min(\alpha_k, 1-\alpha_k)}}\right)$$

Non-smooth, non-strongly convex.

More details

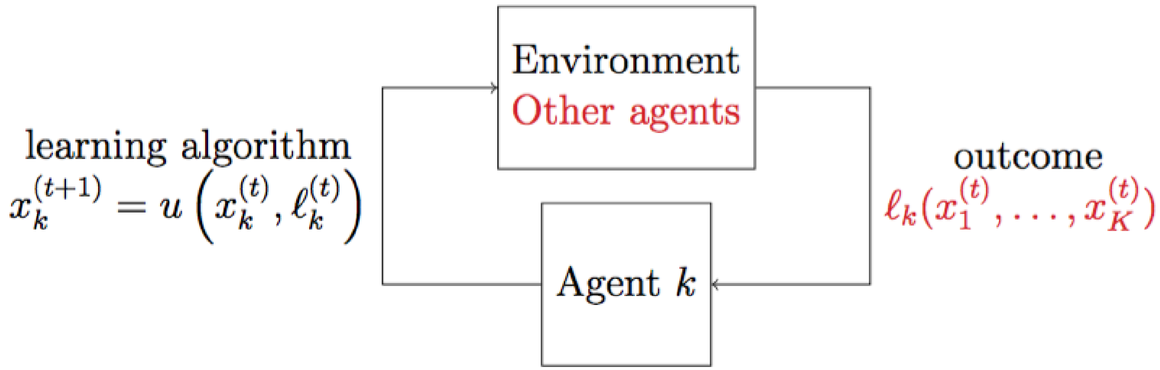
[22]Ohad Shamir and Tong Zhang. Stochastic gradient descent for non-smooth optimization: Convergence results and optimal averaging schemes.
In *ICML*, pages 71–79, 2013

[12] Syrine Krichene, Walid Krichene, Roy Dong, and Alexandre Bayen. Convergence of heterogeneous distributed learning in stochastic routing games.
In *53rd Allerton Conference on Communication, Control and Computing*, 2015

Summary

Distributed learning dynamics in routing games

- **Each player routes population *k* according to distribution $p \sim x_k^{(t)}$ (corresponding to one OD pair)**
- **At each iteration, the population *k* discovers their outcome $\ell_k^{(t)}$**
- **The routing of population *k* at the next step is subsequently updated according to the following law $x_k^{(t+1)} = u_k\left(x_k^{(t)}, \ell_k^{(t)}\right)$**



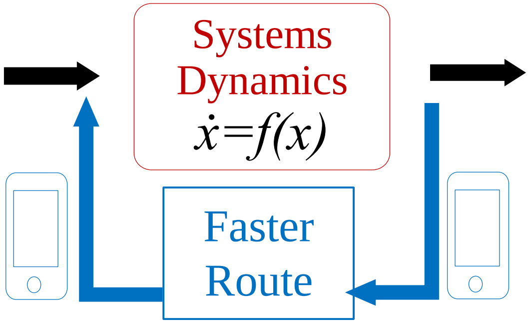
- Regret analysis: convergence of $\bar{x}^{(t)}$
- Stochastic approximation: almost sure convergence of $x^{(t)}$
- Stochastic convex optimization: almost sure convergence, $\mathbb{E}\left[f(x^{(t)})\right] \to f^\star$, $\mathbb{E}\left[D_\psi(x^\star, x^{(t)})\right] \to 0$, convergence rates.

Summary : Matriochka problems

Faster Route

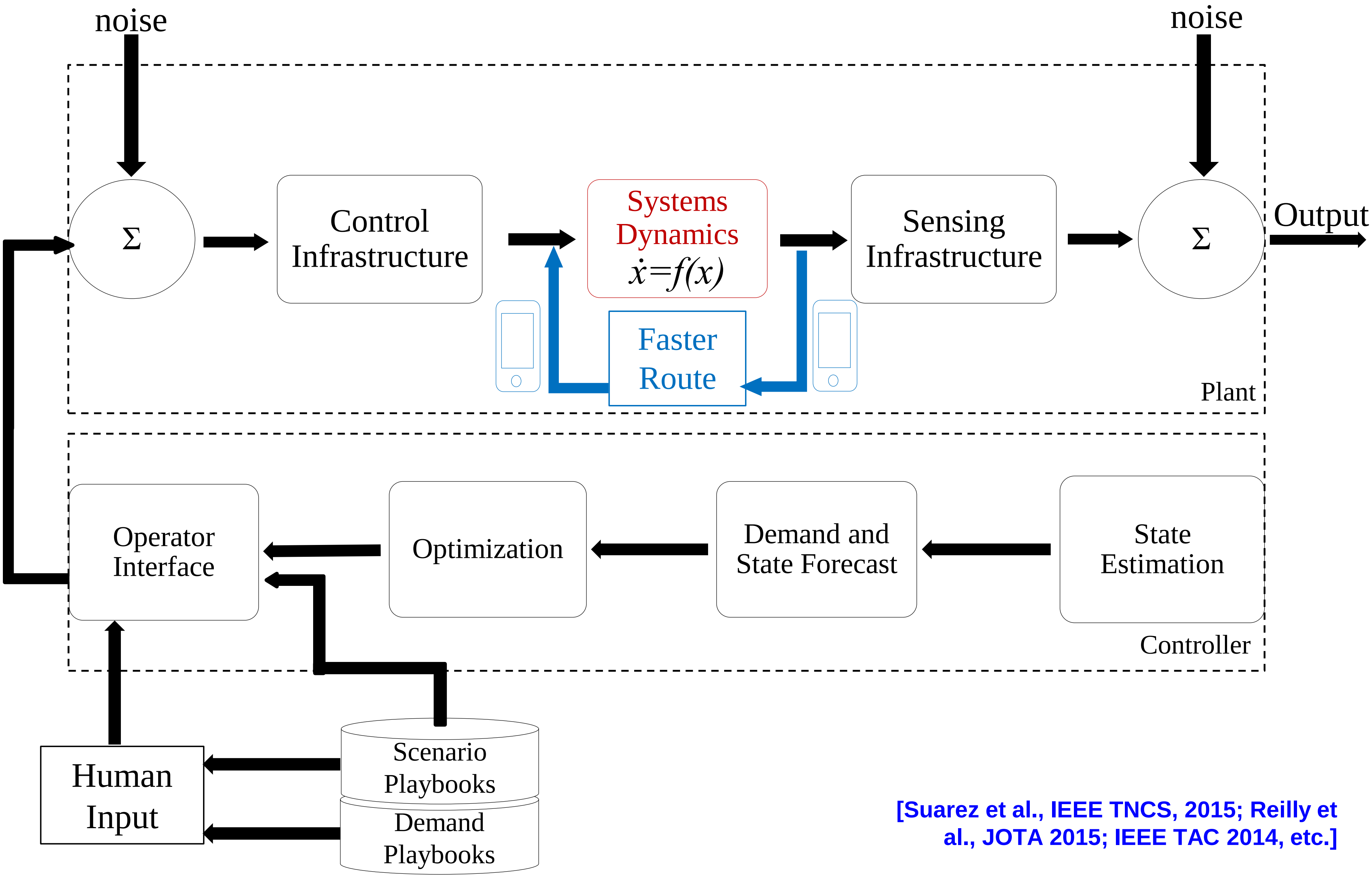
[Samaranayake, Bayen, IEEE ITSC 2011, TR-C, 2012, ALANEX 2014, AATMO 2012]

Summary : Matriochka problems



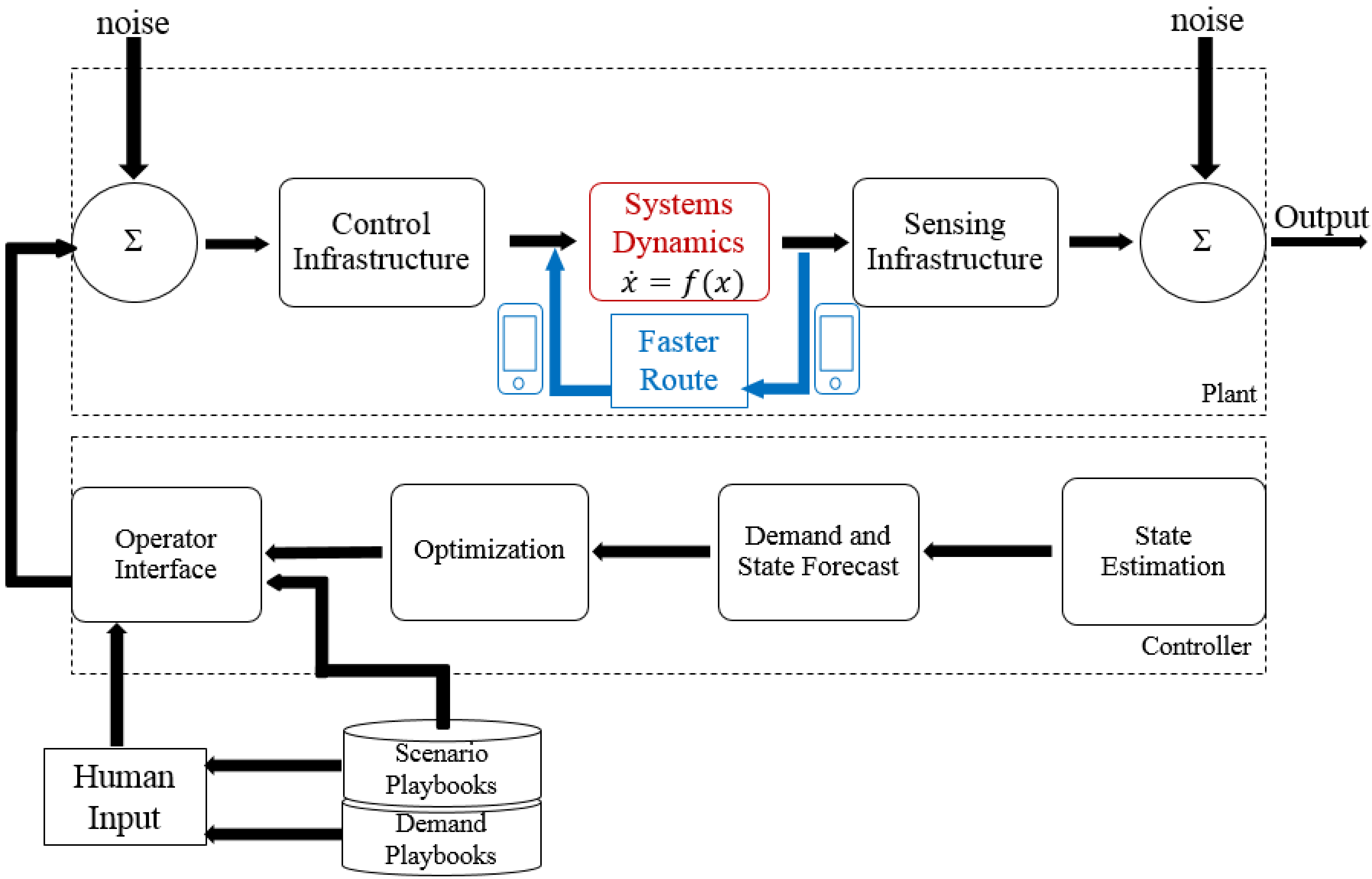
[Krichene, Reilly, Amin, Bayen, et al. IEEE TAC 2014, CDC 2014, Allerton 2013, CDC, 2012]

Summary : Matriochka problems

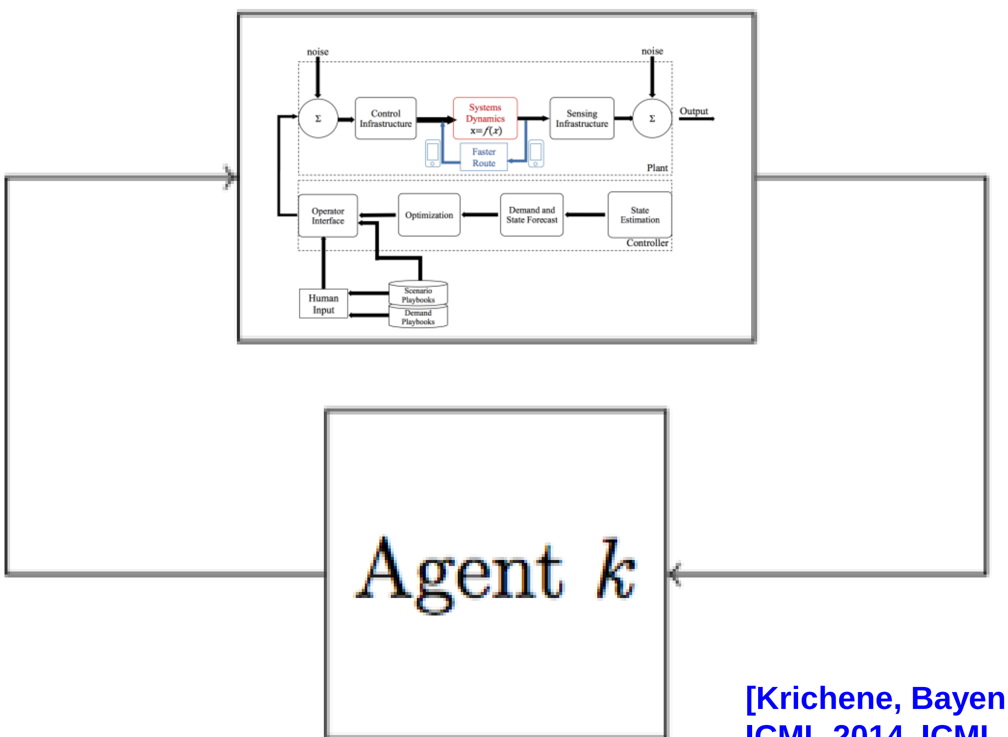


[Suarez et al., IEEE TNCS, 2015; Reilly et al., JOTA 2015; IEEE TAC 2014, etc.]

Summary : Matriochka problems



Summary : Matriochka problems



learning algorithm

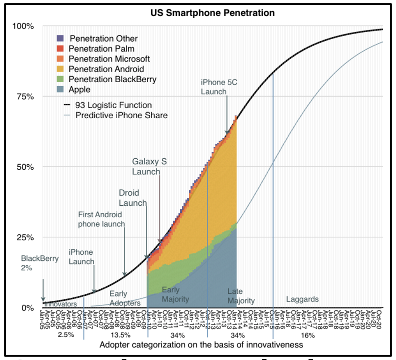
$$x_k^{(t+1)} = u\left(x_k^{(t)}, \ell_k^{(t)}\right)$$

outcome

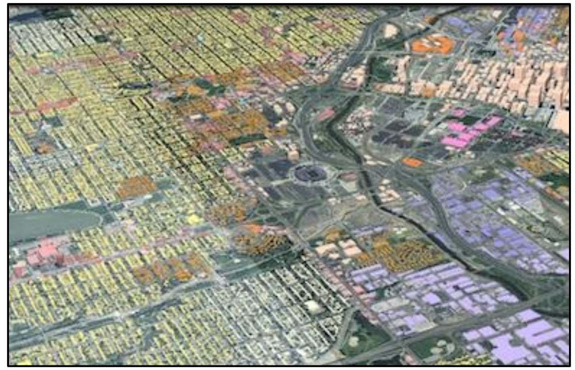
$$\ell_k^{(t)}$$

[Krichene, Bayen, Drighes et al.NIPS 2015, ICML 2014, ICML 2015, ICCPS 2015, ECC, 2015, SIAM 2015, IEEE TNCS 2016]

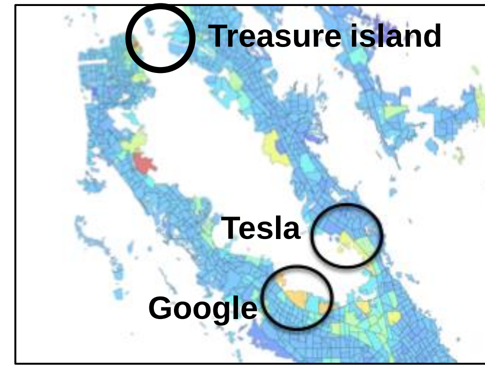
Summary : Matriochka problems



Smartphone explosion



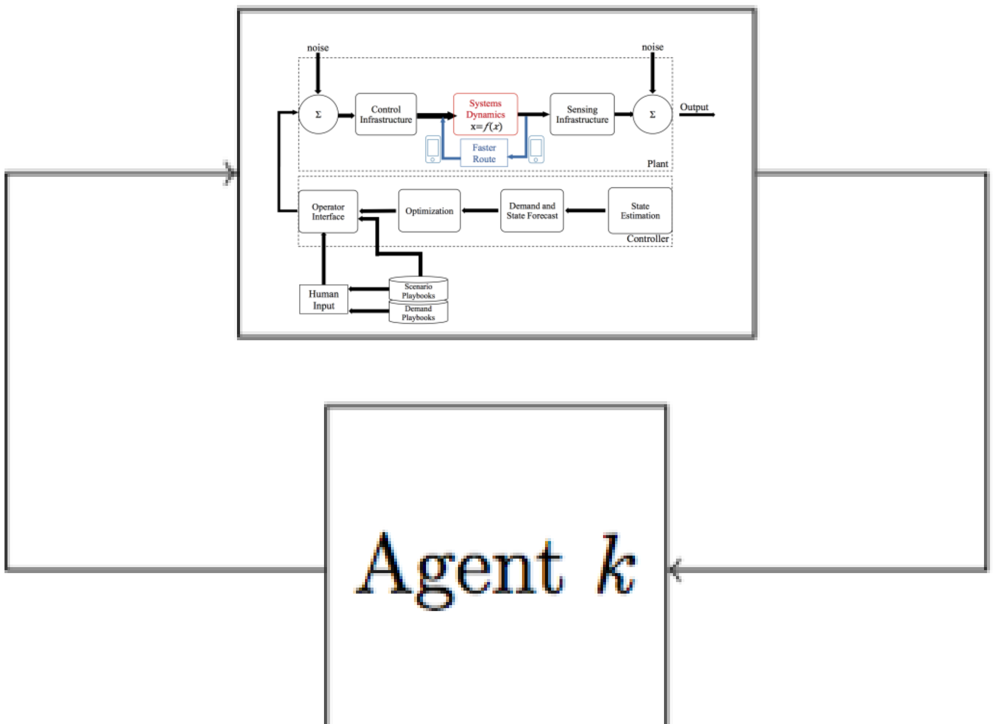
Demographic growth



Landuse disruptions

learning algorithm

$$x_k^{(t+1)} = u\left(x_k^{(t)}, \ell_k^{(t)}\right)$$



outcome

$$\ell_k^{(t)}$$

Summary: Research focus

Dynamics governed by nonlinear and distributed dynamics

Multimodal, mode choice
Behavioral modeling
Automation

(De)centralized, distributed sensing, control, optimization

Resilience
Cybersecurity

Infrastructure CPS

Non cooperative players

[Two sided] markets
Incentivization
Route choice models
MaaS system, shared economy

Feedback loops that integrate humans and keep learning

Information aware DTA
Information forecast

In grey: not covered in this talk

Outline

1. **General framework for traffic operations**

1. **Inference problems**
    1. **Demand inference**
    2. **Traffic estimation**

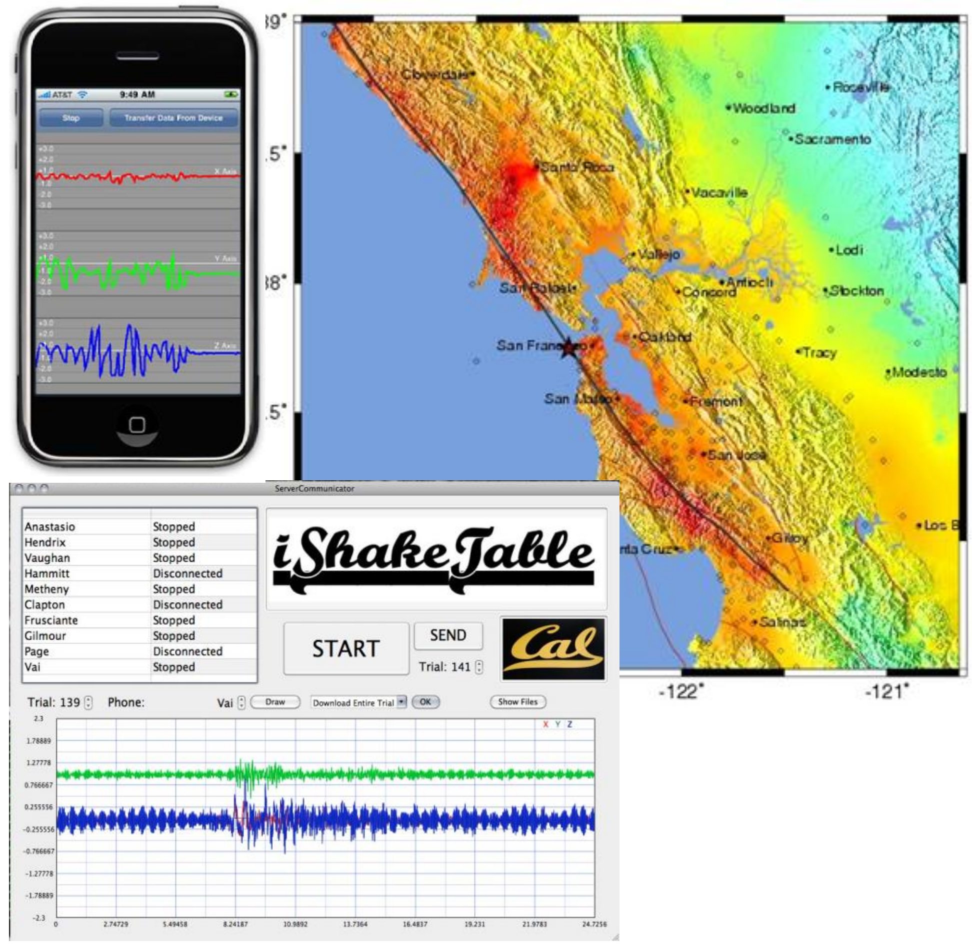
2. **Heterogeneous games**
    1. **Heterogeneous game, Nash-Stackelberg solutions**
    2. **Learning dynamics in repeated games**

3. **Other mobile sensor and data and CPS education**

Using smartphones as seismometers

iShake project

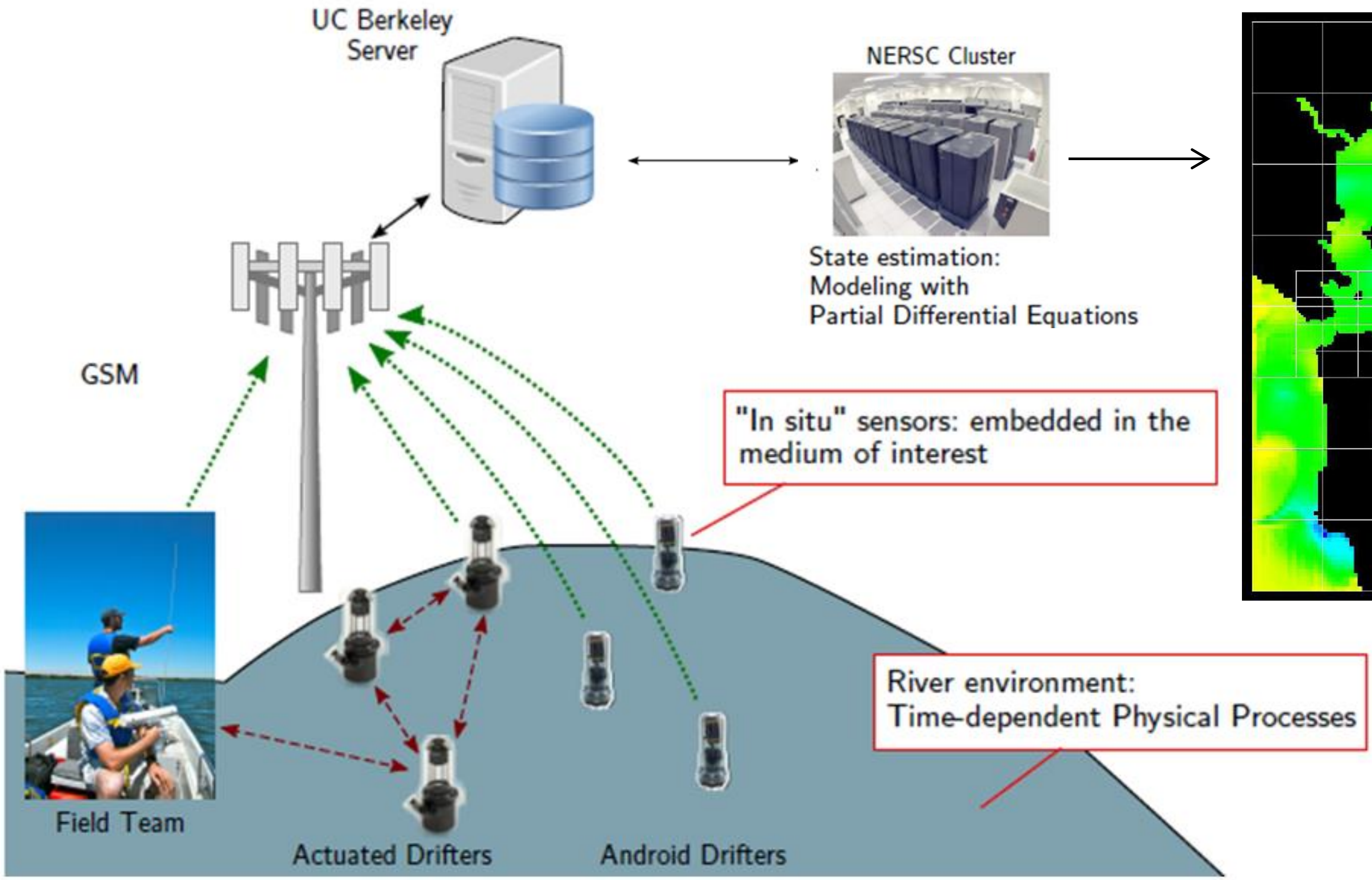
- One of the first shake monitoring apps on the iPhone (2010)
- Scaled up by SeismoLab with T-Mobile / Deutsche Telekon
- Several 100K downloads in first week of existence

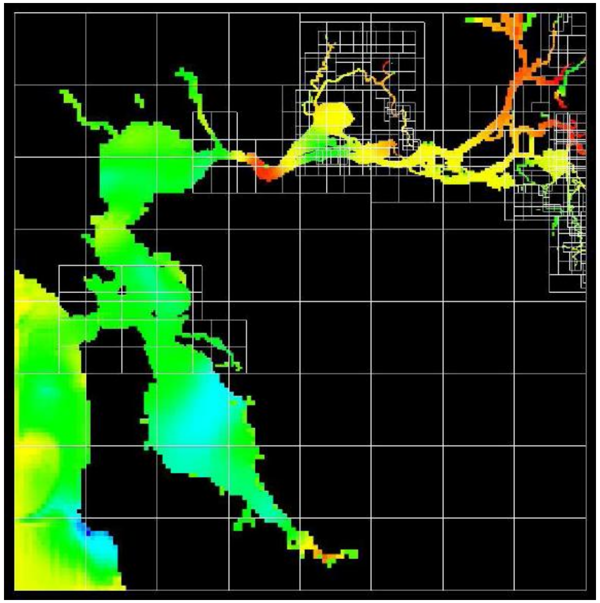


Floating sensor network

Inverse modeling, data assimilation, inference, estimation

- **Real-time, online (with streaming data)**
- **Running two dimensional shallow water models (LBNL REALM)**
- **Using Ensemble Kalman Filtering, statistical inference methods**
- **Running on 500 nodes of the Magellan / NERSC cluster at LBNL**





Floating sensor network

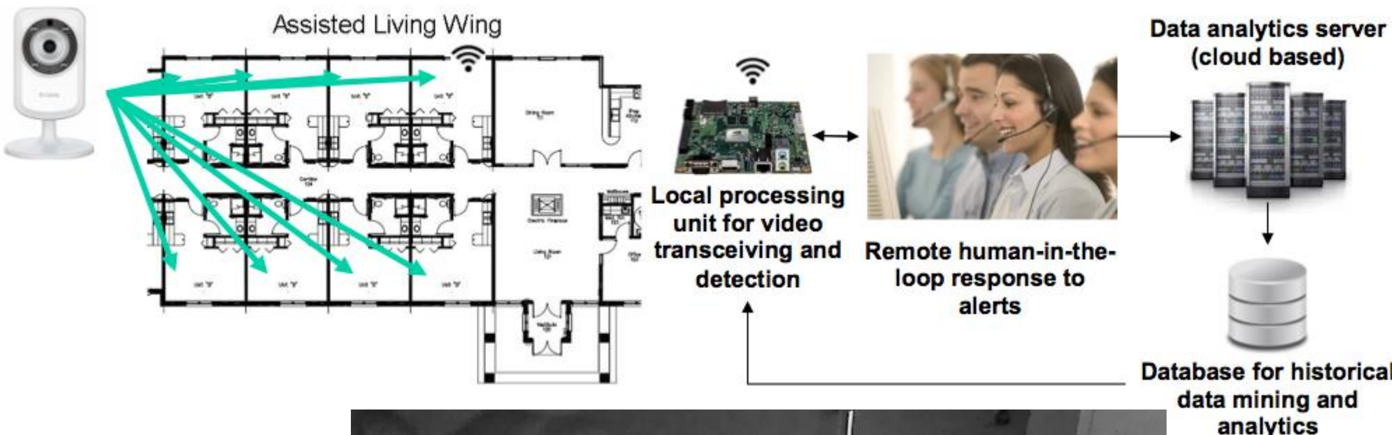
Experimental deployments

- **100 floating units motorized and passive**
- **Experimental deployments: Sacramento Delta (CA), Stillwater (OK), Bordeaux (France)**

Vision-based and mobile/static sensing

Monitoring Alzheimer patients in memory care facilities and homes

- **Homes: Android & SmartWatches, sensors: 18 patients**
- **Memory care facilities: cameras: 100 patients**



Outline

1. **General framework for traffic operations**

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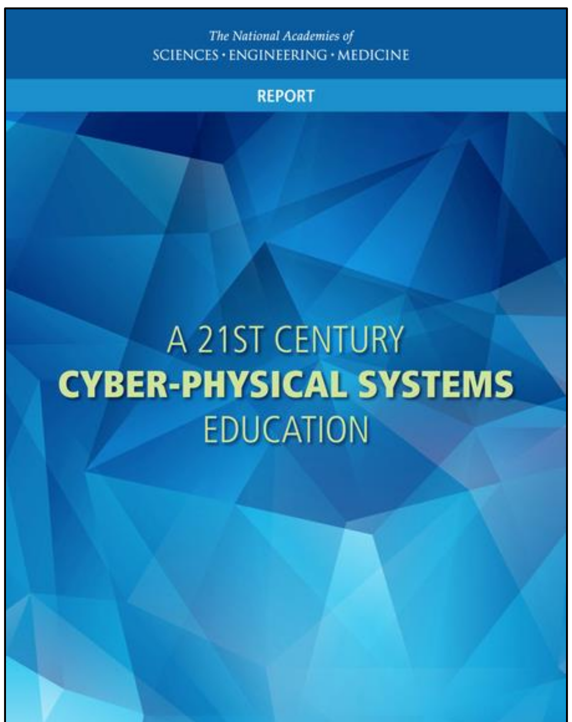
    1. **Heterogeneous game, Nash-Stackelberg solutions**
    2. **Learning dynamics in repeated games**

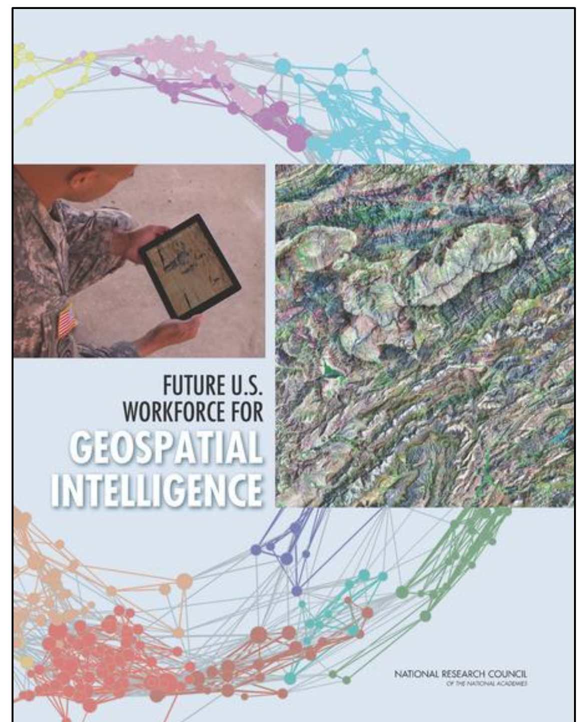
3. **Other mobile sensor and data and CPS education**

CPS education

A changing time for many disciplines

- **Disciplines based on physical sciences**
  - **Civil engineering (structural, geotechnical, transportation)**
  - **Environmental engineering (hydrodynamics, chemistry)**
  - **Mechanical engineering (thermo., fluid mech.)**
- **Modeling-based disciplines**
  - **Economics, behavioral science**
  - **Epidemiology, physical and human geography**

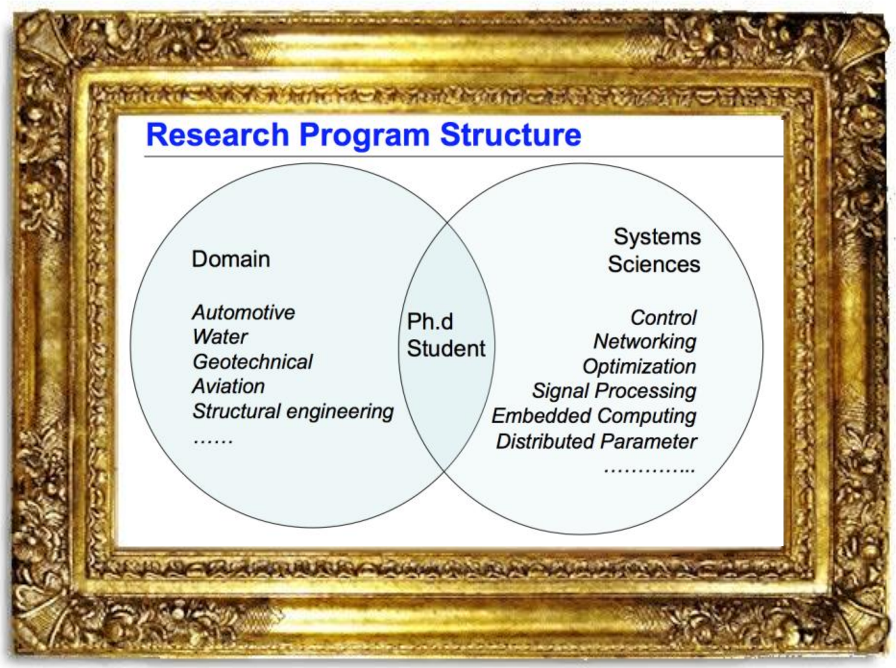


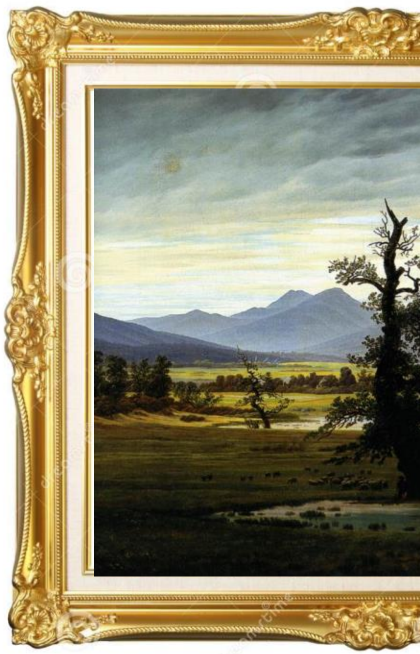


CPS education in CEE

Systems Engineering program (2003 – present)

- **Within CEE, 6 faculty (3 emeriti)**
- **Initial vision:**
  - **one “physical” discipline,**
  - **one methodological discipline**
- **Initially a graduate program**
  - **100s graduate students since 2003 (MS, MEng, PhD)**
  - **20+ alums faculty (MIT, Cornell, GT, UMich, UIUC, Purdue)**
- **One fully integrated curriculum**





CPS education in CEE

Graduate education (student chooses 3 core out of 6 + 5 free)

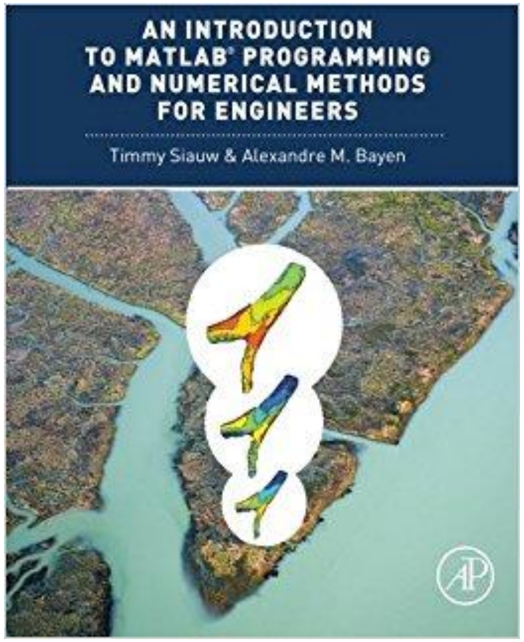
- **2003: CE271: sensor and signals**
- **2004: CE290: control and information manangement**
- **2006: CE291: control of distributed parameter systems**
- **2009: CE264: behavioral modeling**
- **2012: CE263: scalable data analytics**
- **2013: CE295: energy systems and control**

Undergraduate education (one lower div. 2 core electives)

- **2003: CE191: intro to systems analysis [optimization]**
- **2013: CE186: design of cyber physical systems**
- **2016: CE88: data science for smart cities**

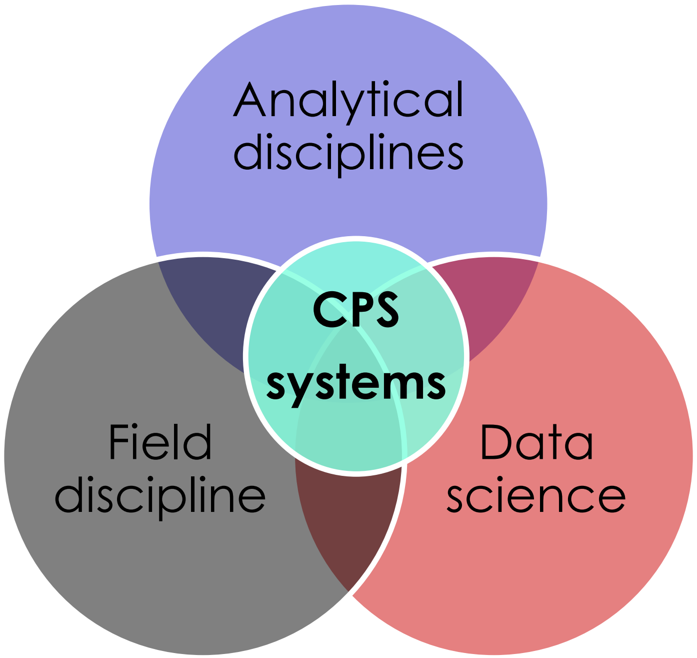
In addition (undergraduate)

- **E7: introduction to numerical analysis and programming**
- **CE93: data analysis (statistics)**



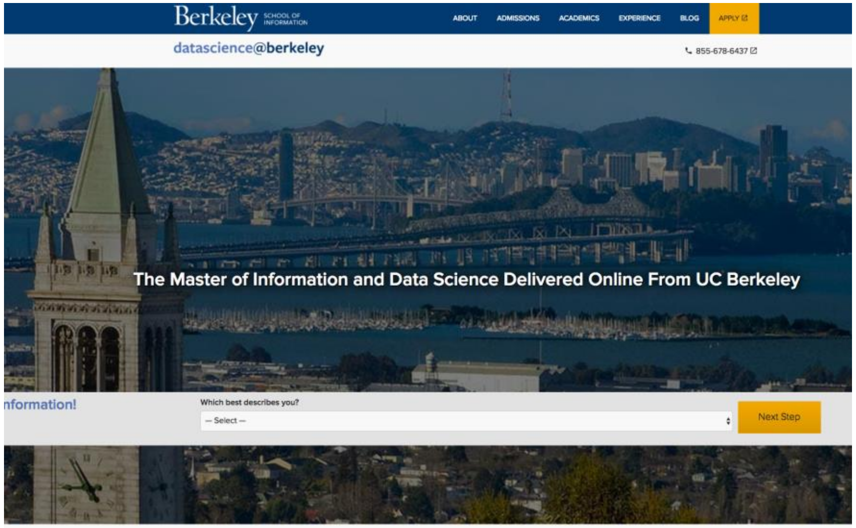
CEE systems vision for UG (and grad) curriculum

Field disciplines
- Environmental Engineering
- Geotechnical Engineering
- Structural Engineering
- Material Science
- Transportation Engineering
- Project Management
- Energy engineering

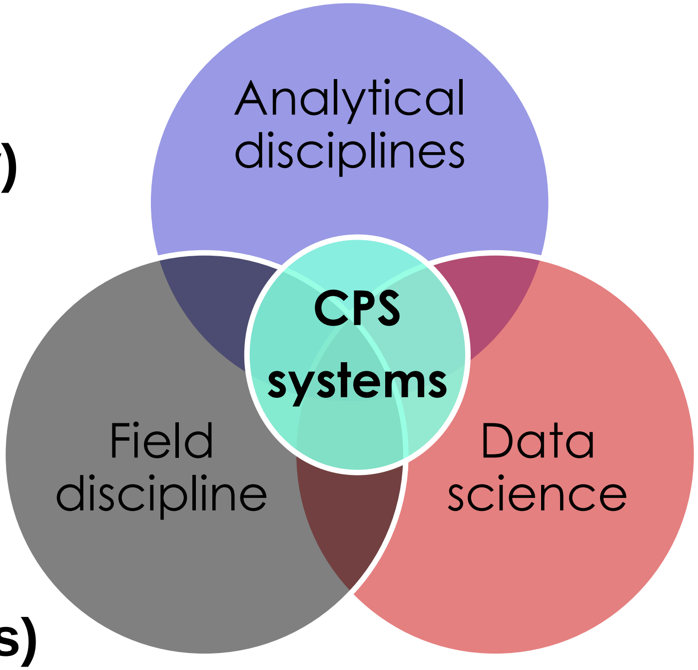


Analytical disciplines
- Mathematical modeling
- Model based control
- Optimization
- Signal processing
- Economics

Data science
- Statistics
- Machine learning
- Programming
- Architecture



Broader questions and CEE links to IDSS



Societal dimensions

- Social and economic networks
- Human models (ex. mobility, energy)
- Markets (ex. energy, transportation)
- Environment (ex. incentivization)
- Infrastructure (ex. electrification)
- Economics (ex. airlines, freight)

Changing landscapes

- Autonomy (incl. flight)
- Electrification
- Shared economy (two-sided markets)
- Rapid urbanization (land use)

Open questions (a few of many)

- Incentivization in networks (ex. transportation)
- Privacy (and security), for ex. in crowdsourcing
- Decision support systems for fully automated districts
- Connection and interactions of networks (ex. energy + water)

CEE systems vision for UG (and grad) curriculum

Field disciplines
- Environmental Engineering
- Geotechnical Engineering
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Analytical disciplines
- Mathematical modeling
- Model based control
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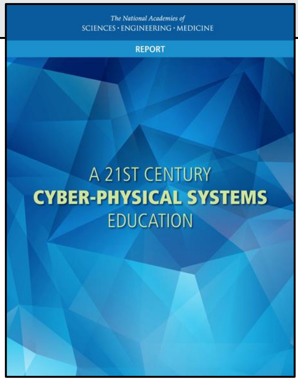
- Signal processing
- Economics

Data science
- Statistics
- Machine learning
- Programming
- Architecture

BOX 3.4
One Model for 4-Year, 40-Course Undergraduate Degree in CPS

Math and Natural Science (10 courses)
- Calculus I and II
- Differential Equations
- Linear Algebra
- Probability and Statistics
- Logic
- Physics I (Mechanics and Dynamics)
- Physics II (Electrical Circuits)
- Chemistry or Biology
- Discrete Math

CPS Core (12 courses)
- Introduction to CPS (Freshman Laboratory Course)
- Computer Programming
- Data Structures and Algorithms
- Programming Physical Systems
- Software Engineering
- Model-Based System Design
- Heterogeneous Models of Computation
- Formal Methods and Synthesis
- Resource-Aware Real-Time Computing
- Control Systems
- Optimization
- Digital Signal Processing



CEE systems vision for UG (and grad) curriculum

Field disciplines

- Environmental Engineering
- Geotechnical Engineering
- Structural Engineering
- Material Science
- Transportation Engineering
- Project Management
- Energy engineering

Analytical disciplines

- Mathematical modeling
- Model based control
- Optimization
- Signal processing
- Economics

Data science

- Statistics
- Machine learning
- Programming
- Architecture

CPS-Related Courses in Current CE Curricula (3 courses)

- Computing for Engineers
- Civil Engineering Systems (needs to be developed with CPS focus)
- Capstone Design (with CPS-focused project)

Technical Electives (6 courses)

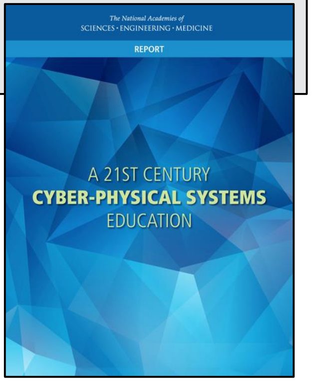
Current CE curricula have few undergraduate elective courses that focus on CPS concepts. If redesigned, some current elective courses could incorporate CPS principles, examples include the following:

- Geographic Information Systems
- Transportation Planning and Design
- Infrastructure Rehabilitation
- Environmental Geotechnology
- Subsurface Characterization
- Environmental Systems Design
- Building Information Modeling
- Conceptual Structural Design
- Computational and Graphical Tools for Structural Engineering
- Structural System Testing and Model Correlation

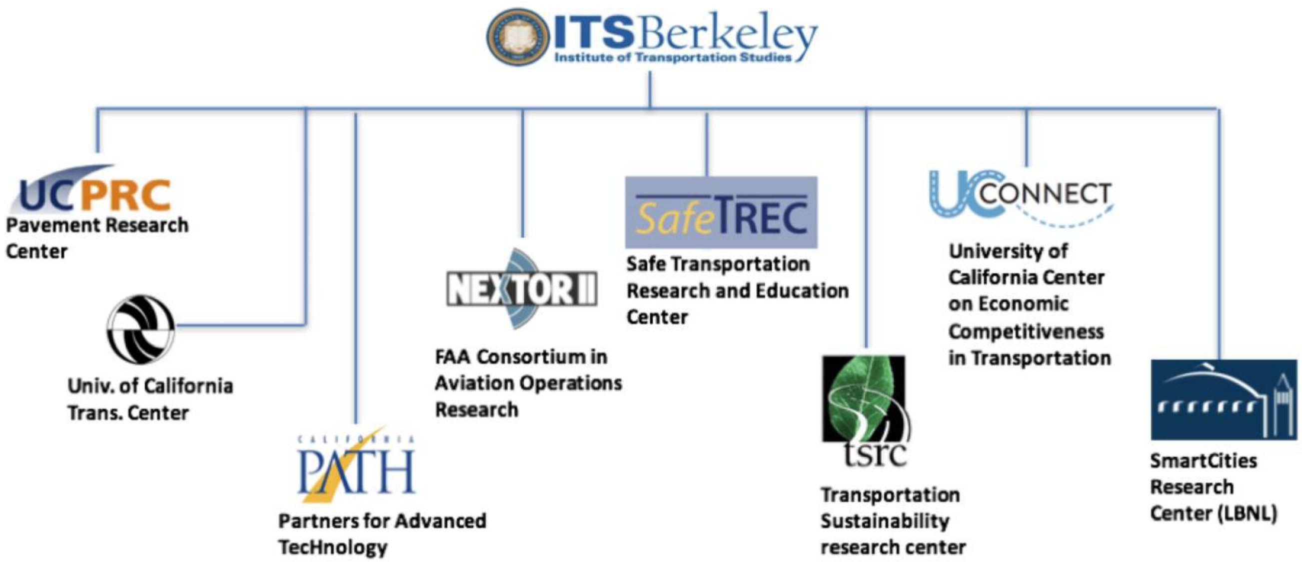
Proposed new CPS-centric electives:[1]

- Principles of CPS: Sustainable Infrastructure
- Principles of CPS: Urban Planning
- Signals and Systems
- Sensor Networks for Civil Engineering Systems
- Model-Based Systems Engineering
- Structural Health Monitoring

Social Science, Economics, Humanities (8 courses)



Links with the Institute of Transportation Studies



7 Centers of research within ITS

$25M Research income each year, leveraged from ~$1M in core funds

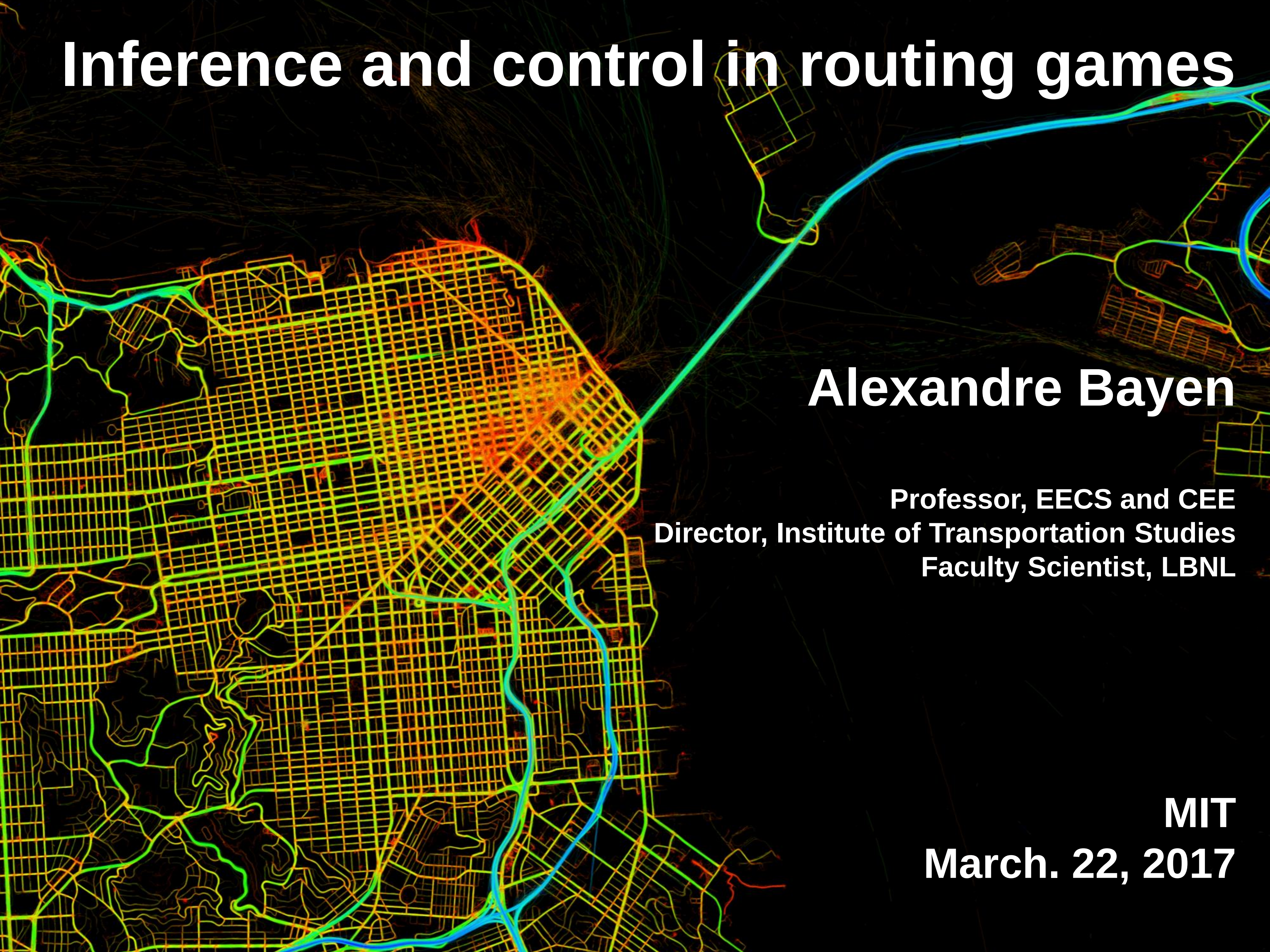
200 Faculty, Staff and Students

10+ Successful start-up companies in the last 5 years

Missions
- Research
- Teaching
- Tech transfer
- Library

Spans
- 7 Departments
- 4 Colleges
- LBNL

Inference and control in routing games



Alexandre Bayen

Professor, EECS and CEE
Director, Institute of Transportation Studies
Faculty Scientist, LBNL

MIT
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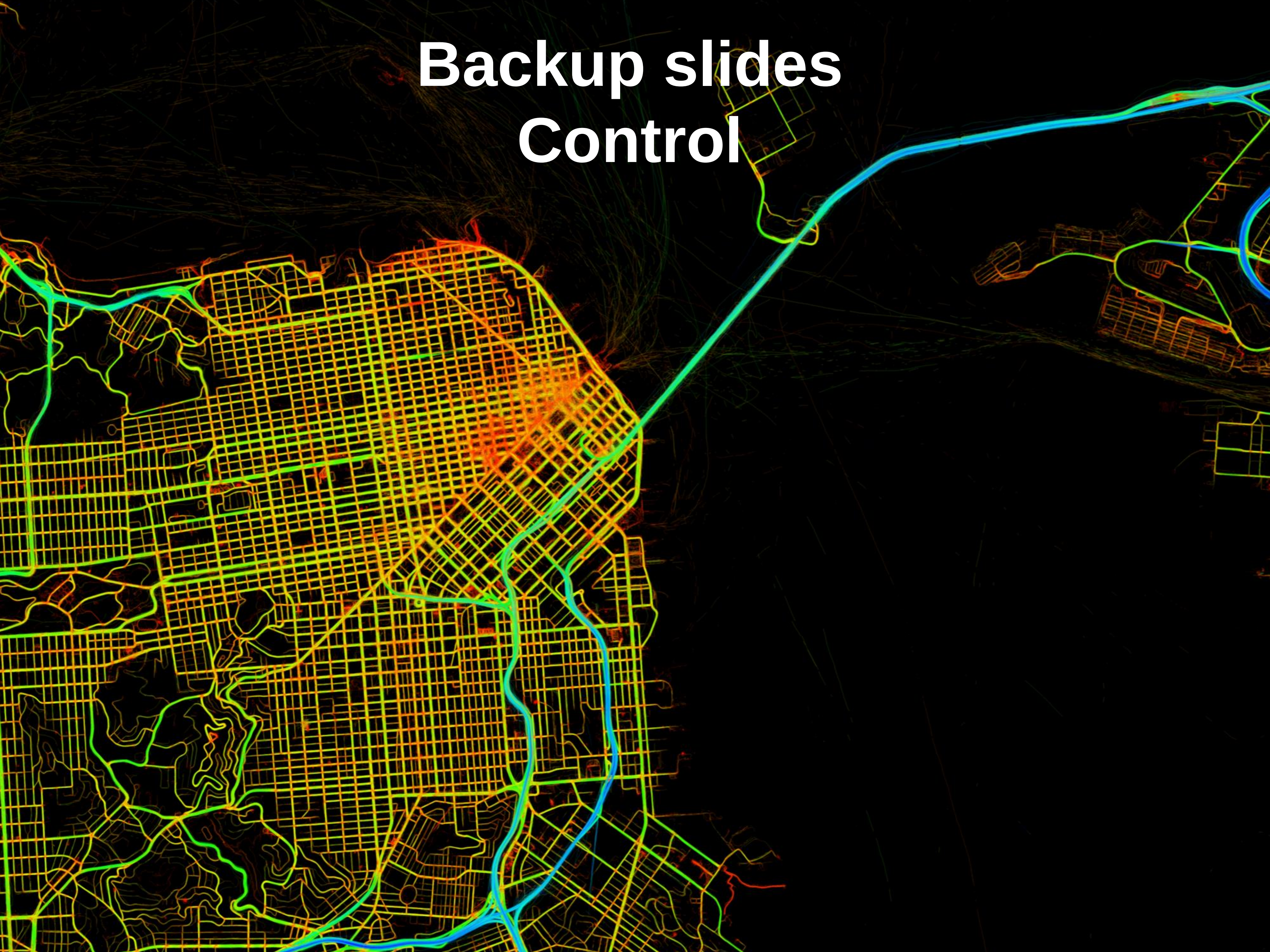
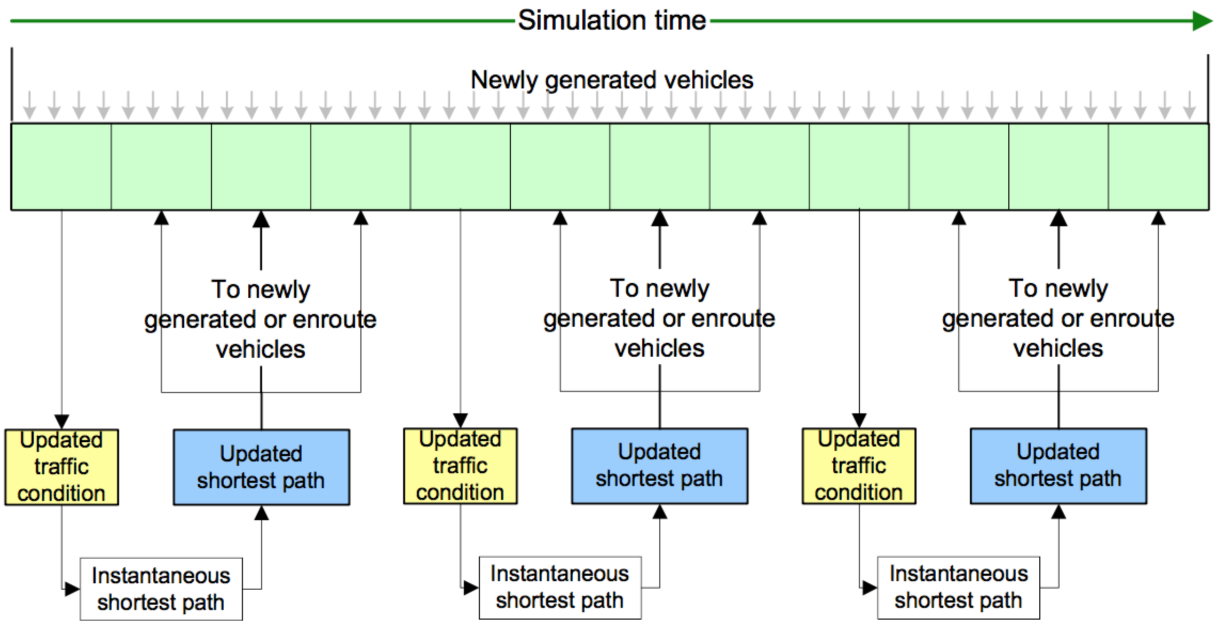
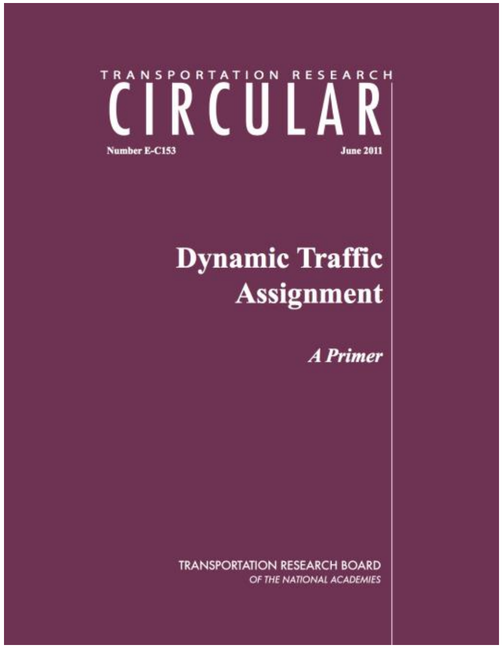
Backup slides
Control



Nonlinear dynamics of DTA





Heterogeneous population

- “Non-app-enabled” users
- App enabled users
  - Some act on info
  - Some do not
- Various versions of shortest time
  - Dijkstra and extensions
  - SOTA
  - Driver preferences
  - Clock update

Travel time forecast

- Historical
- Static (instantaneous)
- Historical (statistical) forecast
- DTA (model based) forecast
- Information aware forecast (i.e. incorporating user’s reaction to information)

Finite-horizon Optimal Control Problem

Model predictive control formulation

- **Nonlinear, nonsmooth, nonconvex optimization program**
- **Objective function: arbitrary velocity target on freeways**
- **Dynamics: LWR PDE, discretized by Godunov scheme**
- **Optimization:**
  - **Adjoint based method**
  - **ADMM**
  - **BFGS**

$$\min_{\mathbf{u}\in U} \underbrace{\sum_{t=1}^{T-1}\sum_{i=1}^{N} f\left(u_{i,t},\rho_{i,t}\right)}_{\text{Running Cost}} + \underbrace{\sum_{i=1}^{N} f_T\left(u_{i,T},\rho_{i,T}\right)}_{\text{Terminal Cost}}$$

subject to system dynamics:

$$\begin{aligned} \rho_{i,0} &= \rho_i^0 \\ \rho_{i,t+1} &= \rho_{i,t} + \frac{\triangle t}{\triangle x}\left(G\left(\rho_{i-1,t},\rho_{i-1,t},u_{i,t}\right) - \right. \\ &\quad \left. G\left(\rho_{i,t},\rho_{i+1,t},u_{i,t}\right)\right) \\ &\quad \forall i \in [1,N], \forall t \in [1,T] \end{aligned}$$

$$\min_{\mathbf{u}\in U} J\left(\mathbf{u},\rho\right)$$

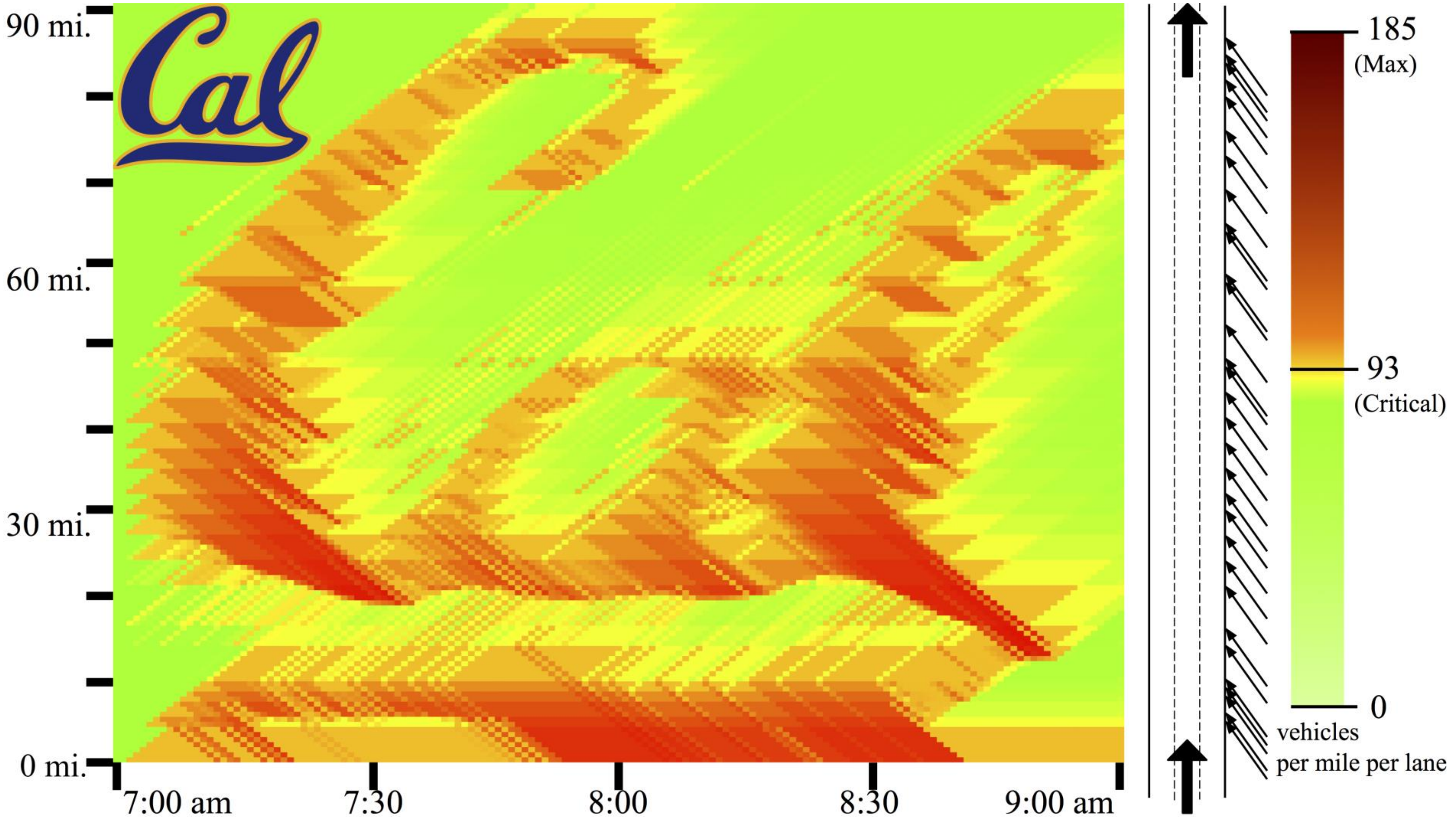
$$\text{s.t. } H\left(\mathbf{u},\rho\right) = 0$$

* **Non-linear**
* **Non-smooth**
* **Non-convex**

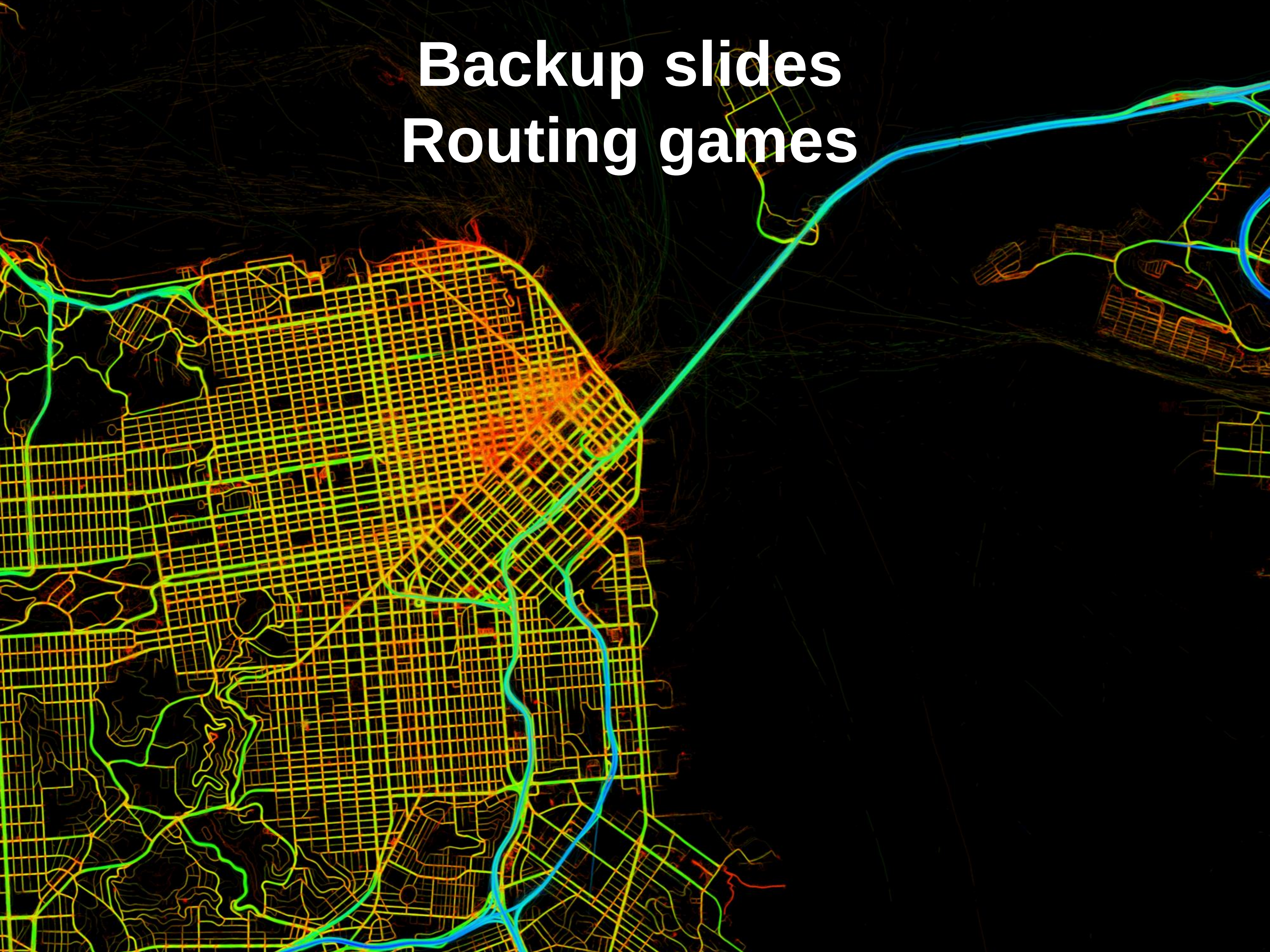
Control

Optimal control problem in freeway operations management

- **Minimize arbitrary cost function with boundary control (inflow at on ramps)**



Backup slides
Routing games



Application to the routing game

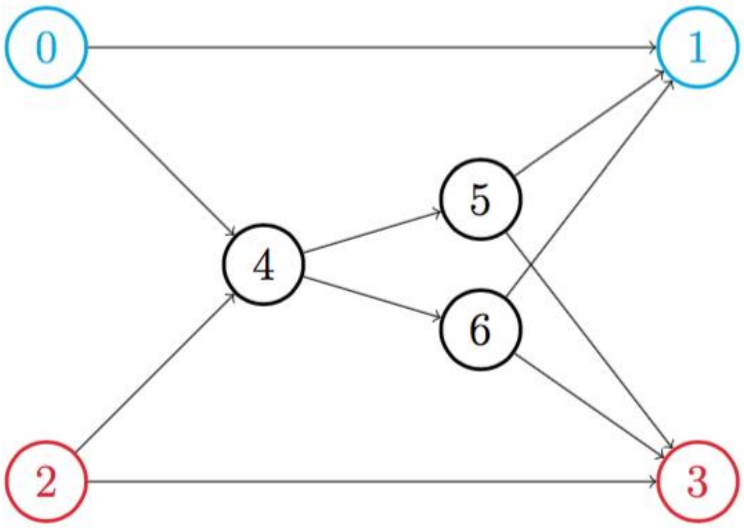
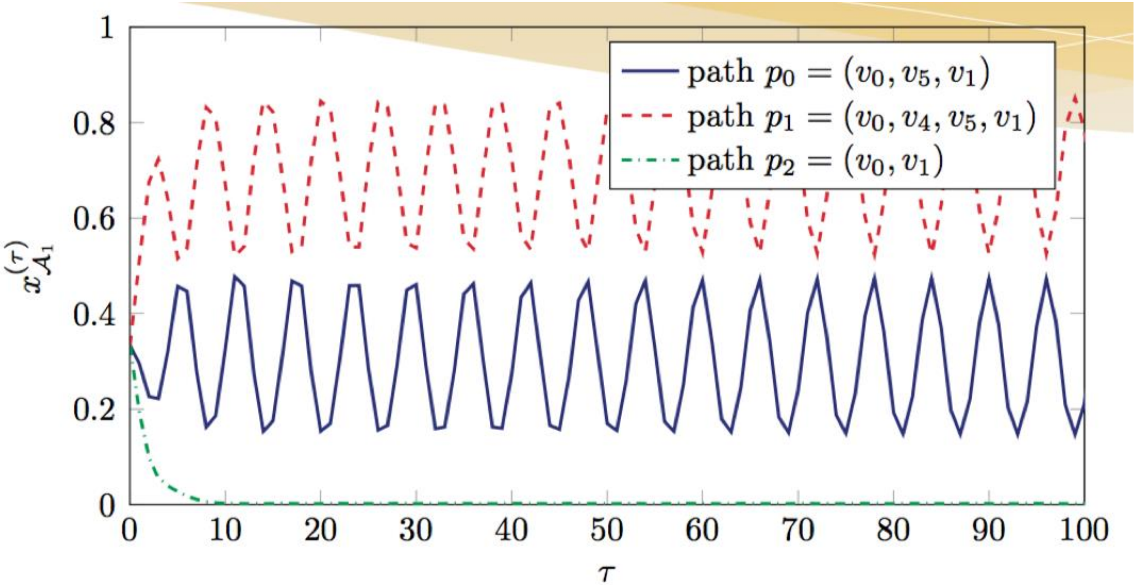


Figure: Example with strongly convex potential.

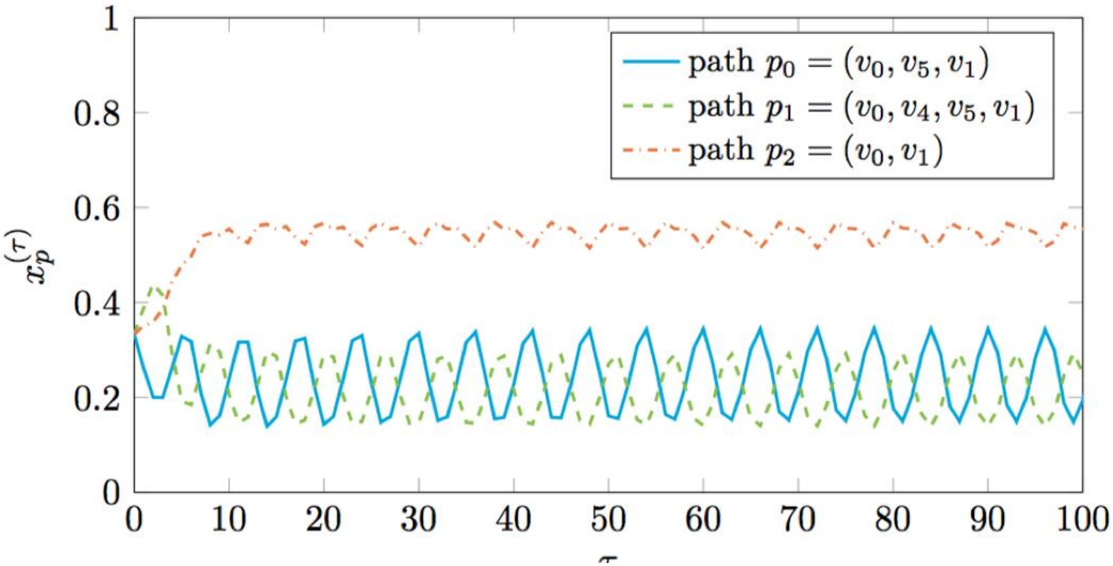
- Population 1: Hedge with $\eta_t^1 = t^{-1}$
- Population 2: Hedge with $\eta_t^2 = t^{-1}$

Convergence on average

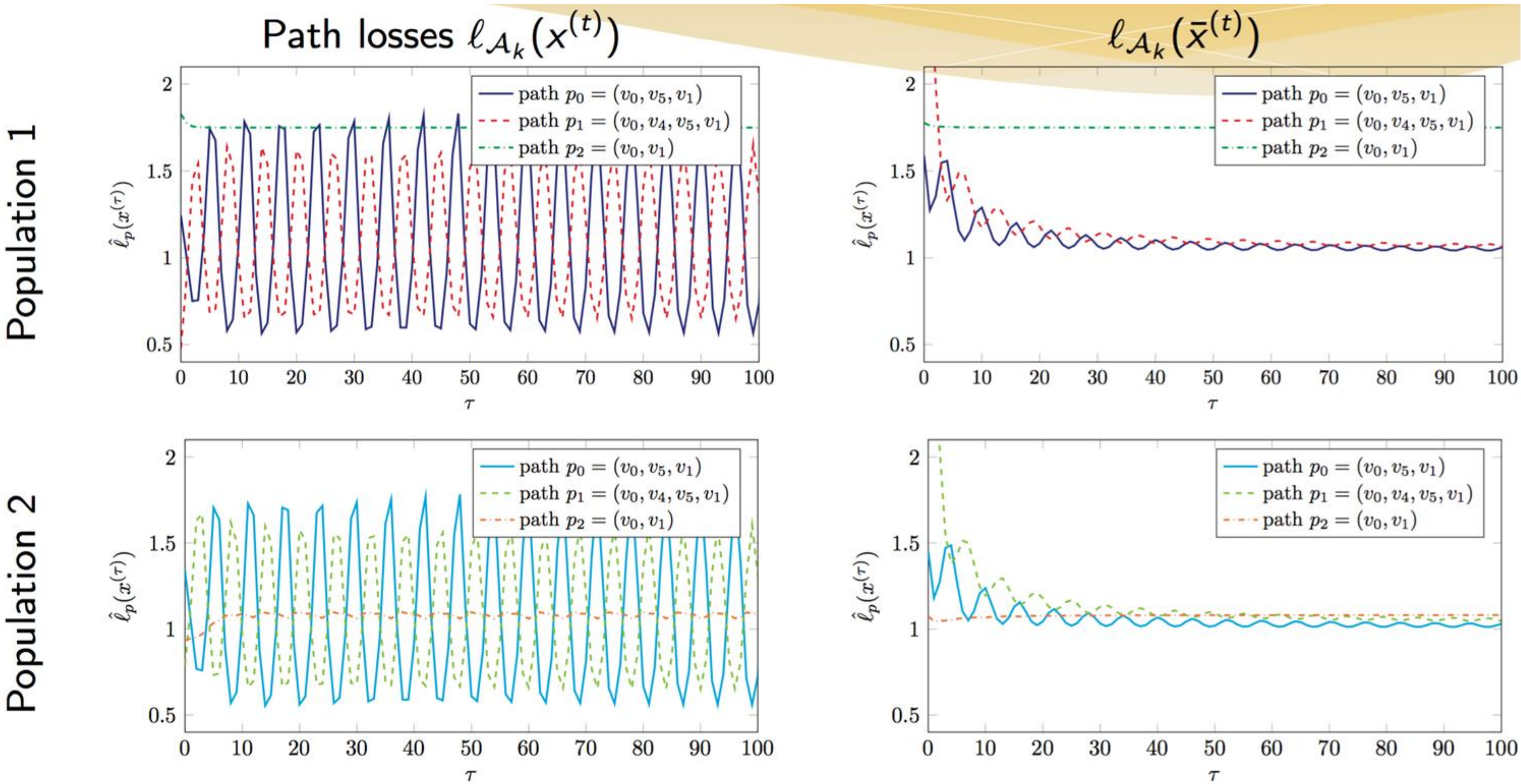
Population 1



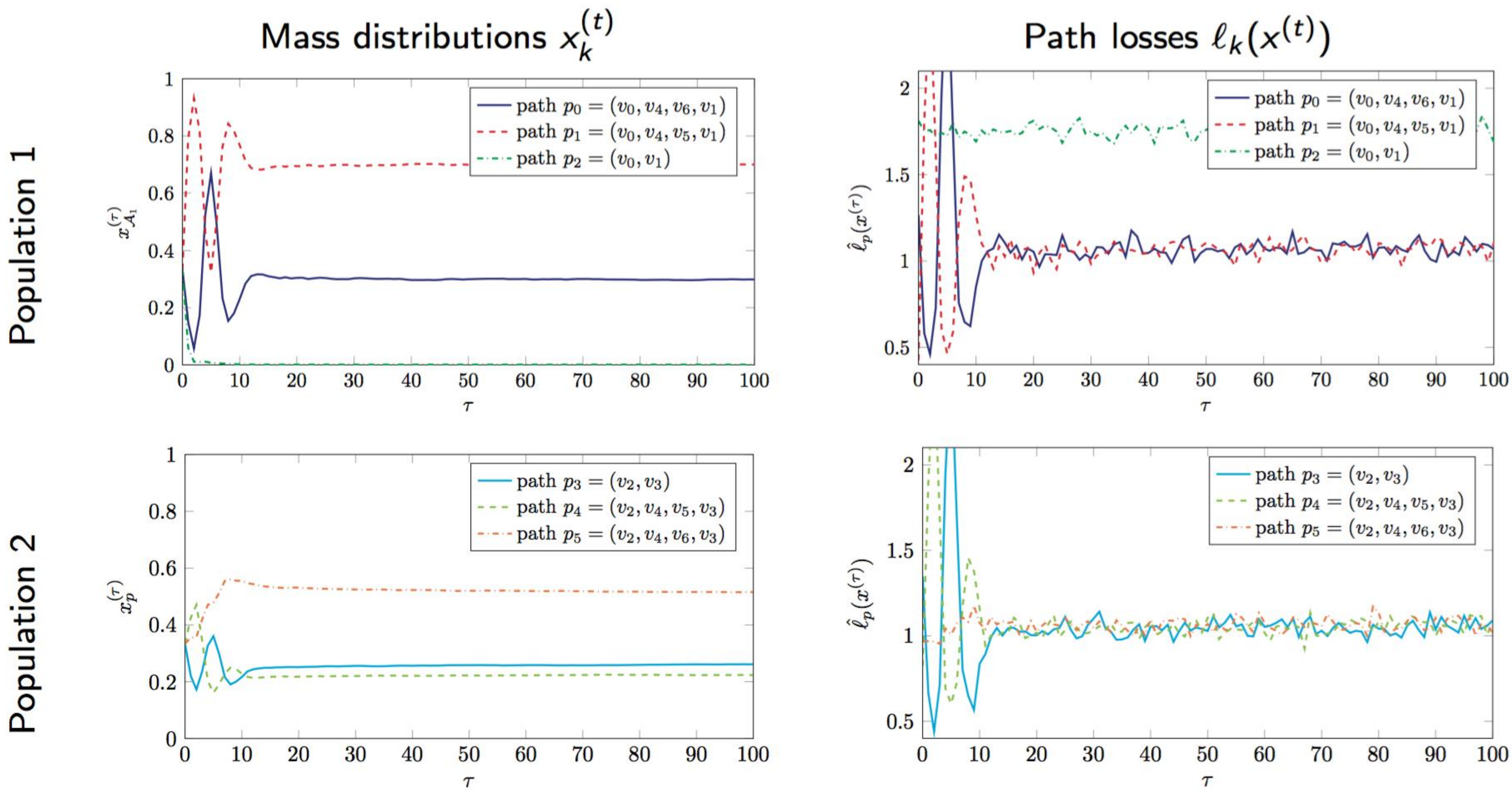
Population 2



Convergence on average



Routing game with strongly convex potential



Routing game with strongly convex potential

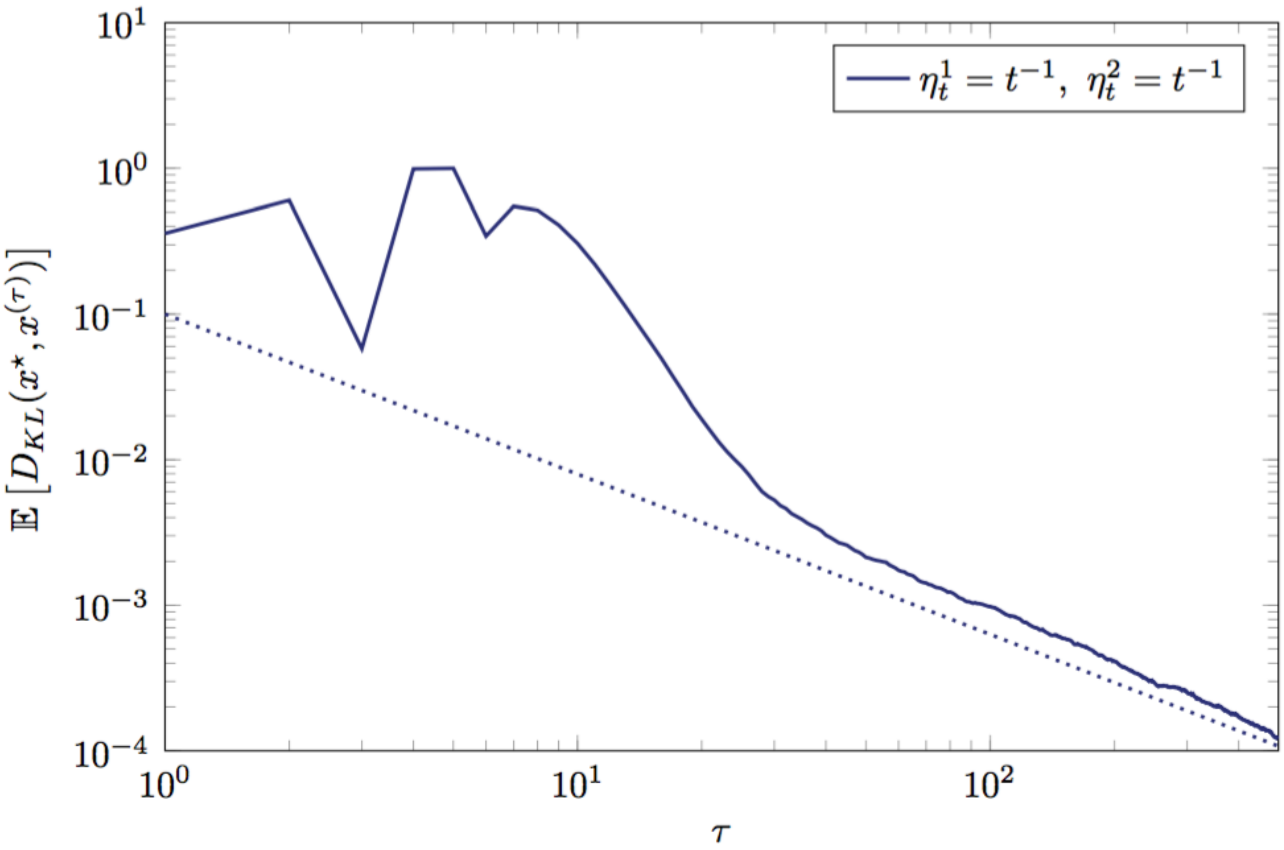


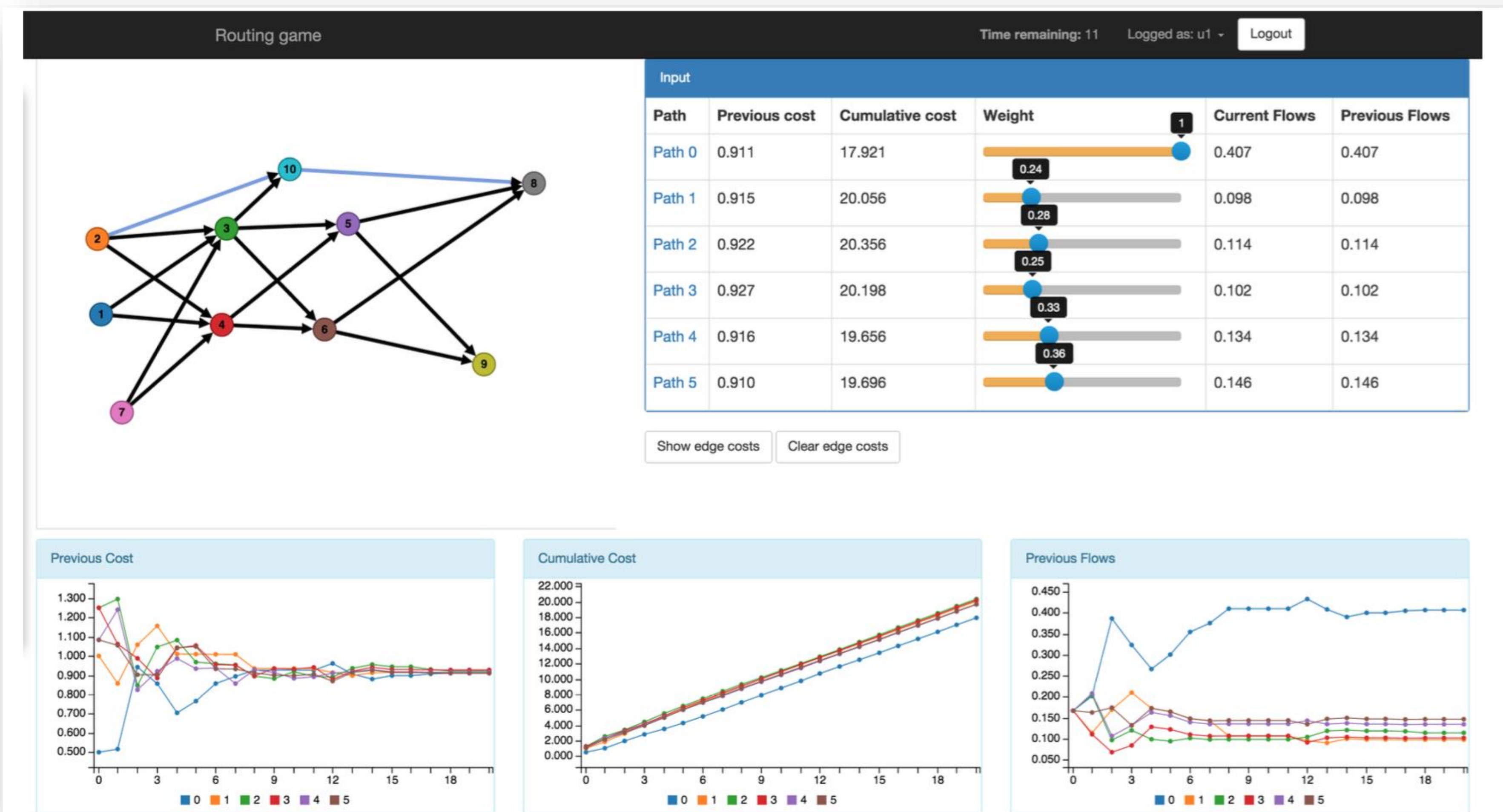
Figure: Distance to equilibrium.

For $\eta_t^k = \frac{\theta_k}{\ell_f t^{\alpha_k}}$, $\alpha_k \in (0,1]$, $\mathbb{E}\left[D_\psi(x^\star, x^{(t)})\right] = O(\sum_k t^{-\alpha_k})$

Practical game implementation: field experiment

Idea of the game: study non-cooperative behavior of routing applications "managers"

- **As if Google was "playing against" Apple, INRIX etc.**
- **Study evolution of distribution over successive iterations**



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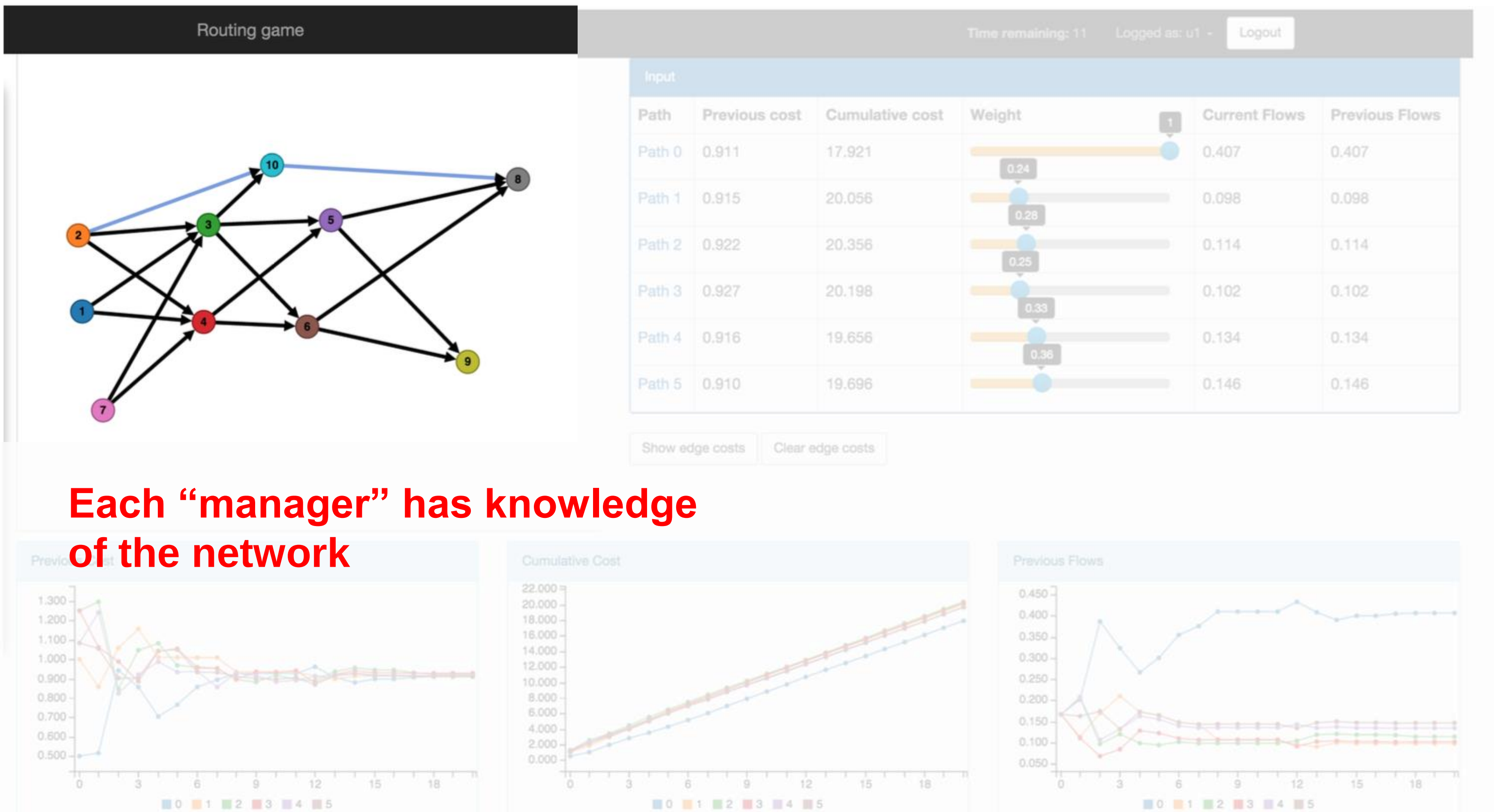

Each "manager" has knowledge of the network

Practical game implementation: field experiment

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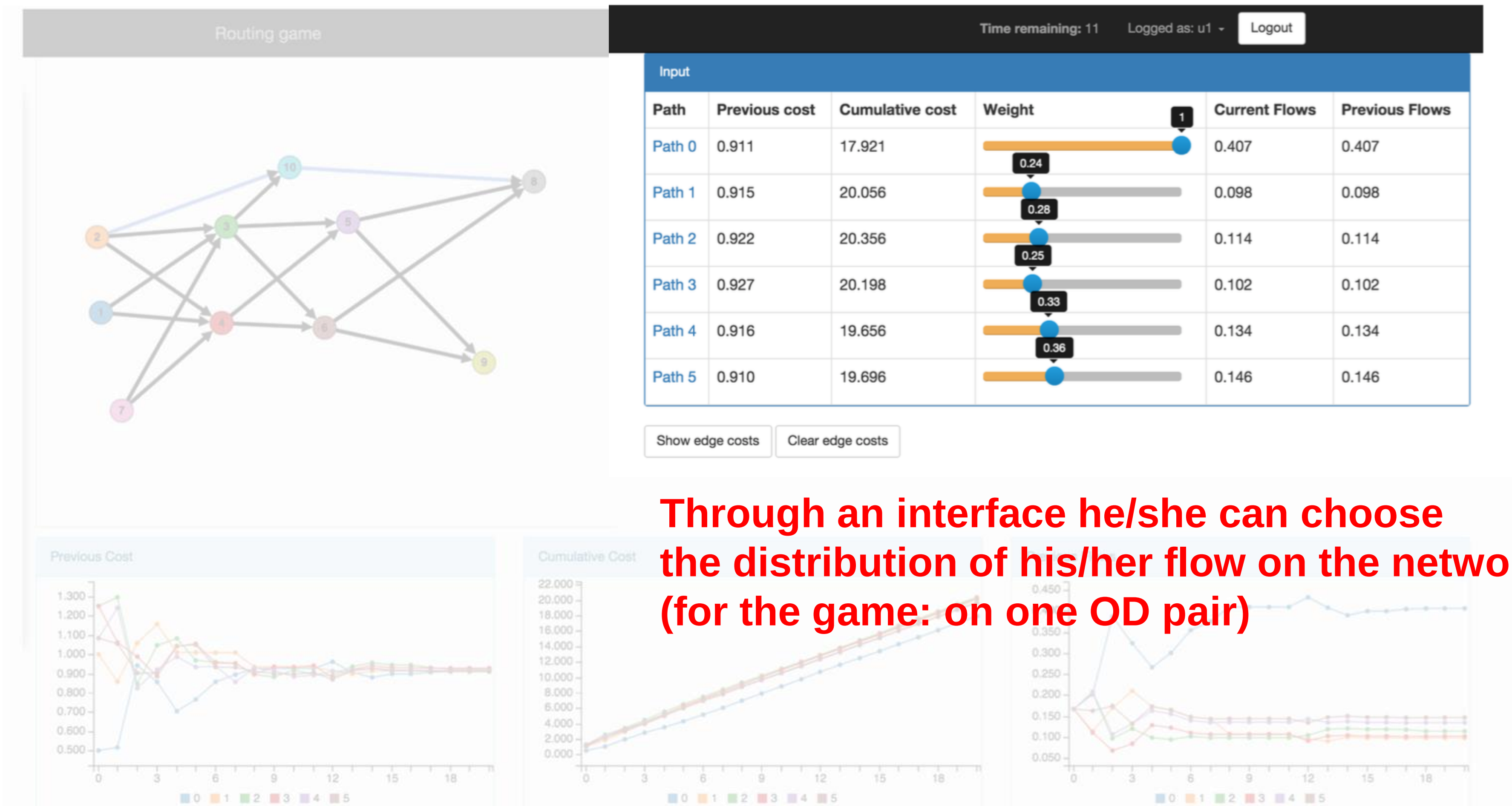

Through an interface he/she can choose the distribution of his/her flow on the network (for the game: on one OD pair)

Practical game implementation: field experiment

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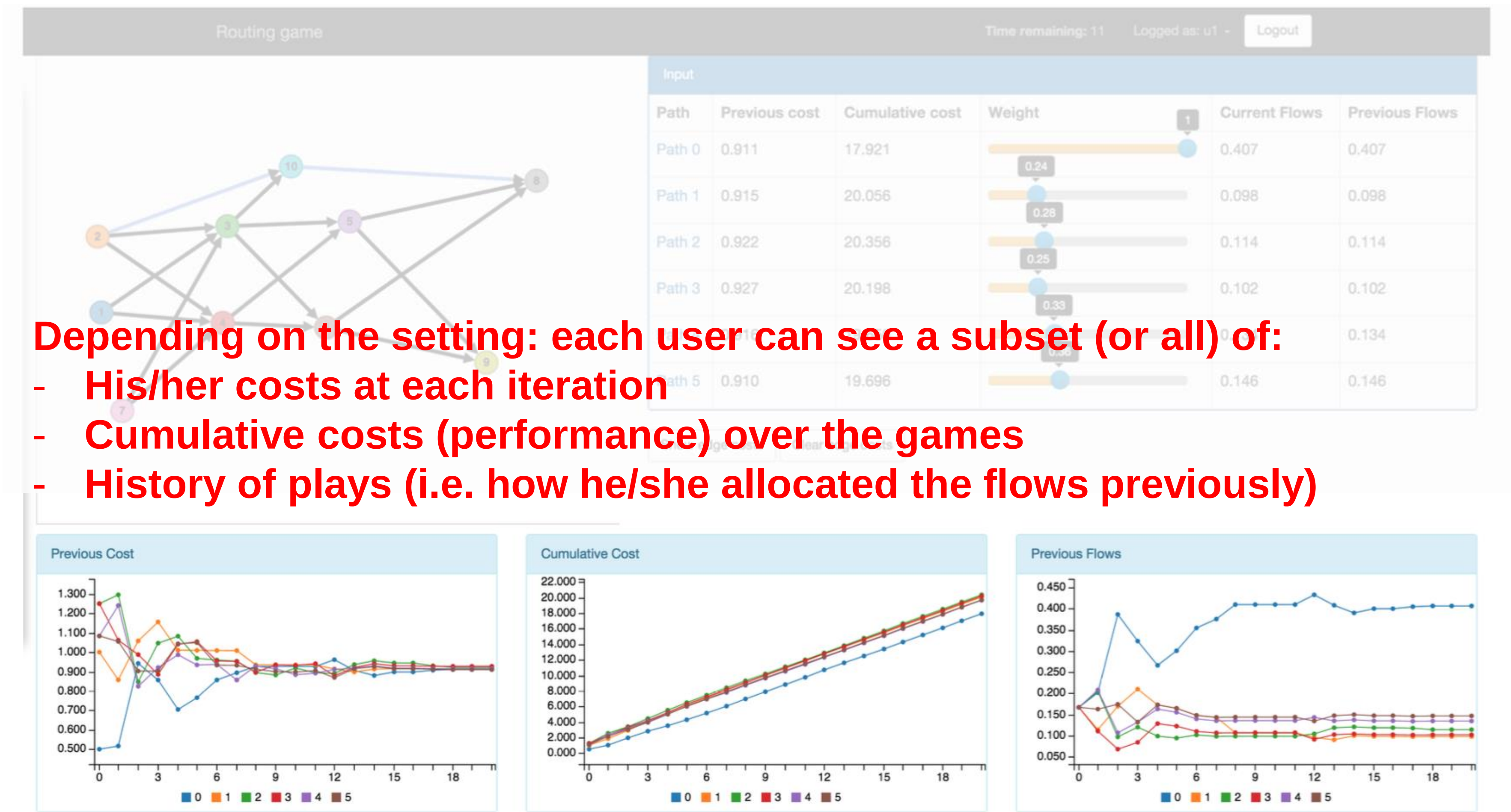
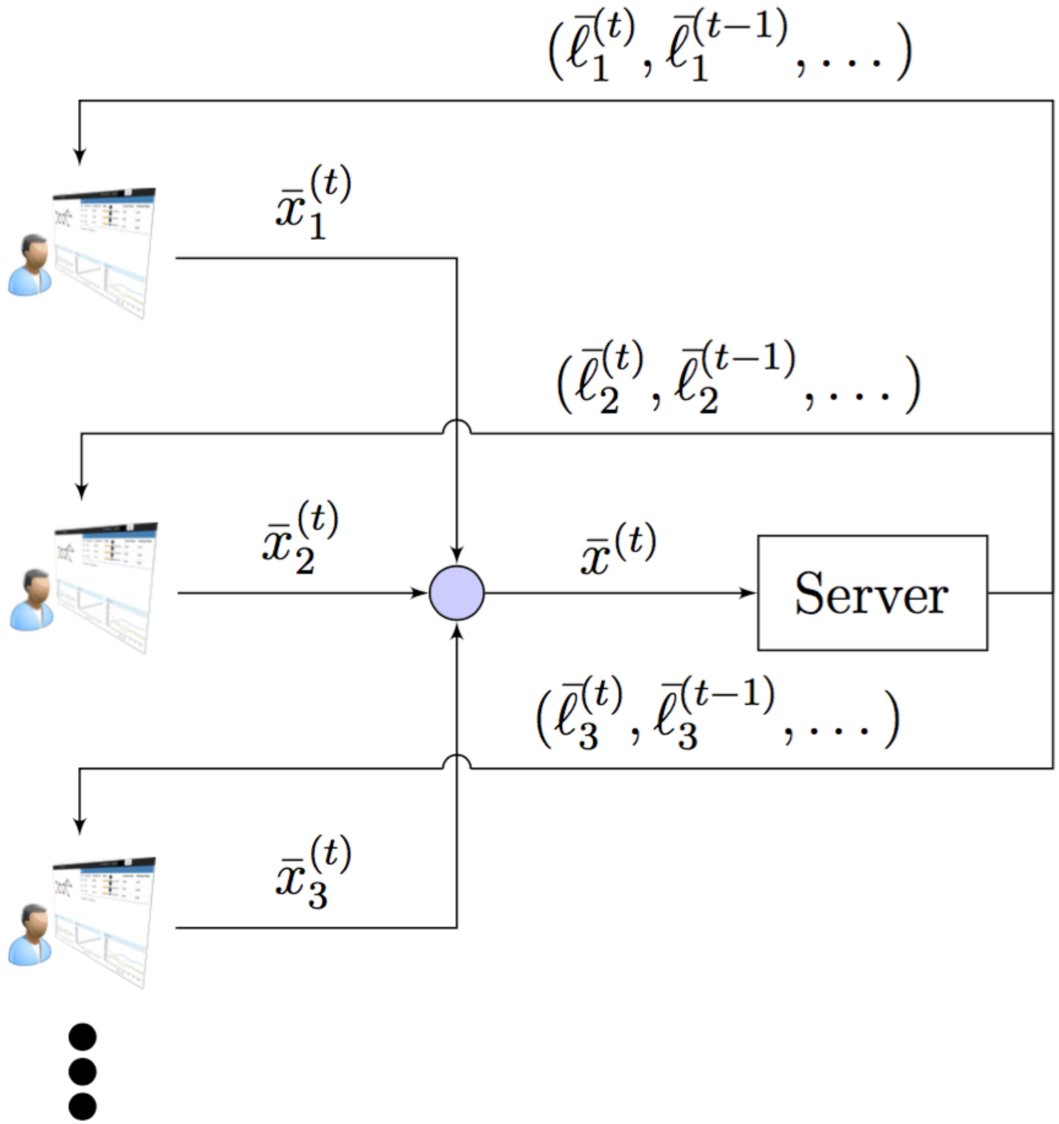


Depending on the setting: each user can see a subset (or all) of:

- **His/her costs at each iteration**
- **Cumulative costs (performance) over the games**
- **History of plays (i.e. how he/she allocated the flows previously)**

Game process



Learning how players learn

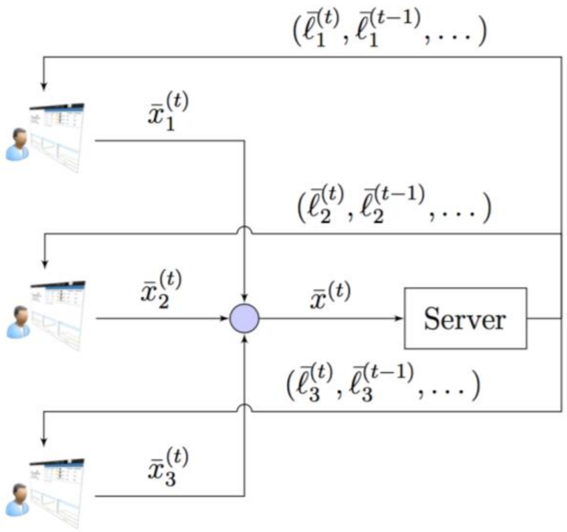
- We observe a sequence of player decisions ($\bar{x}^{(t)}$) and losses ($\bar{\ell}^{(t)}$).
- Can we fit a model of player dynamics?

Mirror descent model

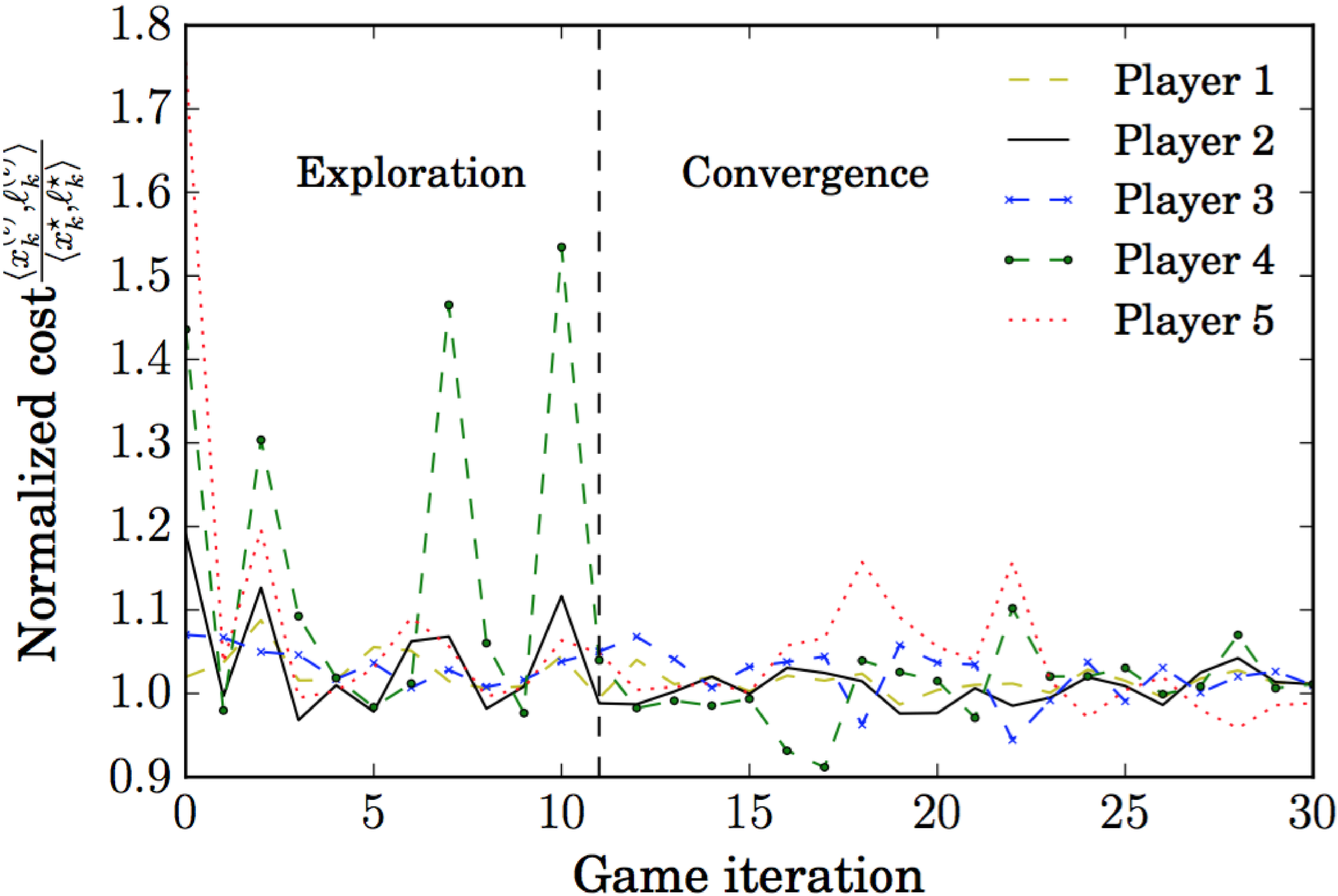
Estimate the learning rate in the mirror descent model

$$x^{(t+1)}(\eta) = \arg\min_{x \in \Delta^{\mathcal{A}_k}} \left\langle \bar{\ell}^{(t)}, x \right\rangle + \frac{1}{\eta} D_{KL}(x, \bar{x}^{(t)})$$

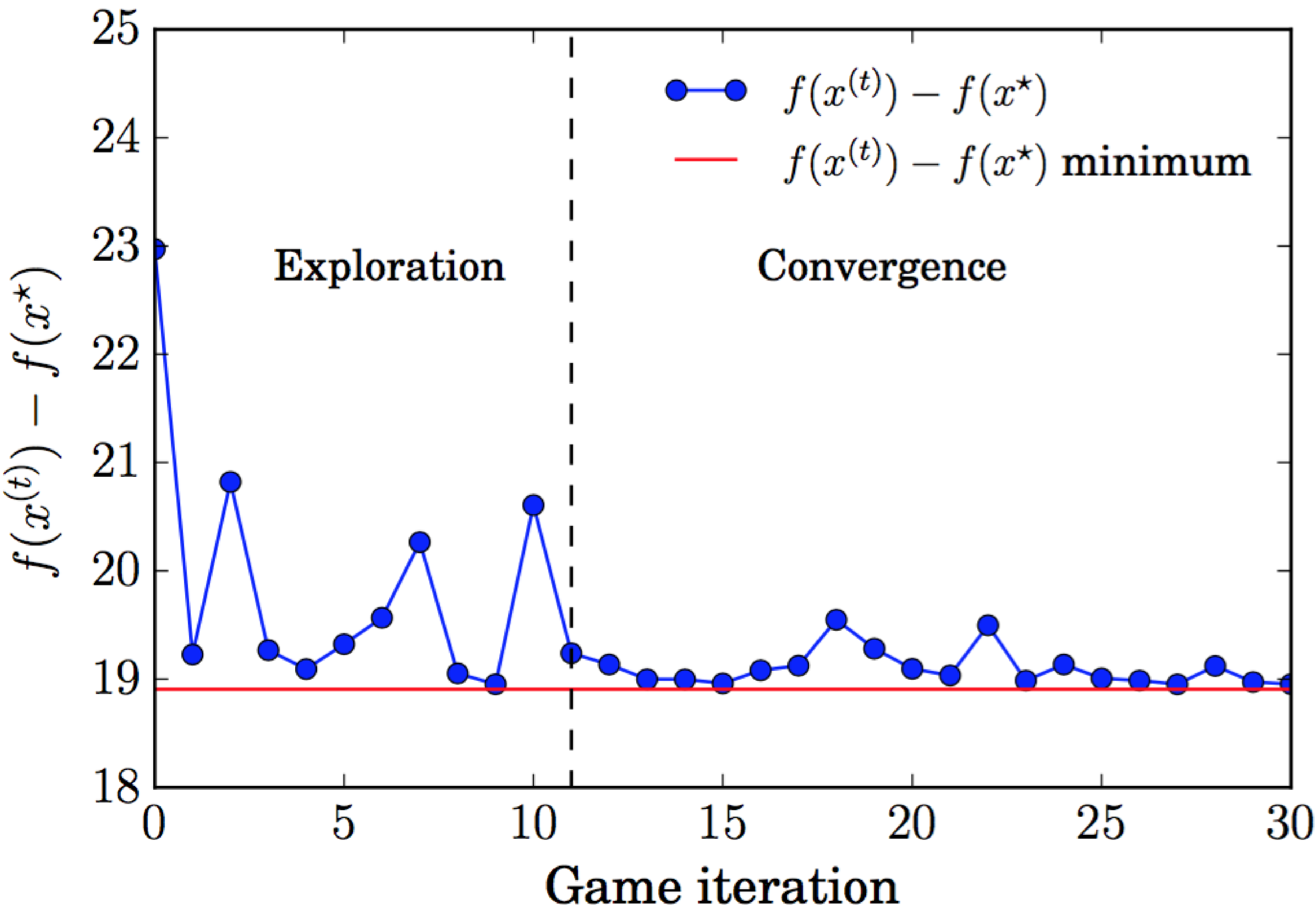
Then $d(\eta) = D_{KL}(\bar{x}^{(t+1)}, x^{(t+1)}(\eta))$ is a convex function. Can minimize it to estimate $\eta_k^{(t)}$.



Cost of each player (normalized by eq. cost)



Potential function $f(x^{(t)}) - f^\star$



Average of KL divergence

Average KL divergence between

- **Predicted distributions**
- **Actual distributions**

As a function of the prediction horizon *h*