Risk-Limiting Dynamic Contracts for Direct and Indirect Load Control

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Variability and uncertainty in power systems

[wikipedia.org]

Renewables
Variability & Uncertainty

→

Grids
frequent generator ramping

Markets
high price volatility & high utility's risk

Proposed solution
demand-side flexibility & information technology

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Motivation 1: Utility perspective

Distributed Renewables

Variability & Uncertainty

↓

Volume Risk

large real-time purchase

+

Price Risk

high price volatility

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Utility's financial risk is high

Question: Can we develop retail tariffs (contracts) that mitigate utilities' financial **risks** in real-time markets?

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Motivation 2: Customer perspective

Real-Time Pricing (RTP)

pass wholesale prices to customers

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high renewable production	+	low renewable production
low wholesale price		high wholesale price

↓

(+) balancing out renewables variability

(-) direct transfer of risks to customers

Question: Can we develop retail tariffs (contracts) that capture the benefits of RTP, but also manage customers' financial **risks**?

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high renewable production — low wholesale price

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low renewable production — high wholesale price

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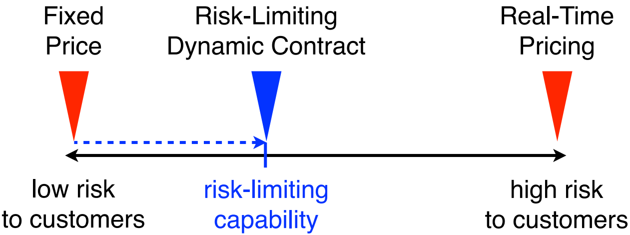
Risk-Limiting Dynamic Contracts: Towards financial risk-sharing



Key Idea: Direct/indirect load control + Contract

Goal:

- Capture the benefits of real-time pricing
- But also manage concerns over financial risks (measure: variance)

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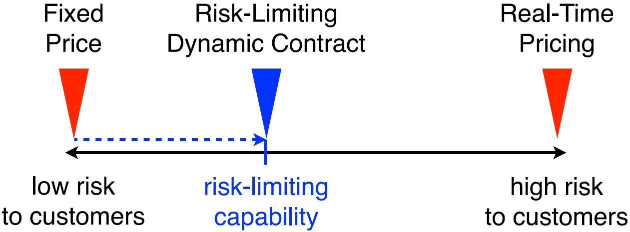

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Relevant Work

Prior work: Electricity-specific demand management work

- (In)direct load control based on physical models: peak demand reduction, energy arbitrage, ancillary services [much activity here]
- Energy contracting work based on demand-side risk-sharing capability [Kaye, Outhred, Bannister, 1990], [Chao, Wilson, 1987], [Tan, Varaiya, 1993], [Bitar, Low 2012]

Our work does risk management, but with physical load models

Prior work: "Dynamic contracts" or "Principal-Agent problems" that specify a compensation scheme and a control strategy [Cadenillas, Cvitanić, Zapatero, 2007], [Sannikov, 2008]

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- requires customer's utility function

Our work: "variance constraint (risk-limiting capability)" + "dynamic programming (flexibility)"

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Motivation/Goal — "Renewables friendly" retail tariffs

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- **Approach**: (In)direct load control for risk management
- **Framework**: Dynamic contracts w/ risk-limiting capability
- **Theory**: DP-based sol'n to mean-var constrained-stochastic control

Demonstration using data (Austin, Texas)

at least 50% mitigation of utility's risk

- beneficial to both utility and customers
- works well under both flat and real-time pricing retail tariffs

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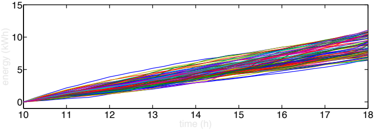
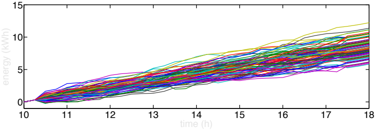
Setting: Demand-side models

- u_t^i: power consumption by customer i's load in DLC/ILC
- $l_i(t)$: forecast of customer i's loads other than u_t^i

Customer energy consumption: e_t^i is the total energy consumption up to time t by customer i

$$de_t^i = (l_i(t) + u_t^i)dt + \tilde{\sigma}_i(t)dW_t^i$$





(more validation to come)

Load state dynamics:

$$dx_t^i = f_i(x_t^i, u_t^i)dt$$

Example: First-order temperature dynamics for air conditioning

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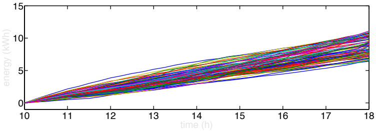

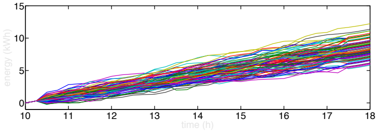

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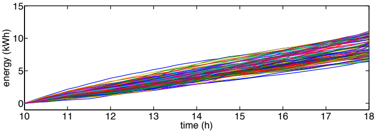

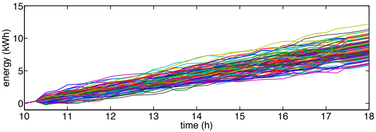

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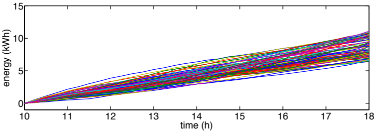

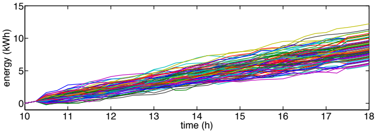

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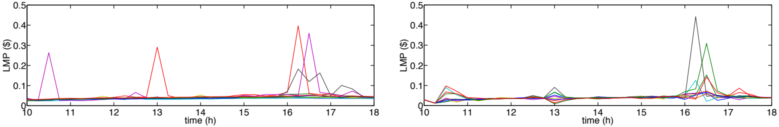
Setting: Electricity price dynamics

- λ_t: energy price in the real-time market

- mean-reverting model [Deng, Johnson, Sogomonian, 2001], [Kamat, Oren, 2002]

$$d\lambda_t = r_0(\nu(t) - \ln \lambda_t)\lambda_t dt + \sigma_0(t)\lambda_t dW_t^0$$

- data (ERCOT LMP) vs. identified model



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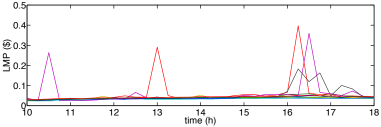

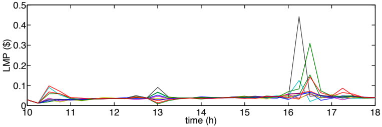

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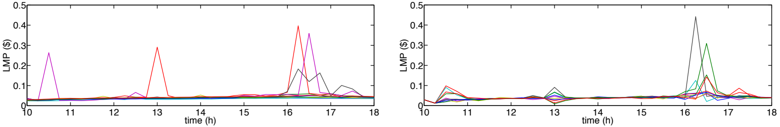

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Setting: Utility payoff function

- C^i: compensation paid to customer i
- $\mu_i(t)$ (retail price) could be time varying, but not necessary

Utility ('P'rincipal) payoff =
(net revenue) + (customer comfort level) − (compensation payment)

$$J^P[C,u] := \sum_{i=1}^{n} \left(\int_0^T \underbrace{\mu_i(t)[(u_t^i + l_i(t))dt + \tilde{\sigma} dW_t^i]}_{\text{revenue from retail customer}} + \underbrace{r_i(x_t^i, u_t^i)dt}_{\text{customer ``comfort''}} + \int_0^T \underbrace{\lambda_t[(p_i(t) - (u_t^i + l_i(t)))dt - \tilde{\sigma} dW_t^i]}_{\text{cost of excess procured power for customer } i} - C^i \right)$$

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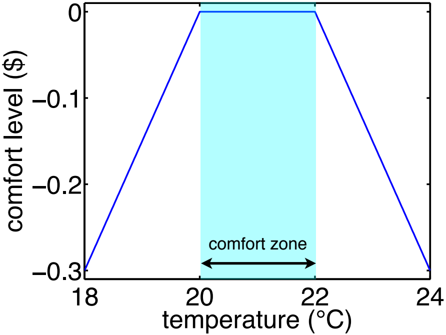
Customer comfort level



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Proposed process: Risk-Limiting Dynamic Contracts

Contract: $(C^i, \{u_t^i\}_{0 \le t \le T})$ (Note: closed-loop (feedback) strategies)

1. Each customer offered a contract menu (= a set of (b_i, S_i)):
   - Participation payoff condition

$$\mathbb{E}[\underbrace{J_i^A[C^i, u^i]}_{\text{customer's payoff}}] \ge b_i$$

   - Risk-limiting condition (risk measure: variance)

$$\text{Var}[\underbrace{J_i^A[C^i, u^i]}_{\text{customer's payoff}}] \le S_i$$

2. The utility does the following on a daily basis (period T):
   - Builds load model
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Contract: $(C^i, \{u_t^i\}_{0 \le t \le T})$ (Note: closed-loop (feedback) strategies)

1. Each customer offered a contract menu (= a set of (b_i, S_i)):
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- Utility's risk management: risk-sensitive control

$$\max_{C,u} \quad -\frac{1}{\theta}\log \mathbb{E}[\exp(-\theta \underbrace{J^P[C,u]}_{\text{utility's payoff}})]$$

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- Utility's risk management: stochastic Stackelberg differential game

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The theoretical problems

How to solve mean-variance constrained risk-sensitive control (design C^i and u^i for direct load control)?

- Variance inequality constraint
- Stochastic maximum principle: local solution

How to solve mean-variance constrained Stackelberg differential game (design C^i and $u^{r,i}$ for indirect load control)?

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High-level description of proposed solution method

1. The risk-limiting condition (variance)
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   (Intuition) $\gamma_t^{i,1}$: portion of price risk passed through to customer
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Reformulated problem

$$\max_{u,\gamma,\zeta} \quad -\frac{1}{\theta}\log \mathbb{E}\left[\exp(-\theta \bar{J}^P[u,\gamma,\zeta])\right]$$

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\text{subject to} \quad & d\lambda_t = r_0(\nu(t) - \ln \lambda_t)\lambda_t dt + \sigma_0(t)\lambda_t dW_t^0 \quad \text{(Price)} \\
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& v_0^i = b_i \qquad \text{(Participation-payoff)} \\
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Theorem (Optimality)

Let (u^*, γ^*, ζ^*) be the solution to the reformulated problem. Define

$$C^{*i} := v_T^{*i}.$$

Then (C^*, u^*) is an optimal risk-limiting dynamic contact.

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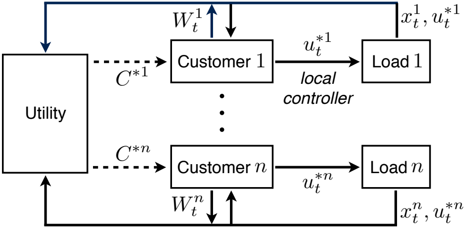
Practical implementation: decentralized control + central monitoring



- Smart meter, smart thermostat
- Low-latency data connection, broadcast of wholesale price
- Local controller programmed with u^i
- Direct: opt-out
  Indirect: opt-out opt-in, no monitoring of control and temperature

Numerical experiments: data and setting

Data

- customer's energy consumption (without air conditioning) model (data: Austin, Texas, Jun. – Sep. 2013)
- energy price model (data: ERCOT, locational marginal price in Austin, Jul. 1 – Jul. 10, 2013)
- outdoor temperature profile (data: Austin, Texas, Jul 5, 2013)
- air conditioner parameters from PNNL
- contract period: $[10\text{am}, 6\text{pm}]$

Options for baseline customer retail tariff:

1. Flat (not time-varying)
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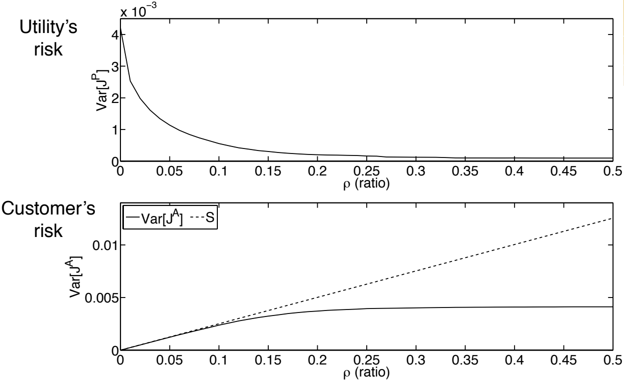

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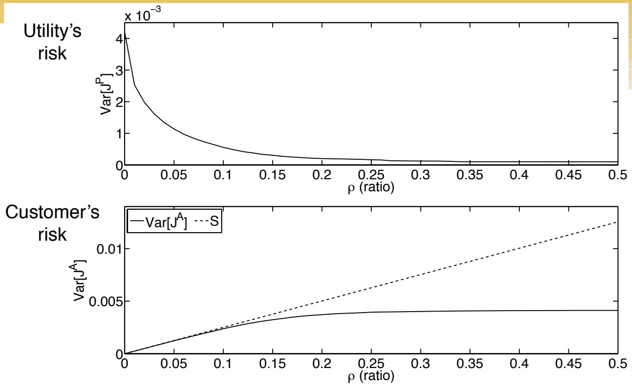

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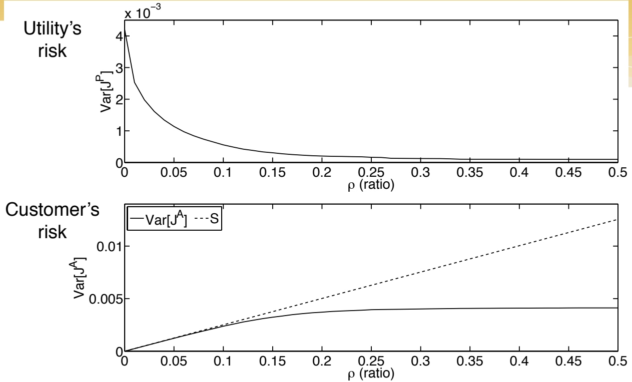

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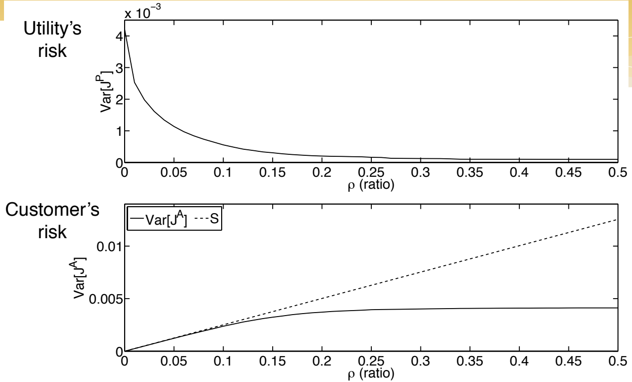

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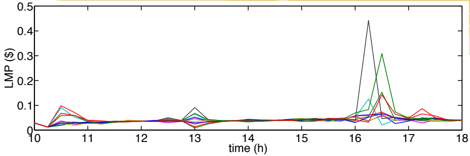
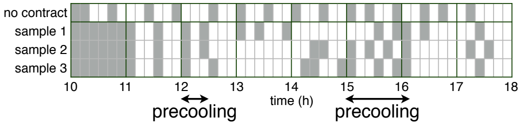
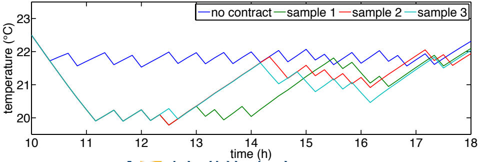
Risk management: price volatility

- wholesale price is highly volatile from 4pm to 5pm



- control w/o contract: no precooling
  control w/ contract: precooling from 3pm to 4pm





Conclusions and Future Work

Conclusions & Contributions:

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- New dynamic contract frameworks with risk-limiting capability
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- System operators' long-term benefits
- Emergency DR contracts (grid resilience)
- Optimal dispatch for risk-limiting dynamic contracts

Manuscript: Yang, Callaway, Tomlin, arXiv:1409.1994 [math.OC]

Conferences: ACC 2014, Allerton 2014 (invited), ACC 2015 (invited)

Working proposal: pilot project with San Diego Gas & Electric

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Conferences: ACC 2014, Allerton 2014 (invited), ACC 2015 (invited)

Working proposal: pilot project with San Diego Gas & Electric

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Conclusions and Future Work

Conclusions & Contributions:

- New demand-side solutions for financial risk management
- New dynamic contract frameworks with risk-limiting capability
- New DP-based solution methods for mean-variance constrained risk-sensitive control

Ongoing & Future Work:

- Real-world experiments: opt-out electricity tariffs with the proposed contracts (San Diego Gas & Electric)
- System operators' long-term benefits
- Emergency DR contracts (grid resilience)
- Optimal dispatch for risk-limiting dynamic contracts

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Thank you

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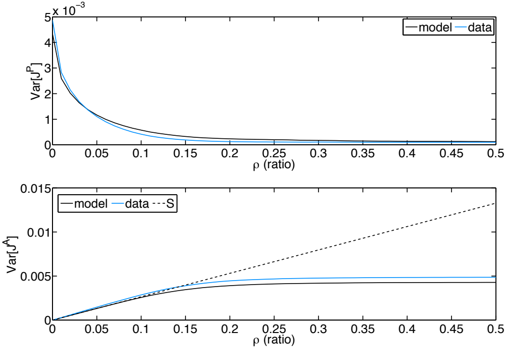
Validation of Brownian motion model using data

Robustness of the proposed contract with respect to the deviation of the demand forecast errors in the data from the Brownian motion model

- Execute the optimal contract over actual load data
- Mean deviation in utility's and customer's payoffs: 0.01%
- No violation of the risk-limiting condition for $\rho > 0.14$; Violation $< 12\%$ for $\rho < 0.14$



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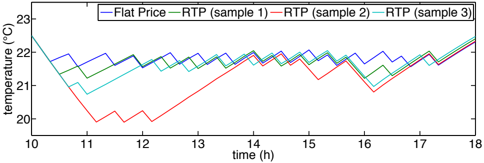
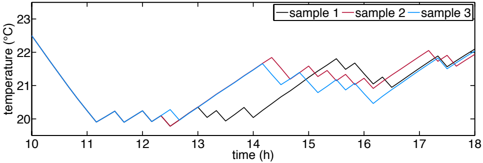
Real-time pricing in retail tariffs

- RTP: $\mu(t) = \lambda_t + \mu_0$ (wholesale price + T&D charge)
- Customer's optimal control with no contracts:



- Optimal control under the contract:



More shifting under the contract

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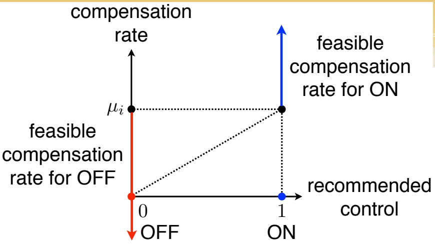
More on indirect load control mechanism



- Recommendation: ON $\Longrightarrow$ compensation rate for (actual load − forecast) greater than retail rate
  - Customer's choice: OFF $\Longrightarrow$ (actual load − forecast) decreases $\Longrightarrow$ total energy bill decreases
- Recommendation: OFF $\Longrightarrow$ compensation rate for (actual load − forecast) lower than retail rate
  - Customer's choice: ON $\Longrightarrow$ (actual load − forecast) increases $\Longrightarrow$ total energy bill decreases

Variance on DP

- U: nonlinear function

Objective functions: $\mathbb{E}[U(x(T))]$ vs $U(\mathbb{E}[x(T)])$

1. $\mathbb{E}[U(x(T))]$: DP is applicable due to "smoothing property"

$$\mathbb{E}[\mathbb{E}[U(x(T))|\mathcal{F}_m]|\mathcal{F}_n] = \mathbb{E}[U(x(T))|\mathcal{F}_n] \quad \forall n \leq m.$$

2. $U(\mathbb{E}[x(T)])$: no analogous relation such as

$$\mathbb{E}[U(\mathbb{E}[x(T)|\mathcal{F}_m])|\mathcal{F}_n] = U(\mathbb{E}[x(T)|\mathcal{F}_n]).$$

Risk-limiting compensation

Theorem (Construction of compensation)

Fix $u^i \in \mathbb{U}^i$. The risk-limiting condition

$$Var\left[J_i^A[C^i, u^i]\right] \leq S_i$$

holds if and only if there exists a unique (up to set of measure zero) $\gamma^i \in \Gamma^i$ such that

$$C^i = \mathbb{E}[J_i^A[C^i, u^i]] - \int_0^T r_i^A(u_t^i, x_t^i)dt - \int_0^T \sigma_i^A(t)dW_t^i + \int_0^T \gamma_t^i dW_t^{(i)}$$

and

$$\mathbb{E}\left[\int_0^T (\gamma_t^i)^2 dt\right] \leq S_i.$$

Remarks

- Risk-limiting condition $\iff$ an expected budget constraint on γ^i
- Design of C^i $\iff$ design of γ^i

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Risk-limiting dynamic contract design (continued)

Theorem (Optimality)

Let (u^*, γ^*, ζ^*) *be the solution to the reformulated problem. Define*

$$C^{*i} := v_T^{*i}.$$

Then (C^*, u^*) *is an optimal risk-limiting dynamic contact.*

Remark:

- Approximate decomposition to n lower dimensional problems: Scalability
- Solution method: dynamic programming for stochastic target constraints