NSF-CPS 1239366 A CPS Framework for Remedial Action Schemes

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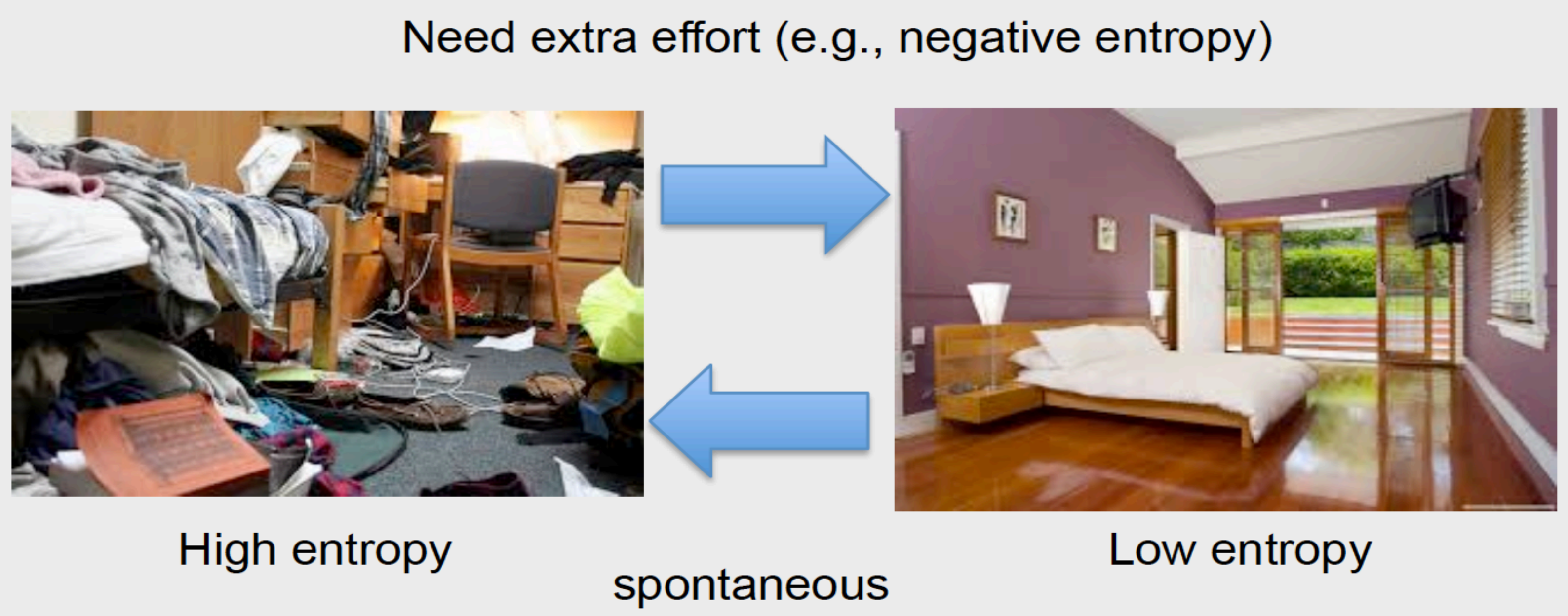
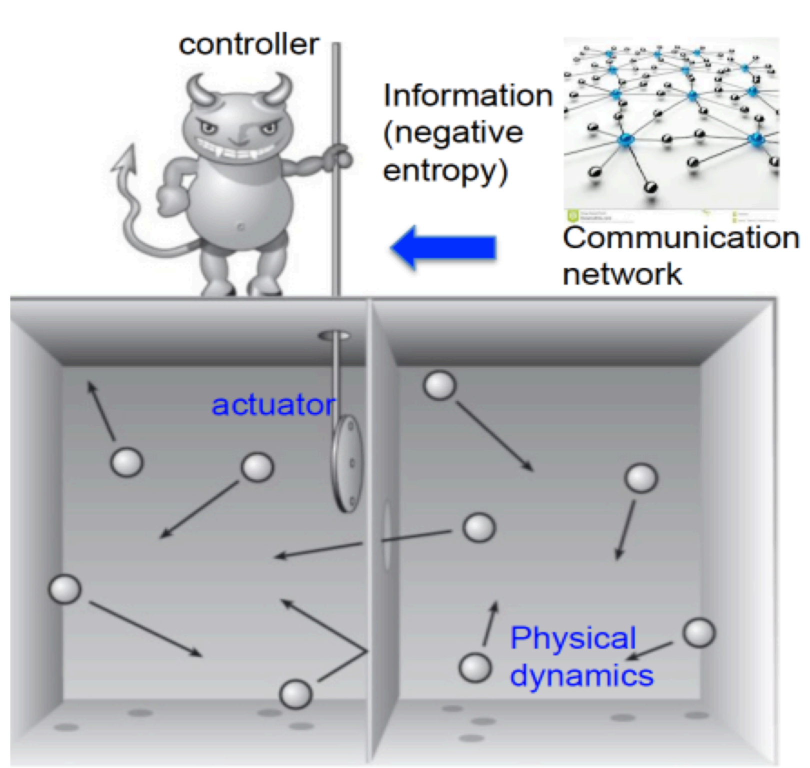
Entropy: The Measure of Uncertainty



- We propose to use entropy to measure the 'fitness' of physical dynamics; the lower, the better.
- A good controller should keep the system state close to the desired one, thus resulting in a low entropy.

Demon, Controller and Communications

- We can consider the controller as a Maxwell's demon, which reduces the entropy of the physical dynamics.
- We can consider the information conveyed by the communication network from the sensor as 'negative entropy', which compensates the entropy generated by random perturbations.
- If we want to reduce the entropy from H_0 to H, do we just need $H_0 - H$ bits of information?
- What if the physical dynamics are networked and the entropy can propagate in the space?

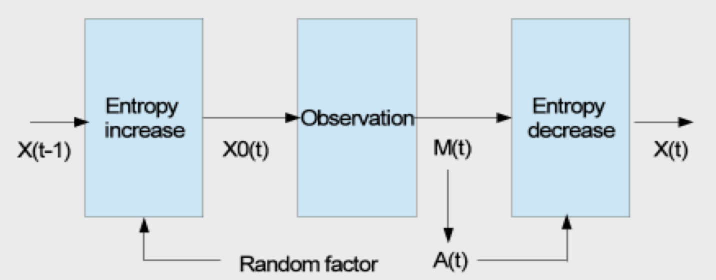


Finite State CPS

Communication Model

- There are finitely many possible messages (denoted by M), whose set is denoted by $\mathcal{M}$ ($|\mathcal{M}| \leq D$).
- The system state X_0 can be observed directly.
- The sensor maps from X_0 to M. We assume that there are no transmission errors.
- The mappings (from observation to message, and from message to control action) are deterministic.

Control Model



- We consider a finite state system with N states.
- Each cycle is divided into three stages: (1) entropy generation (state X_0); (2) observation and communication; (3) control and entropy reduction (state X).
- There are D possible control actions, each of which is a mapping from state to state.

Single Cycle Analysis

Entropy Reduction

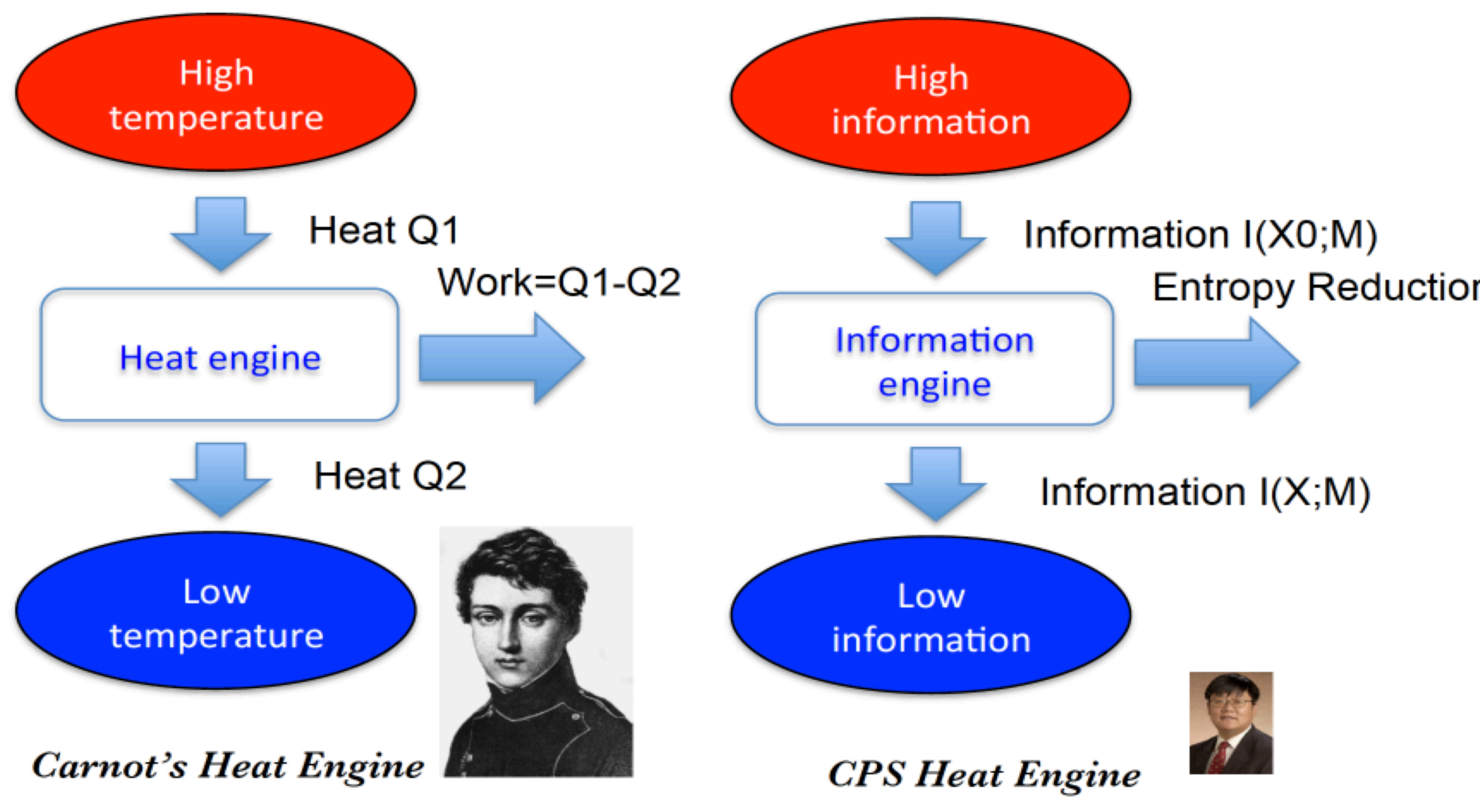
- Assumption: Each control action provides an 1-to-1 mapping.
- Lemma: Consider a single cycle. Given the assumption of injective mapping, we have

$$\underbrace{H(X_0) - H(X)}_{\text{Entropy reduction}} = \underbrace{I(X_0; M)}_{\text{Information}} - \underbrace{I(X; M)}_{\text{Leftover information}}.$$

Remark

- The mutual information $I(X_0; M)$ cannot be completely used to reduce the entropy; some residual, $I(X; M)$, will be unused.
- This is similar to the Carnot heat engine, in which the energy from a high temperature source cannot be completely used to do work.

Information Engine



Information Efficiency: Conditions

Theorem

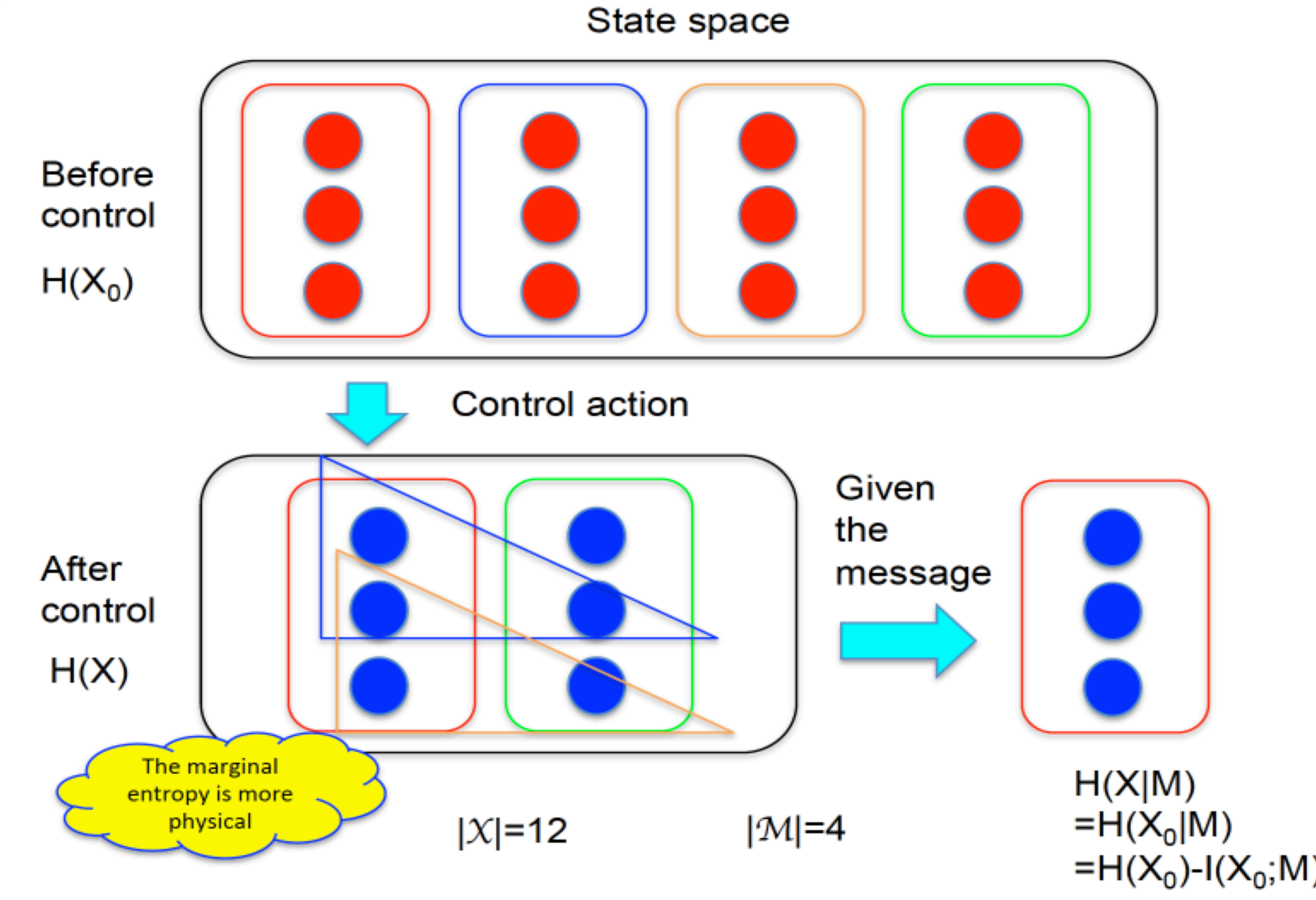
If $|\mathcal{M}|$ (number of messages) does not divide $|\mathcal{X}|$ (cardinality of system state alphabet), we have $\eta_0 < 1$.

Theorem

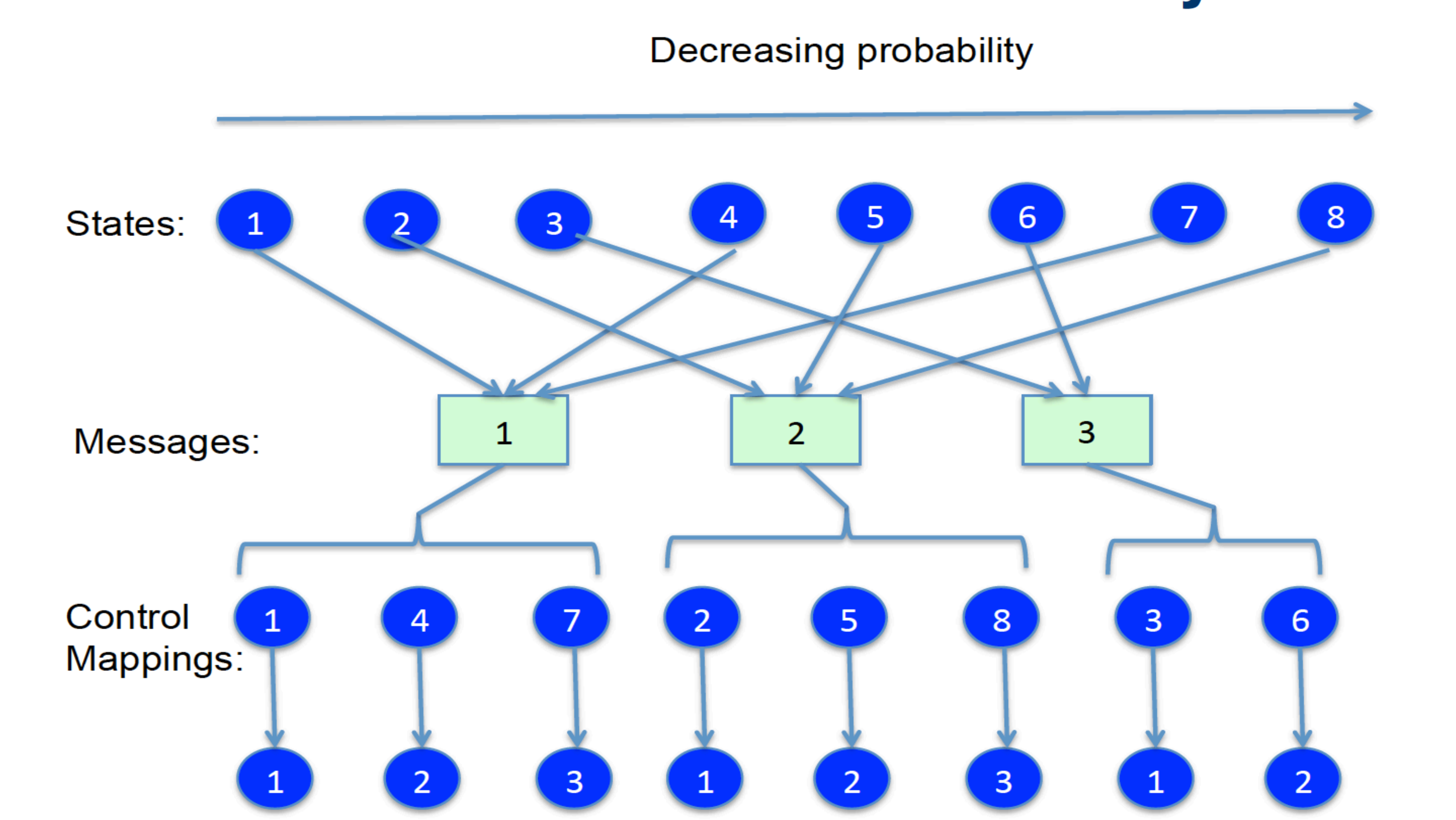
The maximal information efficiency $\eta_1 = \eta_0 = 1$ if and only if the following three conditions hold:

- $K = \frac{|\mathcal{X}|}{|\mathcal{M}|}$ *is an integer.*
- $\mathcal{X}$ *can be partitioned into* K *subsets* $\mathcal{X}_1, ..., \mathcal{X}_K$ *with equal cardinality* $|\mathcal{M}|$. *For* $\mathcal{X}_k = \{x_1^k, \ldots, x_{|\mathcal{M}|}^k\}$, *we assume* $P_0(x_1^k) = P_0(x_2^k) = \ldots = P_0\left(x_{|\mathcal{M}|}^k\right)$, *for all* $k = 1, \ldots, K$.
- *there exist a subset of control actions* $\mathcal{A}_0 = \{a_1, \ldots, a_{|\mathcal{M}|}\} \subset \mathcal{A}$ *and a subset of states* $\mathcal{X}^* = \{x_1^*, \ldots, x_K^*\}$ *such that* $\{a_i^{-1}(x_k^*), i = 1, \ldots, |\mathcal{M}|\} = \mathcal{X}_k$, *for all* $k = 1, \ldots K$.

Illustration and Intuition

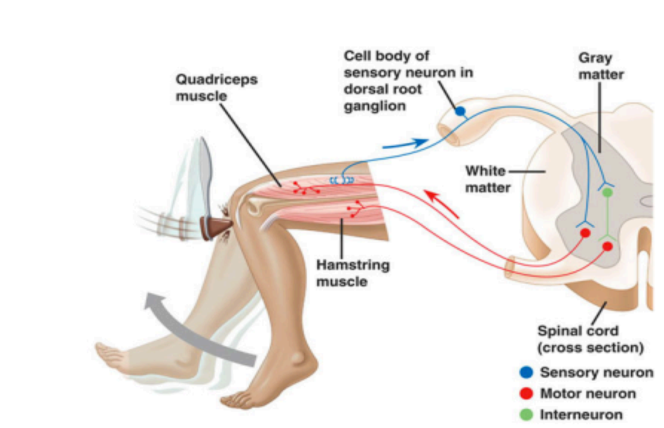


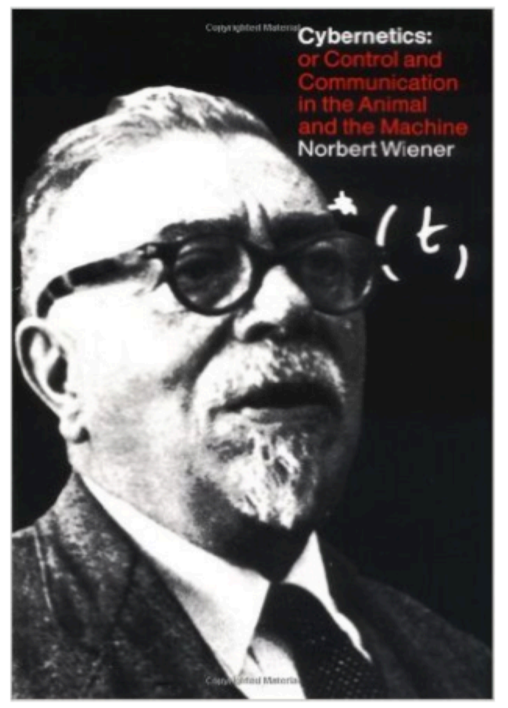
Joint Design of Control and Communications: Finite State Dynamics



If the controller can realize any possible 1-to-1 mapping of states, the above design is optimal for the reduction of entropy.

Human As A Cyber Physical System





- Human being can do communications (through the neural network) and control (muscle), which forms a CPS.
- Can we use the above analysis for human beings?
- Need to study the classical book on cybernetics by Wiener.