A Hybrid Detector Network for Nuclear and Radioactive Threat Detection: NSF CNS-1239509

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Abstract of the Results Reported in This Poster

This post considers a problem of target or source localization based on the measurements from a mobile sensor or a network of sensors. The assumption is that the received signal strength is strictly monotonic with the distance between the target and the sensor. No explicit signal propagation model, neither the structure nor the mathematical description, is assumed or used in localization. It shows that this mere knowledge of monotonicity suffices in locating the unknown target in the absence of noise. Further in the presence noise, robust localization algorithms are developed and analyzed by exploiting various forms of angular, temporal and spatial averages. The asymptotic convergence results have been established in the presence of noise and further the finite step performance results are developed.

Basic Ideas

Let $y^* = \begin{pmatrix} y_1^* \\ y_2^* \end{pmatrix} \in R^2,\ x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \in R^2,\ d^*(t) = \|x(t) - y^*\|,\ s(x(t)) = g(d^*(t)) + v(x(t))$

be unknown source location, sensor location, the distance between the sensor and the source and the received signal strength at sensor x(t). Assume $g(d_1) > g(d_2)\ \ if\ and\ only\ if\ \ d_1 < d_2.$
Then, s'(t) =s'(x(t))> 0 implies that the sensor is getting closer to the source and s'(t) s'< 0 implies that the sensor is moving away from the source.

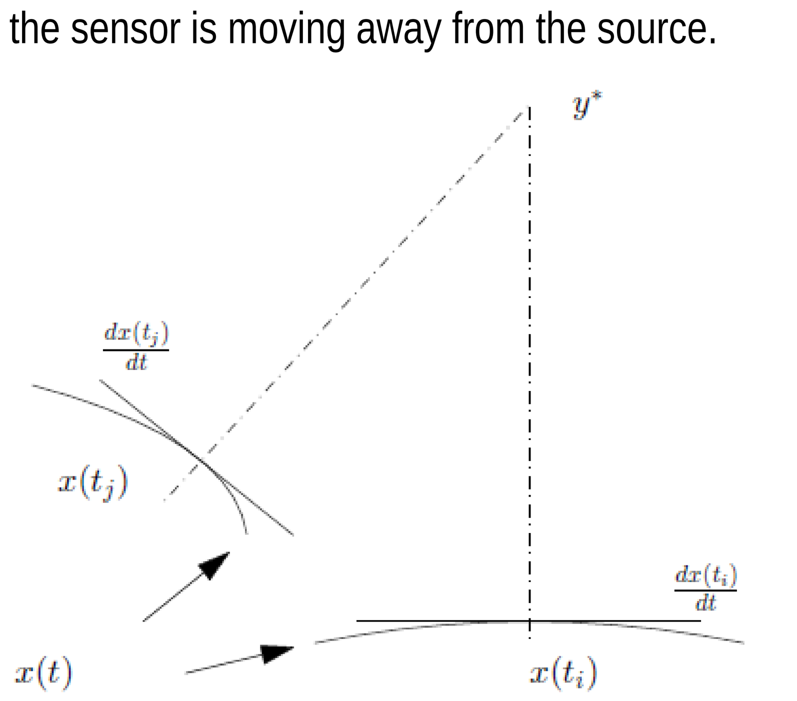

Fig. 1. Local maximum and the source location.

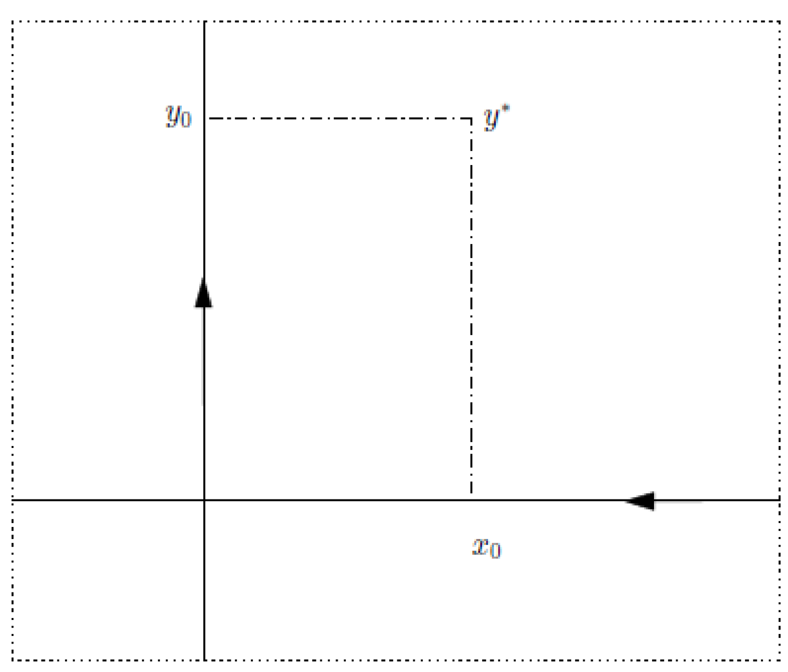

Fig. 2. Illustration of the result.

Result 1: Let x(t) be the sensor path. Assume ds/dx and dx/dt exist and are continuous. Suppose s(x(t)) achieves two local maxima at x(ti) and x(tj) respectively. Further assume that the tangent lines dx(ti)/dt and dx(tj)/dt are linearly independent. Then in the absence of noise, the unknown source location is uniquely determined at the intersection of two normal lines at x(ti) and x(tj) respectively.

The result is illustrated in Fig 2 with two line segments of trajectory.
Now let $z_i(\theta) = \begin{pmatrix} r\cos(\theta) \\ r\sin(\theta) \end{pmatrix} + x(i),\ \theta \in [0, 2\pi),\quad s(z_i(\theta)) = g_i(\theta) = g(\|z_i(\theta) - y^*\|)$

$$z_i(\theta_0(i)) = arg \min_{z_i(\theta)} \|z_i(\theta) - y^*\| \qquad \theta_0(i) = arg \max_{\theta} g_i(\theta).$$

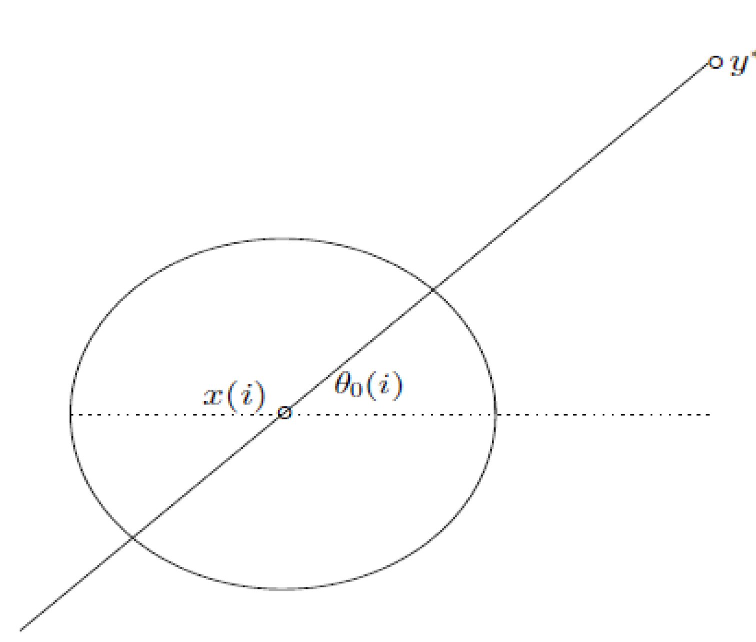

Fig. 3. Illustration of sensor circular movement.

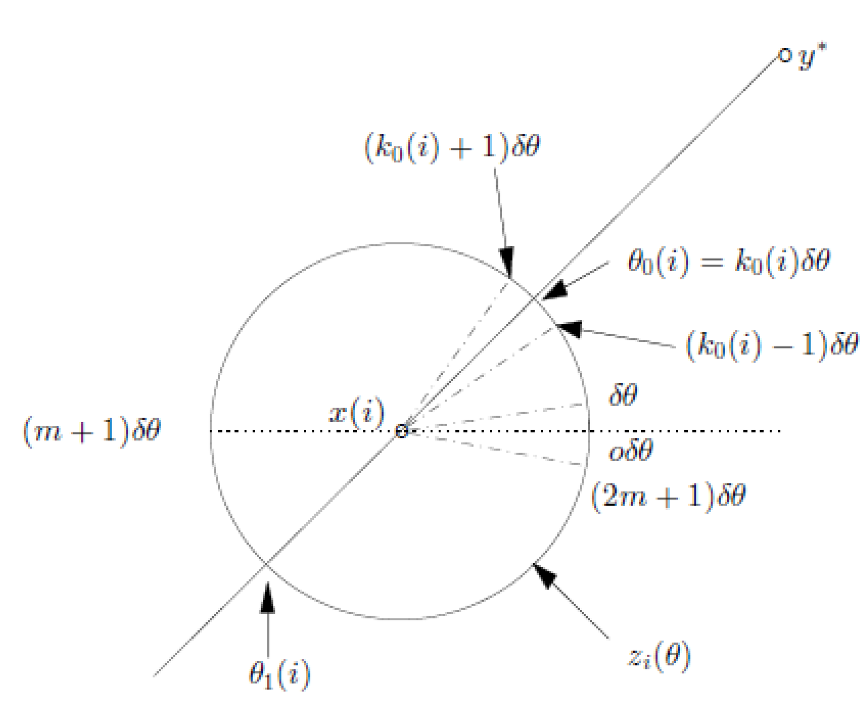

Fig. 4. Illustration of angular averages.

Result 2: In the absence of noise, the line connecting the center x(i) to the source location is uniquely determined.

Robust Algorithms: Angular Average

A problem is that local maxima and consequently, source locations are hard to calculate reliably in the presence of noise. So a key question is how to robustly determine if s(x(t)) achieves a local maximum or not in the presence of noise. Let , as in Figure 4, $\theta_0(i) = k_0(i)\delta\theta,\ \ for\ some\ k_0(i) \in [0, 2m+2),$

$$s_i(k) = s_i(k\delta\theta) = g_i(k\delta\theta) + v_i(k\delta\theta) = g_i(k) + v_i(k).$$

$$\hat{k}_0(i) = arg \min_k \frac{1}{m} \sum_{j=1}^{m} (s_i(k+j) - s_i(k-j))^2,$$

$$k \pm j = k \pm j\ mod(2m+2).$$

$$\hat{\theta}_0(i) = \hat{k}_0(i)\delta\theta.$$

$$\hat{y} = arg \min_{\hat{y} = \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix}} \frac{1}{n} \sum_{i=1}^{n} (\hat{\theta}_0(i) - \arctan \frac{\hat{y}_2 - x_2(i)}{\hat{y}_1 - x_1(i)})^2.$$

Result 3: In distribution as $n \to \infty,\ \ \hat{y} \to y^*$

Robust Algorithms: Temporal Average

The idea is an average of multiple samples to average out the effect of noise. Let the length of the rectangle in the horizontal direction be l(units) and l/n be the sampled step. Then the received signal along the horizontal path can be written as

$$s_h(k - k_x) = s_h((k - k_x)\Delta x)$$
$$= g\left(\left\| \begin{pmatrix} k\Delta x \\ \bar{y} \end{pmatrix} - y^* \right\|\right) + v(k\Delta x),\ k = 0, 1, ..., n$$

and determine
$$k_x = arg \min_k g\left(\left\| \begin{pmatrix} k\Delta x \\ \bar{y} \end{pmatrix} - y^* \right\|\right),$$

by solving
$$\hat{k}_x = arg \max_k s_h(k - k_x) = arg \max_k \{g_h(k) + v(k)\}.$$

To be robust, define

$$s_h^L(k - k_x) = \frac{1}{L} \sum_{j=1}^{L} s_{hj}(k - k_x) \qquad \hat{k}_x = arg \max_k s_h^L(k - k_x)$$

Result 4: The estimate $\hat{k}_x$ converges to k_x in probability.

Quantification of Errors for a Finite L

Quantification of errors can be done by extremely theory. Let $z_n = \max\{\nu_1, \nu_2, ..., \nu_n\}$. Then,

$$F(z) = Prob\{z_n < z\} \to exp(-exp(-\frac{\frac{z-\mu}{\sigma/\sqrt{L}} - a_n}{b_n}))$$

Simulation Results:

$y^* = \begin{pmatrix} 110 \\ 95 \end{pmatrix}$, noise std=0.1, $r = d_{max} = 5$, $m = 61$	
# of sensors $x(i)$'s	
10	$\begin{pmatrix} 112.21 \\ 103.65 \end{pmatrix}$ $\begin{pmatrix} 1982.3 \\ 2712.2 \end{pmatrix}$
20	$\begin{pmatrix} 102.4 \\ 92.22 \end{pmatrix}$ $\begin{pmatrix} 996.59 \\ 1900.6 \end{pmatrix}$
30	$\begin{pmatrix} 104.29 \\ 97.01 \end{pmatrix}$ $\begin{pmatrix} 784.97 \\ 1257.7 \end{pmatrix}$
40	$\begin{pmatrix} 105.91 \\ 97.51 \end{pmatrix}$ $\begin{pmatrix} 706.48 \\ 1927.5 \end{pmatrix}$
50	$\begin{pmatrix} 109.22 \\ 92.56 \end{pmatrix}$ $\begin{pmatrix} 706.54 \\ 1816.5 \end{pmatrix}$
100	$\begin{pmatrix} 108.51 \\ 93.59 \end{pmatrix}$ $\begin{pmatrix} 152.15 \\ 269.86 \end{pmatrix}$
300	$\begin{pmatrix} 108.76 \\ 94.78 \end{pmatrix}$ $\begin{pmatrix} 10.69 \\ 5.68 \end{pmatrix}$

Table 1
The estimates $\hat{y}$'s and their variances in the first and second parentheses respectively.

$\hat{\rho}_x \cdot \hat{\rho}_y$, (estimation variance) and the true but unknown $\rho_x \cdot \rho_y$ ($l_h = l_v = 400$(unit), $\Delta x = \Delta y = 0.1$)				
	$\Delta x \cdot m = \Delta y \cdot m = 8$	$\Delta x \cdot m = \Delta y \cdot m = 10$	$\Delta x \cdot m = \Delta y \cdot m = 15$	$\Delta x \cdot m = \Delta y \cdot m = 20$
$SNR_h = SNR_v = -3dB$				
$L = 3$	.7718(.0752),.8740	.7609(.0765), .9640	.6712(.1015),.9990	.5676(.1109),.9990
$L = 5$	.9326(.0239),.9370	.9106(.0314),.9890	.8185(.0642), 1.0000	.0708(.0967), 1.0000
$L = 10$	.9947(.0012), .9950	.9935(.0018),0.9990	.9680(.0125), 1.0000	.9104(.0340),1.0000
$SNR_h = SNR_v = 0dB$				
$L = 3$	.9533(.0143), .9650	.9467(.0176),.9840	.8903(.0368), 1.0000	.7860(.0709),1.0000
$L = 5$	.9959(.0010), .9910	.9945(.0011),1.0000	.9656(.0133), 1.0000	.9097(.0327),1.0000
$L = 10$	1.0000(0), 1.0000	.9999(0),1.0000	.8903(.0368), 1.0000	.9927(.0014),1.0000
$SNR_h = SNR_v = 5dB$				
$L = 3$	1.0000(0), 1.0000	1.0000(0),1.0000	.9984(.0002), 1.0000	.9917(.0012),1.0000
$L = 5$	1.0000(0), 1.0000	1.0000(0),1.0000	1.0000(0), 1.0000	.9993(.0001),1.0000
$L = 10$	1.0000(0), 1.0000	1.0000(0),1.0000	1.0000(0), 1.0000	1.0000(0),1.0000

Table 2
The estimated probability $\hat{\rho}_x \cdot \hat{\rho}_y$, its variance in parenthesis based on 1000 Monte Carlo runs and the true $\rho_x \cdot \rho_y$.