A Lie Algebraic Model Predictive Control for Legged Robot

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https://www.youtube.com/watch?v=h8rFXjUP_UU

Scan for papers!

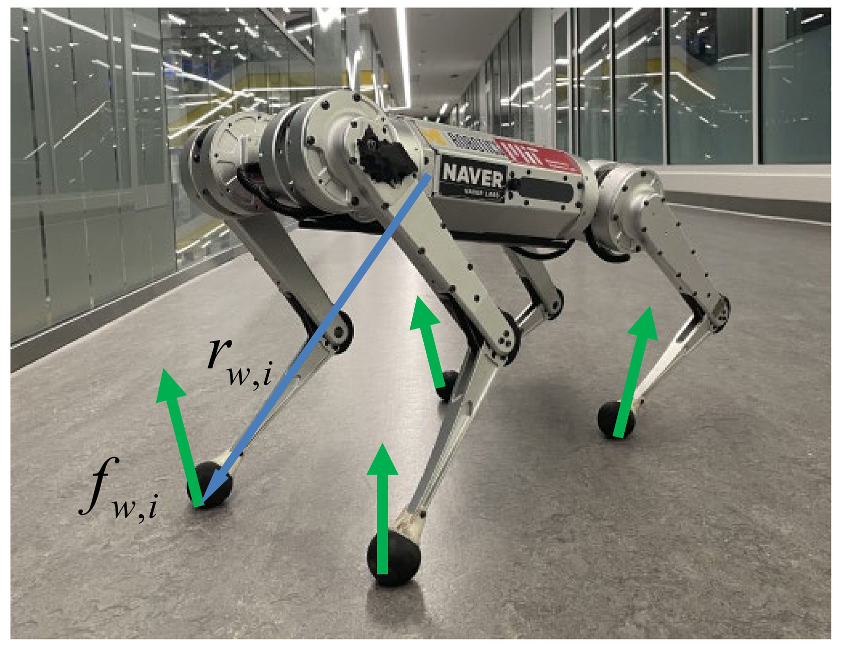
MPC

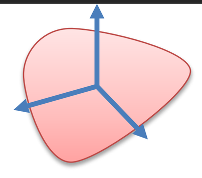
Stability Analysis

This work develops an error-state Model Predictive Control for the legged robot. The centroidal motion of a legged robot can be modeled by a single rigid body on SE(3). The tracking error is linearized in its Lie algebra, thus making the linearized matrix in the MPC independent of the configuration error. A quadratic Lyapunov function in Lie algebra is designed to certify exponential stability.

Motivation and challenges:

- The centroidal dynamics of the legged robot are evolving on SE(3) group that, is nonlinear and not trivial to parameterize.
- The state-of-the-art geometric MPC does not fully utilize the symmetry of the Lie group.
- The well-adopted Frobenius norm-based cost function can not ensure exponential stability when the configuration error becomes large.





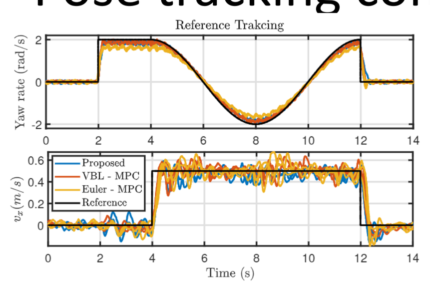
$$X = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \in SE(3)$$

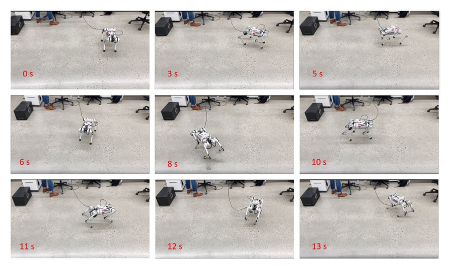
$$\xi^\wedge = \begin{bmatrix} \omega^\wedge & v \\ 0 & 0 \end{bmatrix} \in \mathfrak{se}(3)$$

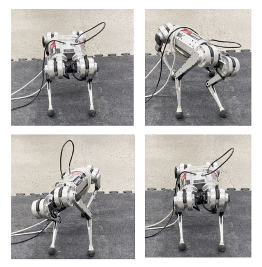
$$J_b = \begin{bmatrix} I_b & 0 \\ 0 & mI \end{bmatrix}$$

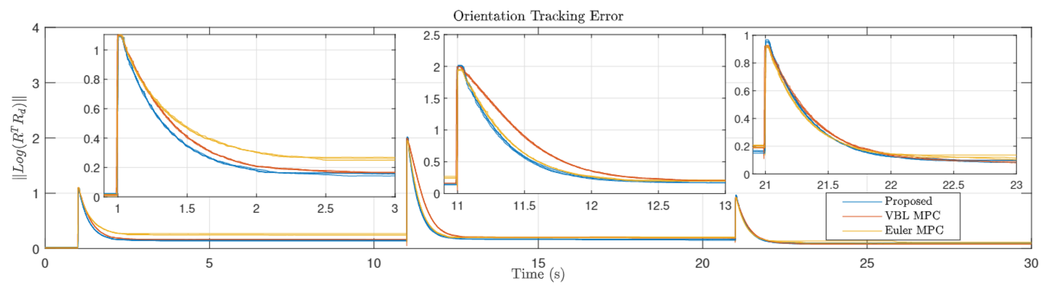
Fast tracking control and trajectory optimization :

Pose tracking control:



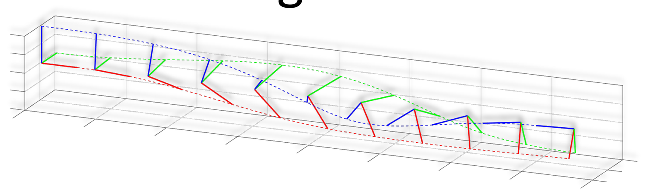


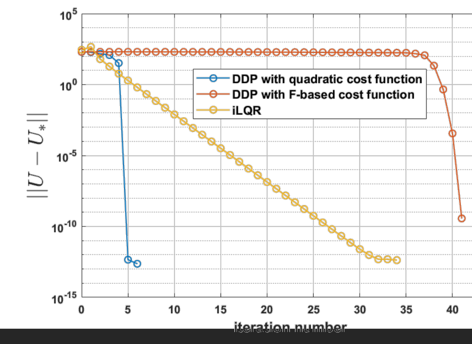




Fast trajectory optimization (TO):

- Cost function in Lie algebra enables faster convergence.
- TO on SO(3) with bad initialization:





Convex MPC with Linearized Error Dynamics on Lie Group

- Trajectory on Lie group

$$\dot{X} = X\xi^\wedge \qquad \dot{X}_d = X_d \xi_d^\wedge$$

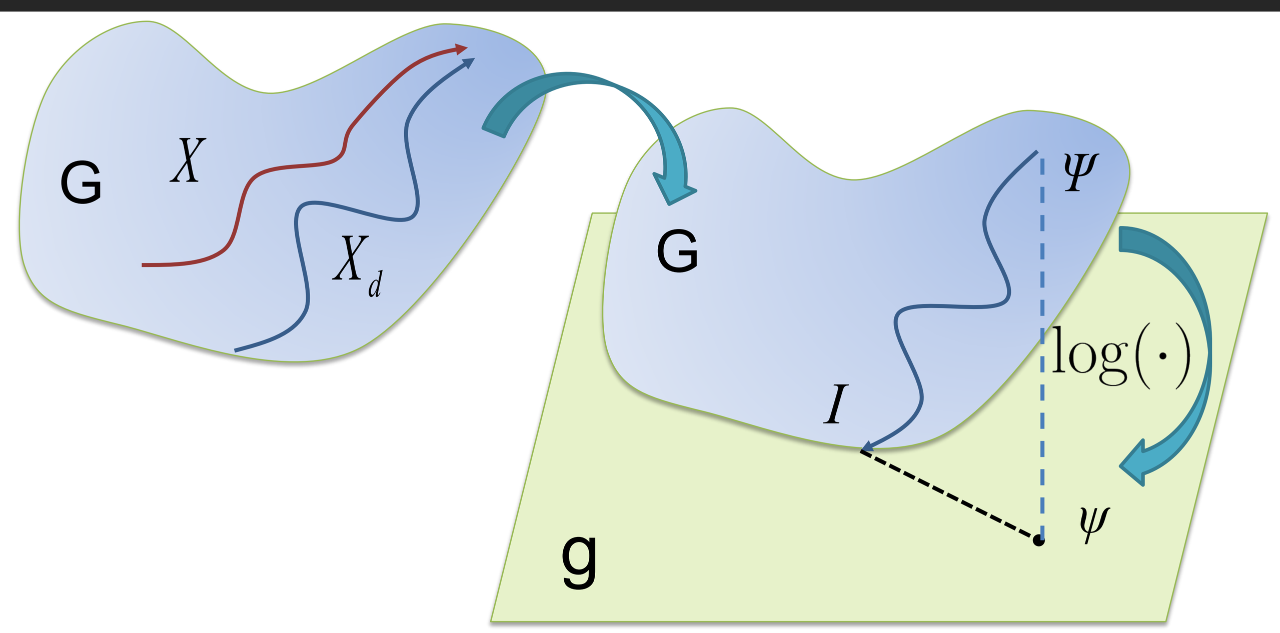
- Tracking error defined by group action

$$\Psi := X_d^{-1} X \in \mathrm{G} \qquad \dot{\Psi} = \Psi(\xi - \mathrm{Ad}_{\Psi^{-1}} \xi_d)^\wedge$$

- Tracking error linearized in the Lie algebra

$$\Psi = \exp(\psi) \approx I + \psi^\wedge$$

$$\dot{\psi} \approx -\mathrm{ad}_{\xi_d} \psi + \xi - \xi_d$$



Euler-Poincare Equation (Dynamics of twists)

$$\dot{\xi} = J_b^{-1} \mathrm{ad}_\xi^* J_b \xi + J_b^{-1} u$$

$$\approx H\xi + J_b^{-1} u + b_t$$

Redefine the state and output

$$x := \begin{bmatrix} \psi \\ \xi \end{bmatrix} \quad y := \begin{bmatrix} I & 0 \\ -\mathrm{ad}_{\xi_d} & I \end{bmatrix} \begin{bmatrix} \psi \\ \xi \end{bmatrix} - \begin{bmatrix} 0 \\ \xi_d \end{bmatrix}$$

Regularize the error and its velocity in the Lie algebra.

Linearized error dynamics independent of X and Ψ

$$\dot{x}_t = A_t x_t + B_t u_t + h_t$$

$$A_t := \begin{bmatrix} -\mathrm{ad}_{\xi_{d,t}} & I \\ 0 & H_t \end{bmatrix}, B_t = \begin{bmatrix} 0 \\ J_b^{-1} \end{bmatrix}, h_t = \begin{bmatrix} -\xi_{d,t} \\ b_t \end{bmatrix}$$

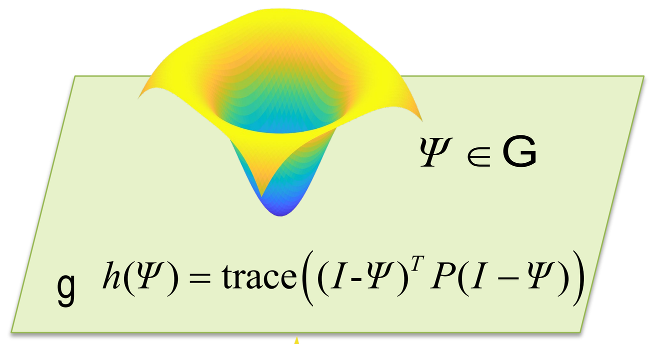
$$\min_{u_k} \quad y_N^T P y_N + \sum_{k=1}^{N-1} y_k^T Q y_k + u_k^T R u_k$$

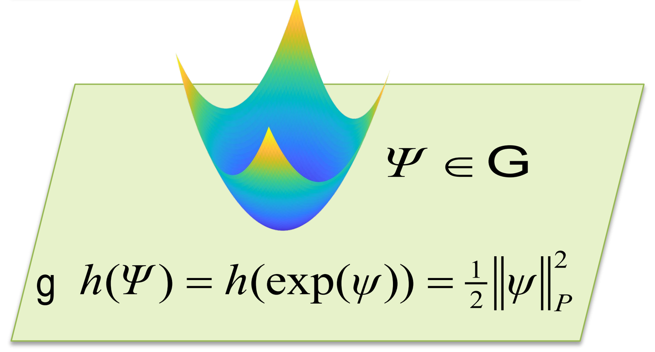
$$s.t \quad x_{k+1} = A_k x_k + B_k u_k + h_k$$

$$u_{\min} \le u \le u_{\max}, k = 0,1,2,\dots,N-1$$

Cost Function Design & Stability Analysis

- A quadratic cost function expressed in the Lie algebra is designed to ensure/certify the exponential stability.
- Gradient is accessible by introducing a left-invariant metric.
- Linear feedback of the configuration error in its Lie algebra ensures exponential stability.





Broader Impact (Outreach and Education)

- The proposed formulation can make Lie group controller design more accessible.
- The trajectory-independent algebraic formulation is suitable for senior undergrad and graduate level robotics and control curriculum.

Broader Impact

- The proposed convex MPC formulation for rigid body systems can be readily implemented using QP solvers.
- The runtime is significantly lower than current nonlinear MPC formulations, making it suitable for real-time implementations.

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