

# A Mathematical Theory of Cyber-Physical Systems

## UC Berkeley

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### Thrusts

- Fixed points, causality, and mixed continuous/discrete systems
- Models of time
- Executable mixed continuous/discrete models
- Fundamental limits** (this poster)

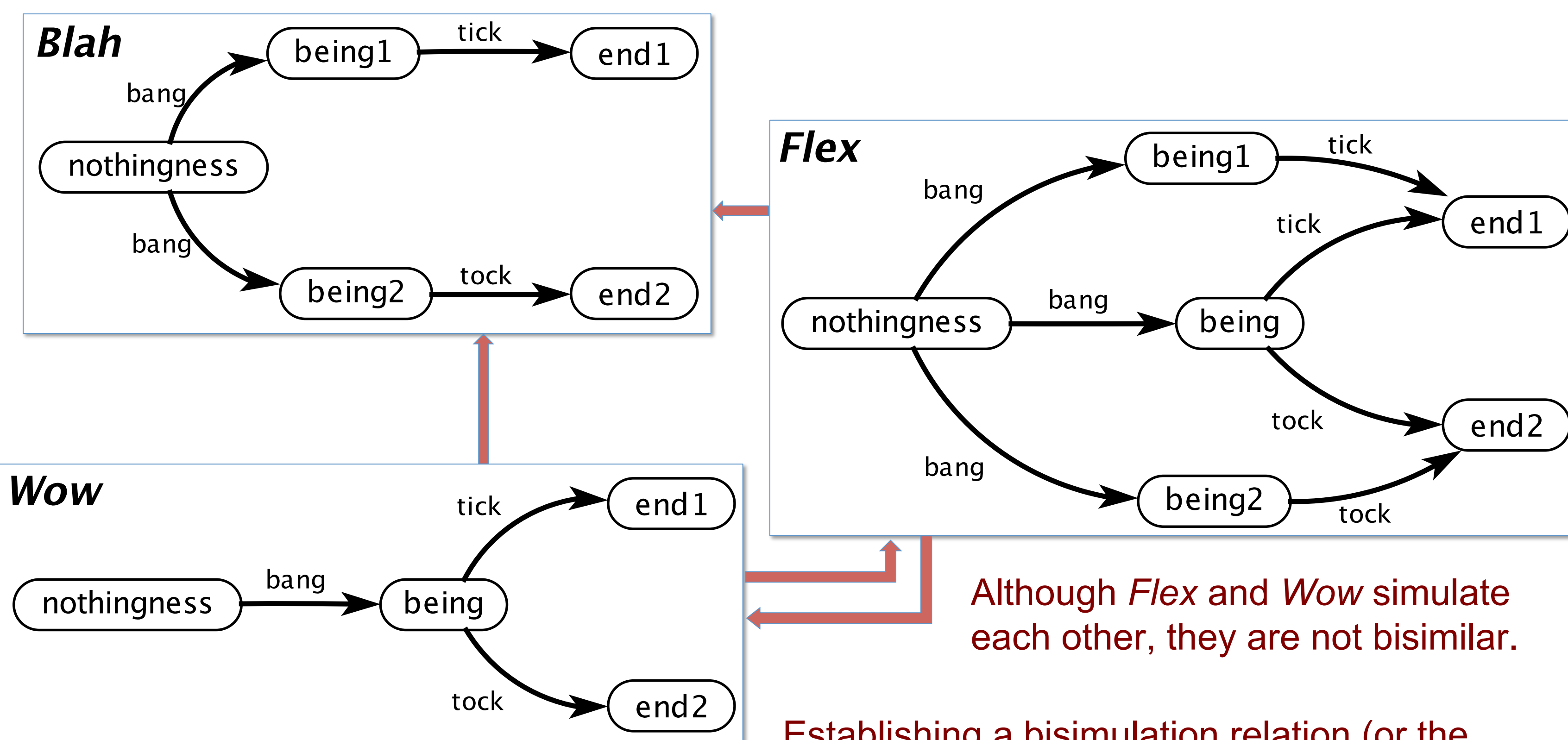
### Interaction is More Powerful than Computation

Cyber-physical systems are interactions between a computational system and a physical environment. Various fundamental results demonstrate that interaction can accomplish things that computation by itself cannot.

### Bisimulation

Interaction is needed to tell the difference between a system in which free will may exist and one in which it cannot.

In a tiny universe where beings can either tick or tock, it is impossible without interaction to tell whether the determination of whether they tick or tock is made at the time of creation or later.

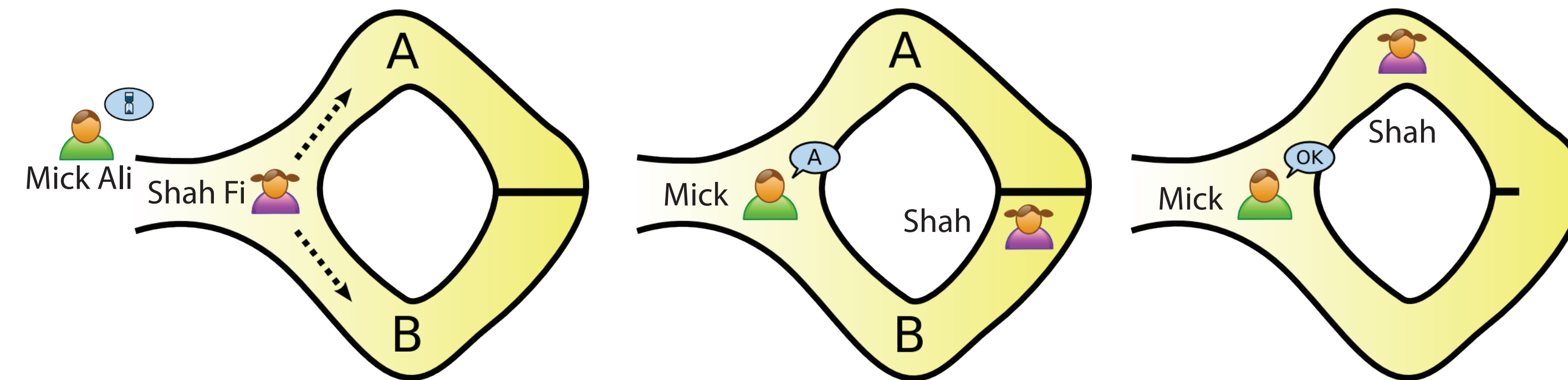


Establishing a bisimulation relation (or the absence of one) requires interaction.

Robin Milner, Turing Award, 1991

### Zero-Knowledge Proofs

First-person interaction is needed to convey a *minimum* amount of information.



Mick waits outside the cave while Shah picks one of two directions, A or B. Mick does not know which direction Shah picked.

Mick calls out at random one of A or B. With probability 1/2, Shah will have to use her password to open the door.

Shah exits via the tunnel identified by Mick, showing that with probability 1/2, she knows the password. Repeating the challenge will raise the probability to 3/4. Repeat to make the probability as close to 1 as you like.

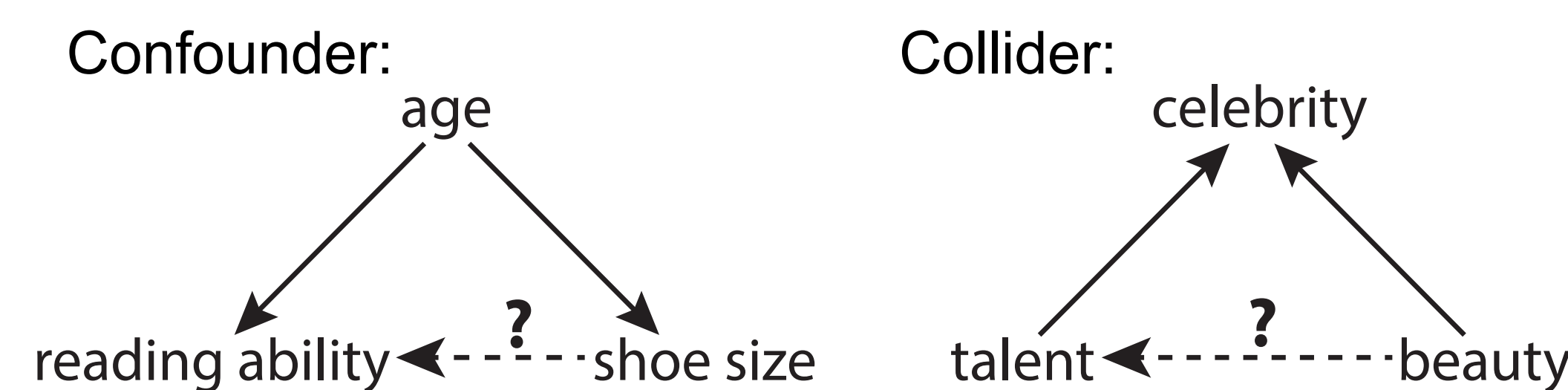
Shah convinces Mick through first-person interaction that she knows the password, but Mick cannot pass along the proof to anyone else.

Goldwasser and Micali, Turing Award, 2012

[Drawings by Dake, via Wikimedia Commons CC BY 2.5]

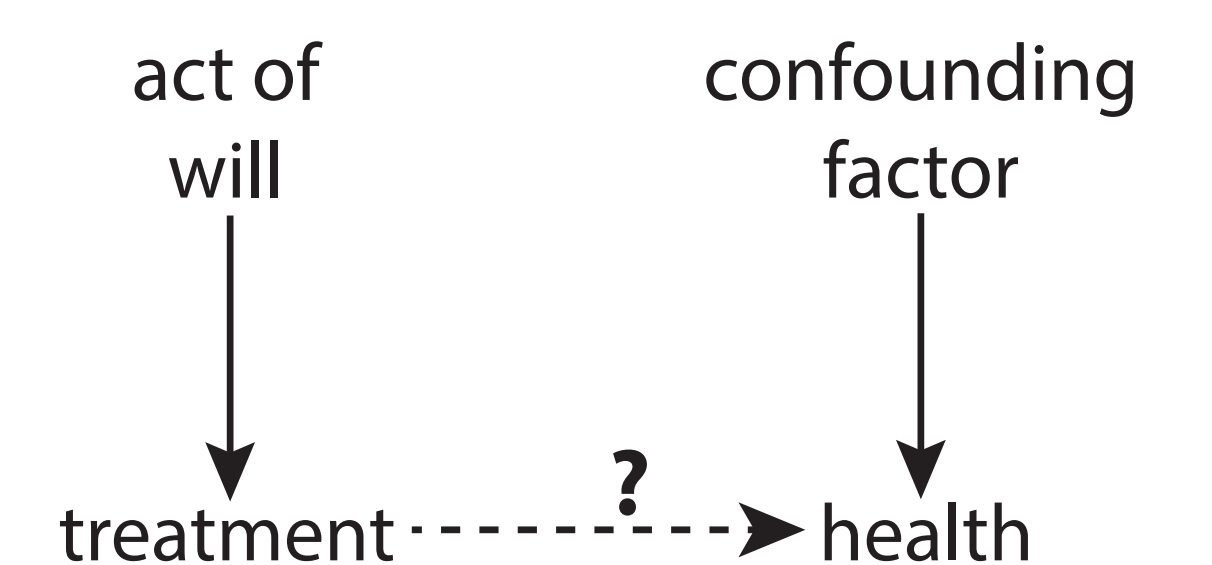
### Causal Reasoning

Interaction is needed to reason about causality.



Using data alone, it is impossible to tell whether shoe size causes reading ability or beauty causes talent.

To use data, we have to control for confounders and *not* control for colliders.



A randomized controlled trial is an intervention in the system to break the effects of both confounders and colliders.

Judea Pearl, Turing Award, 2011

### References

- E. A. Lee, *Living Digital Beings*, MIT Press, 2020 (to appear)

