A Robust Convex Optimization Framework for the Design of Ubiquitous Series Elastic Actuators

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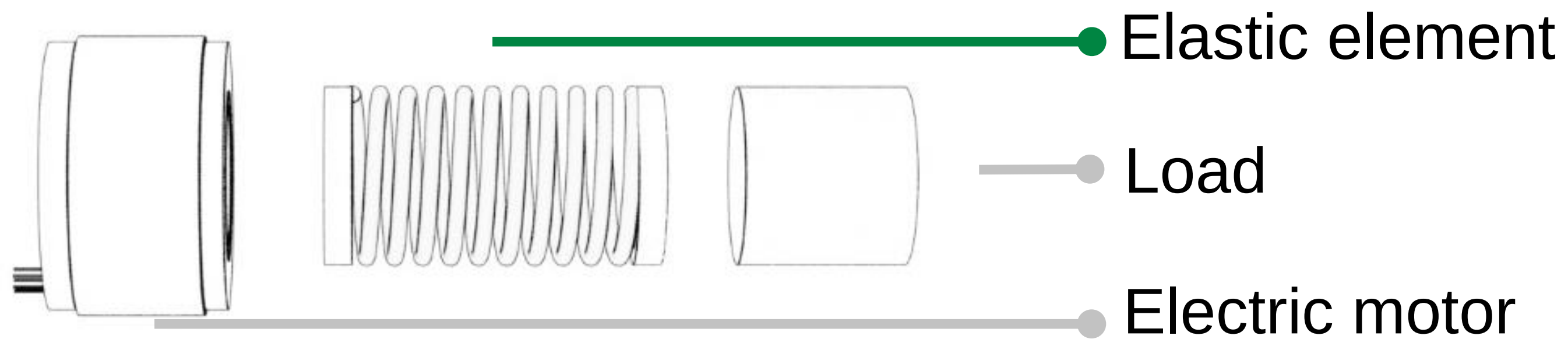
Challenge: No Consensus for Energy Efficient SEA Design

Popular design methods to reduce energy consumption assume a linear spring or a polynomial parameterization of the spring



Linear elastic element

$\tau = k\delta$

Nonlinear elastic element

$\tau = f(\delta)$

Natural dynamics

- \+ Analytical solution
- \- No actuator constraints

Parametric optimization

- \+ Actuator constraints
- \- Limited to parameter space
- \- Locally optimal points

We propose a non-parametric optimization

The effects of nonlinear series elasticity in energy consumption and robustness are unknown.

Solution: A Robust Globally-Optimal Design

Non-parametric optimization of spring profile.

Global and robust optimal design.

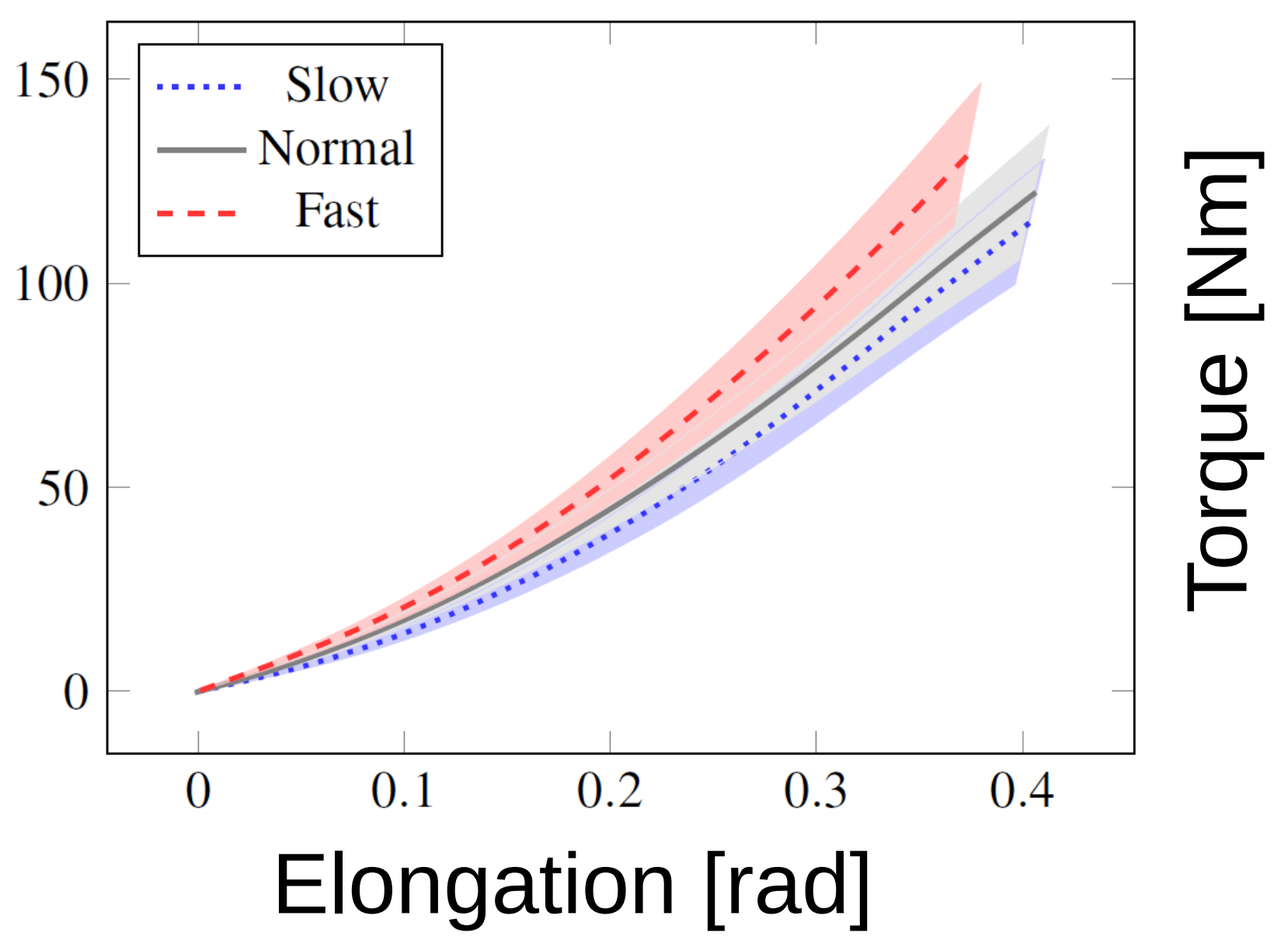
Motor and spring constraints remain feasible despite modeling, manufacturing, and task uncertainty.

Key features:

- Convex optimization framework.
- Solutions computed within polynomial time.
- Solution is robust to uncertainty in:
  - reference kinematics,
  - reference kinetics,
  - efficiency of the transmission,
  - accuracy in manufacturing of spring,
  - and system modeling.

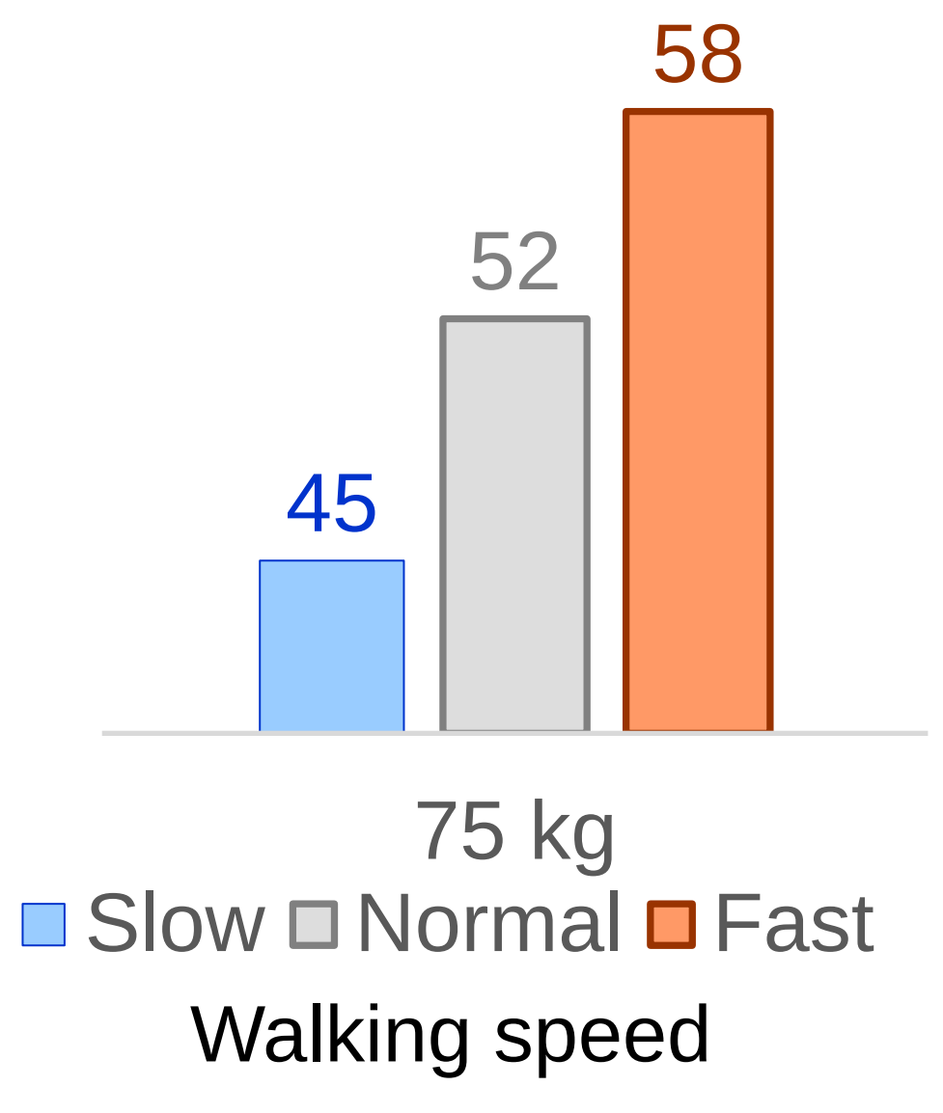
$$\begin{aligned}&\text{minimize } \frac{1}{2}x^TQx + q^Tx + c\\&\text{subject to } Ax \le b, \forall A, b \in \boldsymbol{\mathcal{U}}\end{aligned}$$

Optimal elastic element



Simulation results: SEA for a powered prosthetic ankle

Reduction of dissipated energy [%]

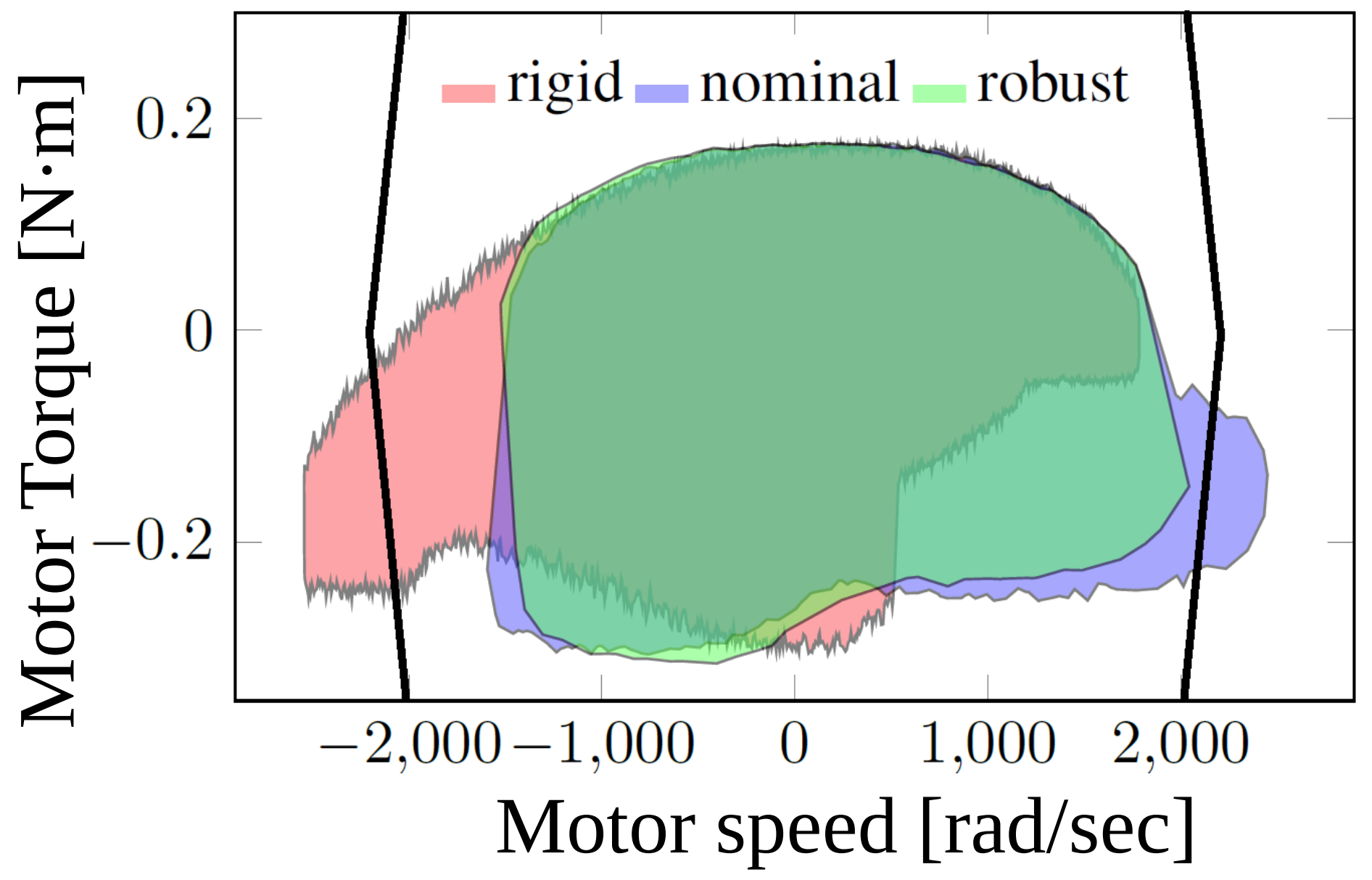


Robust Feasible Design

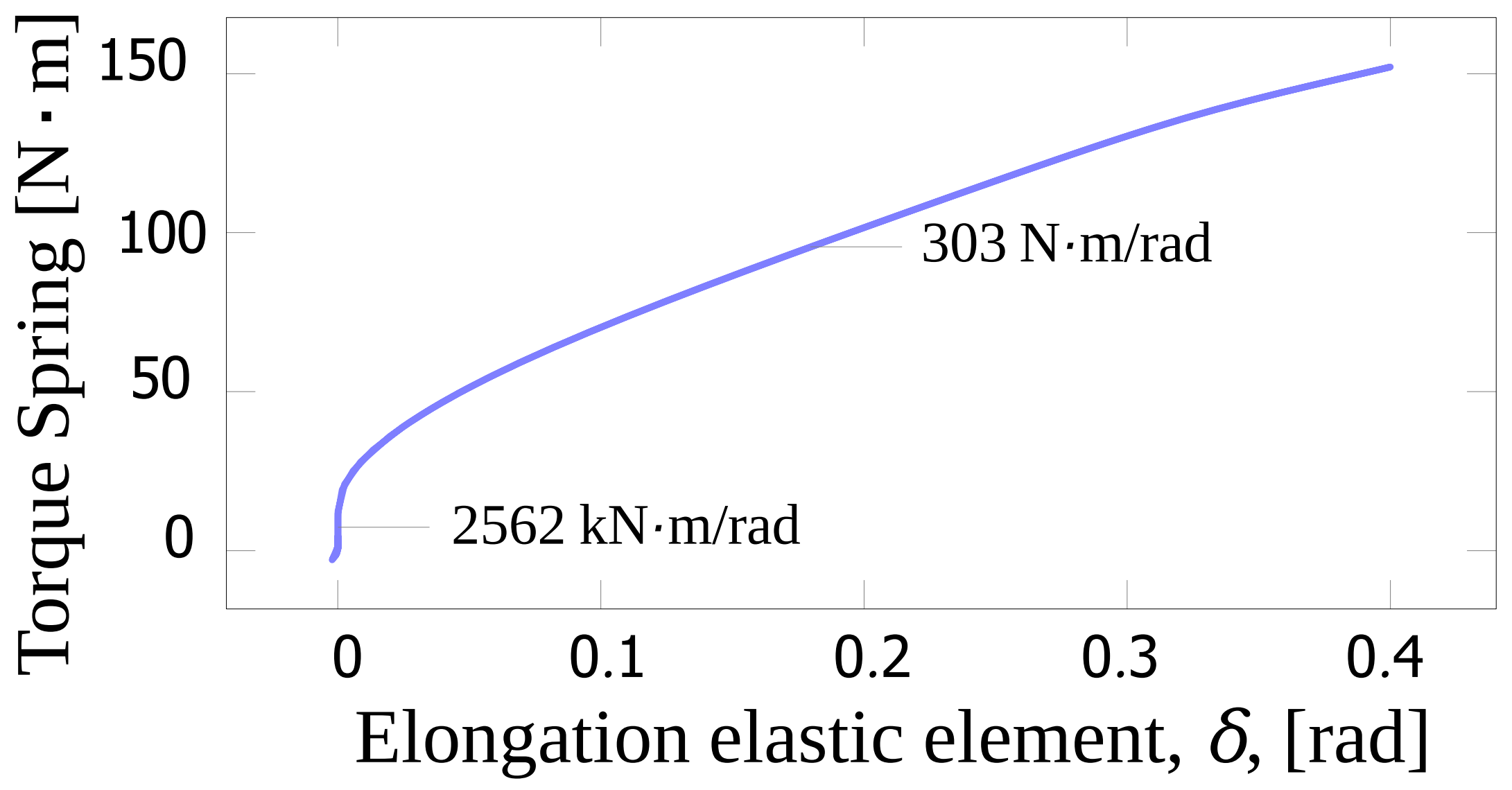
Violating actuator constraints during operation might be unsafe for robots that cooperate with humans.

Stochastic approaches do not consider worst case scenarios and requires a probability distribution.

The solution is not always to make the system as rigid as possible.



Not too rigid, not too soft to guarantee actuator constraints. (Nominal: 217.4 N·m/rad, Robust: 243.4 N·m/rad)



Nonlinear series elasticity satisfies all the constraints, but a rigid actuator and linear SEAs cannot

Application to Powered Prostheses

Quadratic springs could be the global optimal solutions for the ankle joint (for actuators with high ratio transmission).

SEAs in powered prosthesis:

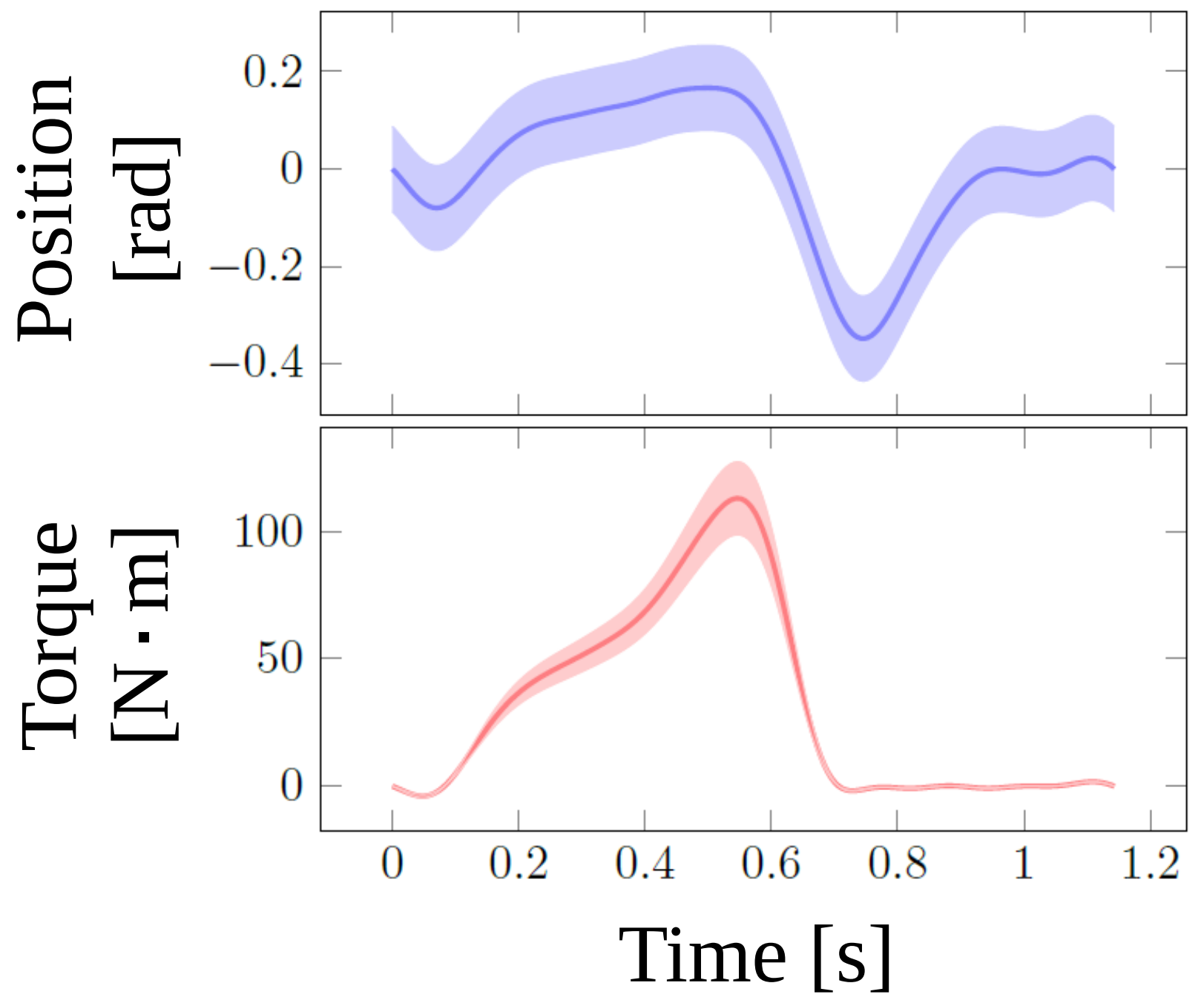
Empower Ottobock

Open Source Prosthetic Leg, Rouse, et al.

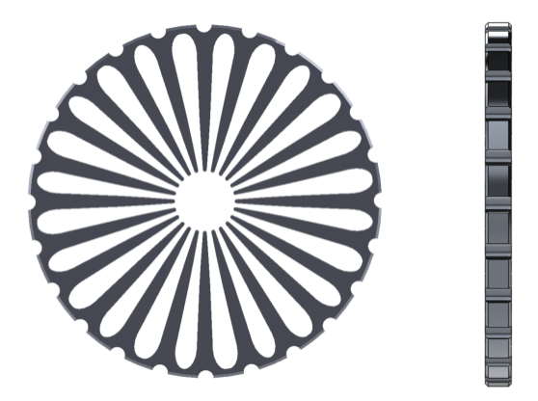
Spring active, Thomas Sugar, et al.

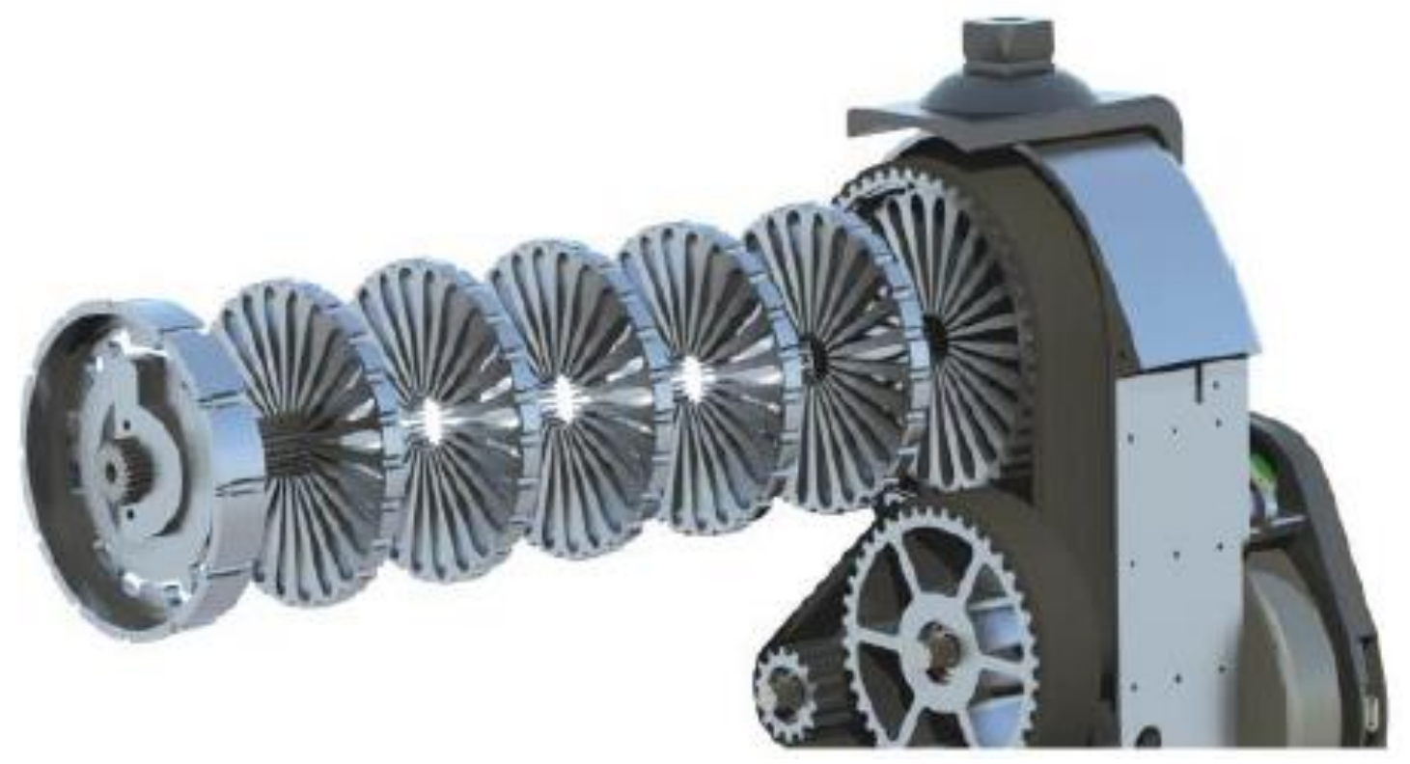
Powered prostheses are prone to uncertainty due to changing kinematics and kinetics.

Actuator should operate despite uncertainty.



Selectable Series Elastic: Individual Spring Disks





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