A Substrate for CPS Design*

Anuradha Annaswamy
Active-adaptive Control Laboratory
Department of Mechanical Engineering
Massachusetts Institute of Technology

* Collaborators: Damoon Soundbaksh, Linh Phan, Oleg Sokolsky, and Leslie Maldonado

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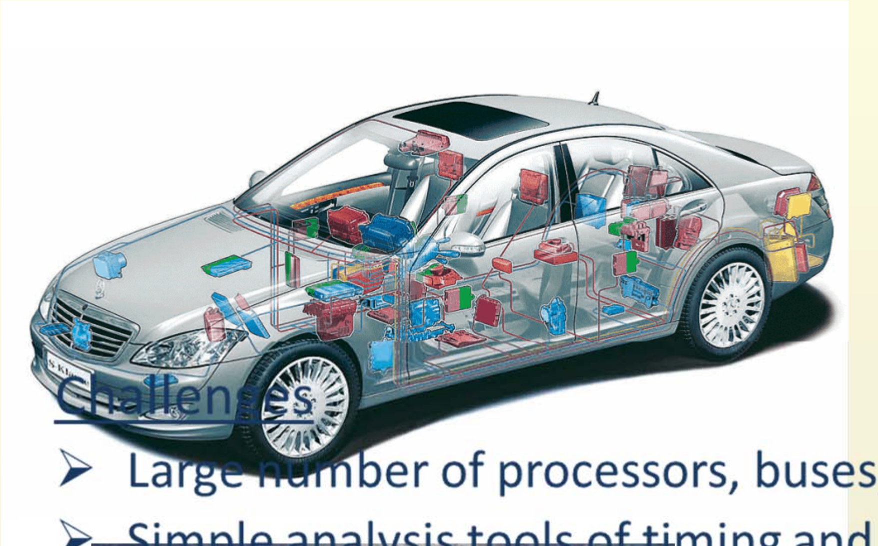
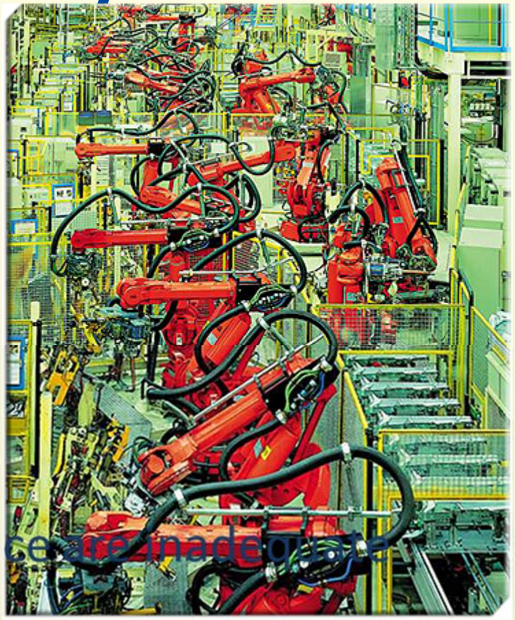
Distributed Embedded Systems





Challenges

- Large number of processors, buses, and gateways
- Simple analysis tools of timing and control performance are inadequate
- Synthesis tools need to be advanced as well
- Tools from Real-time Systems and Control Systems have to be brought together

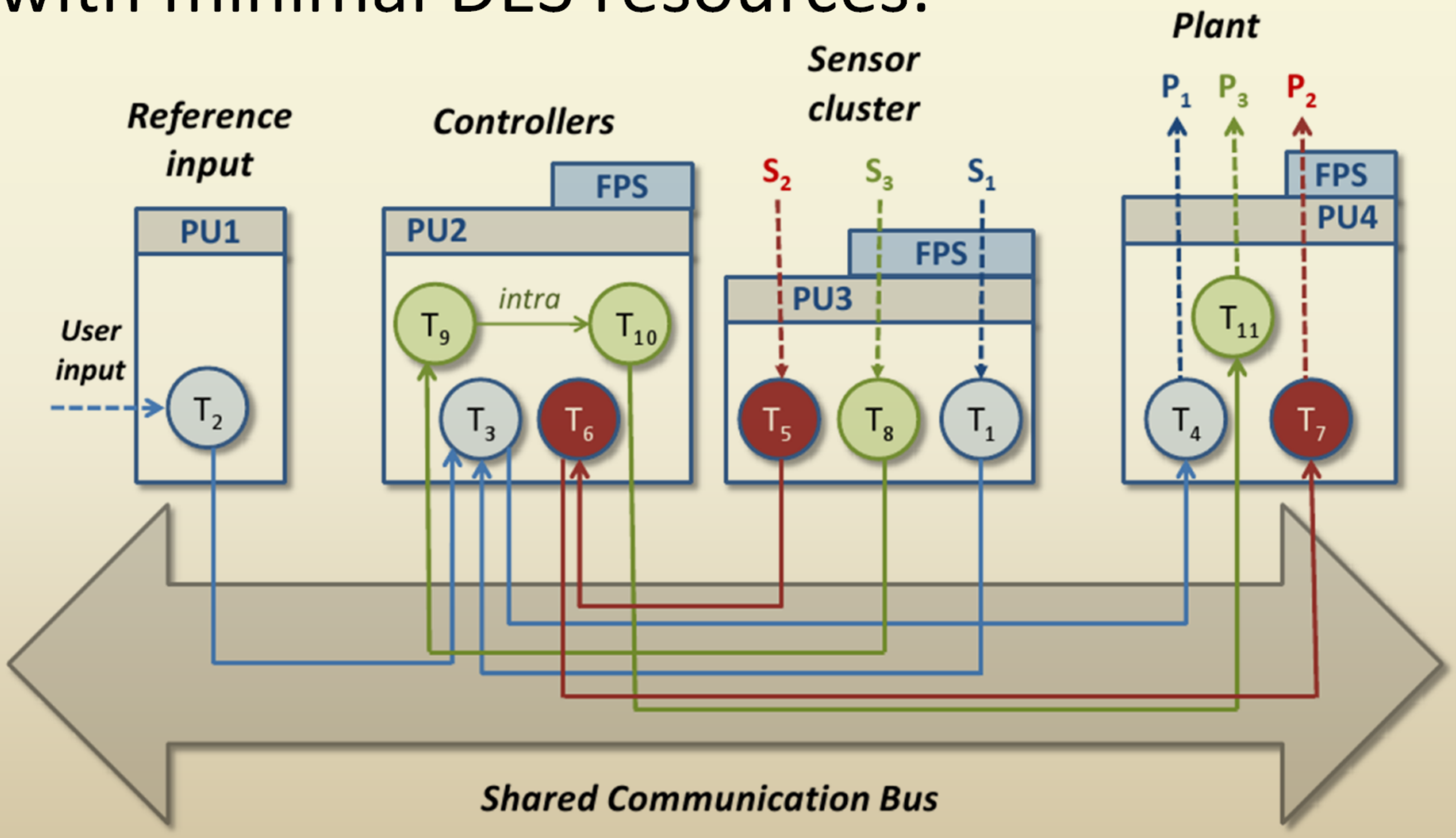
Objective of CPS Design

Efficient implementation of multiple control applications in a Distributed Embedded System (DES) using

- flexibility and transparency in the DES platform
- properties of nonlinear dynamic systems

CPS Design Goals

- Co-design the controller and the DES architecture to control multiple applications with minimal DES resources.



Substrate Components

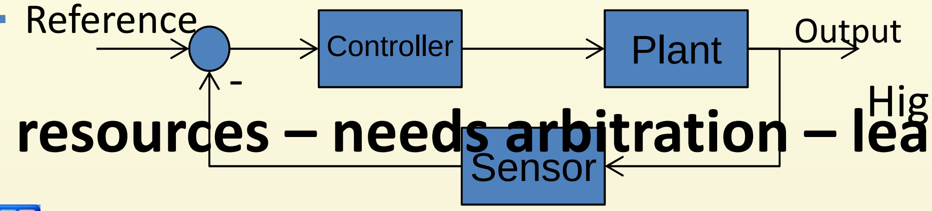
1. Codesign using arbitration

This talk

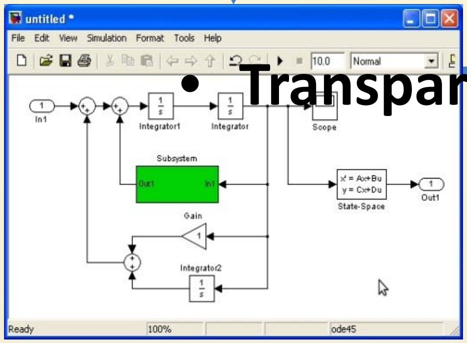
2. Control implementation in multicore processors

Codesign Using Arbitration

- **Shared resources – needs arbitration – leads to delays**
- **Transparencies in delays – used in our codesign**

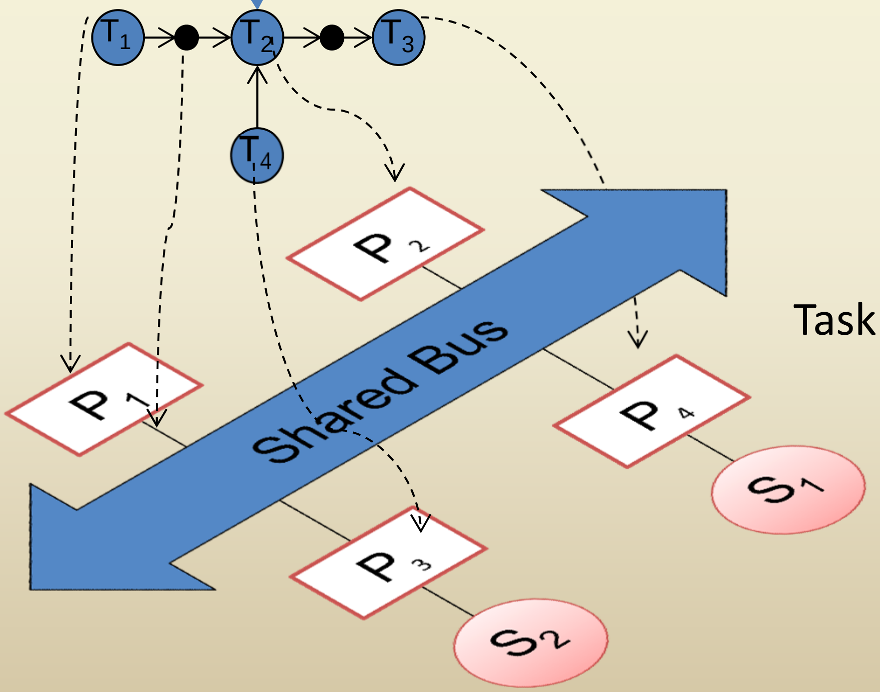


High-level Specification



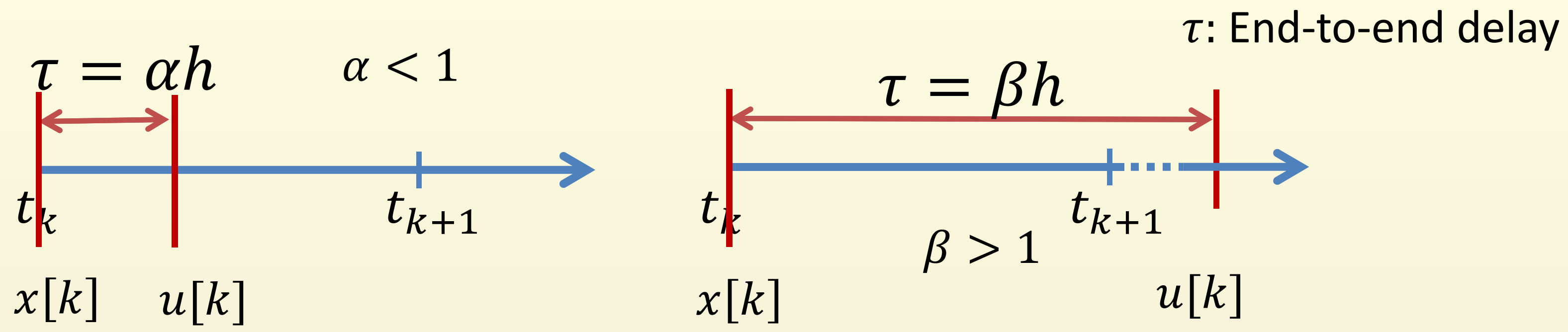
Controller Design

Task Partitioning



Task mapping and
scheduling

A window into embedded control



- At each t_k: Measure $x[k]$, compute $u[k]$after τ
- $\tau = \begin{cases} \alpha h, \\ \beta h, \end{cases}$ - depending on the applications serviced
- Control performance directly depends on τ
- Prior information about τ is highly useful.

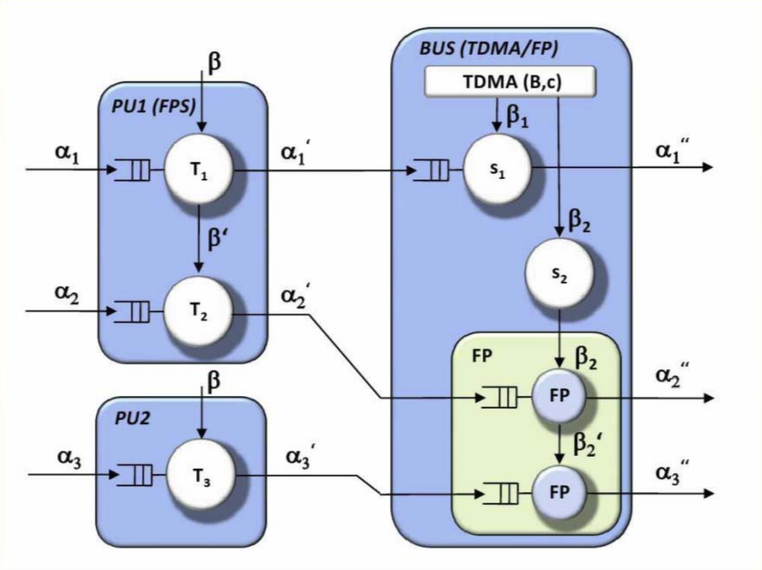
Delay Estimation Tool

- Worst case end-to-end delay: Real-time Calculus
- RTC establishes a link between three areas
  - Max-Plus Linear System Theory dealing with certain class of discrete systems
  - Network Calculus for establishing time bounds in communication networks
  - Real-time Scheduling
- Used for
  - Feasibility analysis
  - Optimal priority assignment for a general task
  - Estimating end-end delay and for co-design

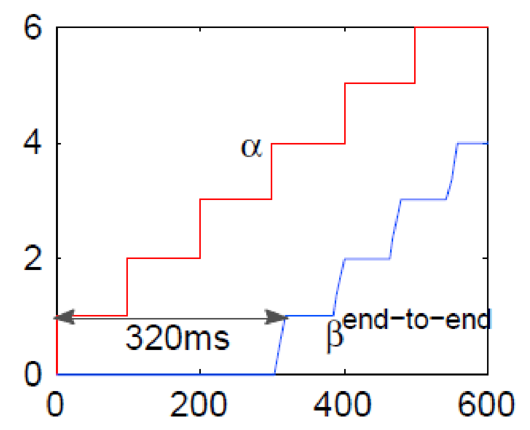
Worst Case End-to-end Delay

- Real-time Calculus
  - Consider message arrival curves α, service curves β, and any interval length Δ.
  - The maximum delay d that is experienced can be computed as:

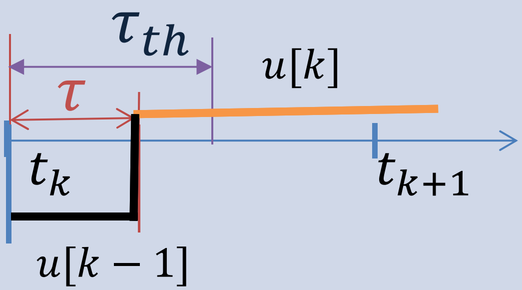
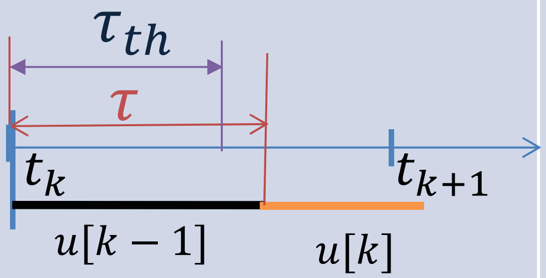
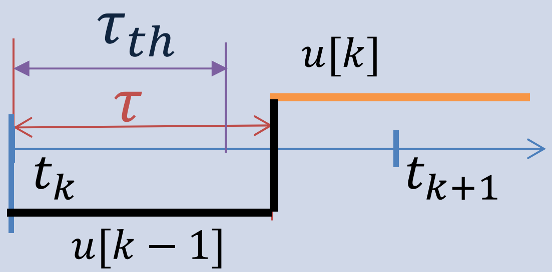
$$d = \sup\{\inf\{\tau \geq 0 \,|\alpha^{u}(\Delta) \leq \beta^{l}(s+\tau)|\, s \geq 0\}$$



System with hierarchical arbitration policy



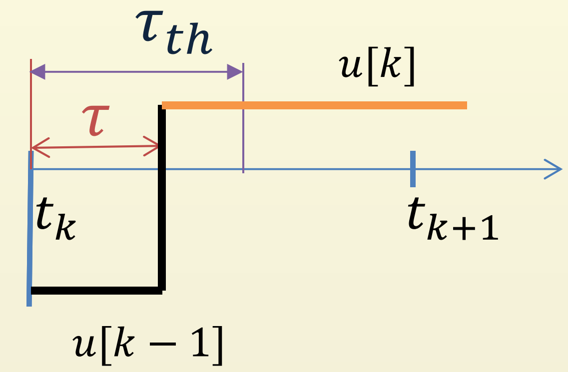
Possible Strategies

Nominal	Abort	Skip
		

Co-design Strategies

1. Nominal + Abort

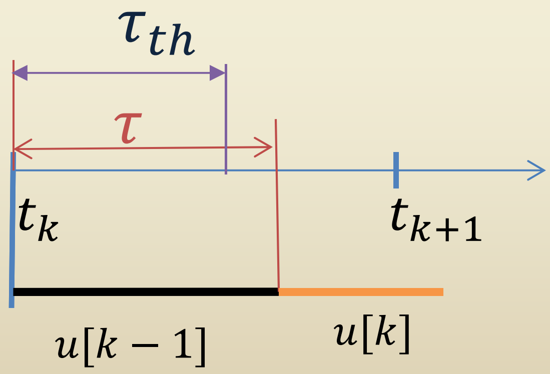
$$\begin{cases} \text{Compute nominal u[k]} & \text{if} \quad \boldsymbol{\tau} \leq \boldsymbol{\tau_{th}} \\ \text{Abort u[k]} & \text{if} \quad \boldsymbol{\tau} > \boldsymbol{\tau_{th}} \end{cases}$$



$u[k]$ is computed (nominal).

2. Drop Compensation Control

$$\begin{cases} \text{Compute nominal u[k]} & \text{if} \quad \boldsymbol{\tau} \leq \boldsymbol{\tau_{th}} \\ \text{Compute drop–based u[k]} & \text{if} \quad \boldsymbol{\tau} > \boldsymbol{\tau_{th}} \end{cases}$$



$u[k]$ is dropped.

Co-design Strategy 1

Nominal + Abort

$$\begin{cases} \text{Compute nominal u[k]} & \text{if} \quad \boldsymbol{\tau \le \tau_{th}} \\ \text{Abort u[k]} & \text{if} \quad \boldsymbol{\tau > \tau_{th}} \end{cases}$$

Leads to:

1. Nominal

$$\begin{cases} x[k+1] = Ax[k] + B_1u[k] + B_2u[k-1] & \text{if} \quad \boldsymbol{\tau \le \tau_{th}} \\ u[k] = K\,x[k] & \end{cases}$$

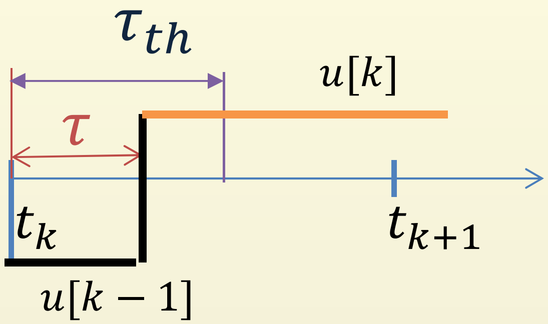
2. 1 Drop

$$\begin{cases} x[k+1] = Ax[k] + (B_1{+}B_2)u[k-1] & \text{if} \quad \boldsymbol{\tau > \tau_{th}} \\ u[k] = u[k-1] & \end{cases}$$

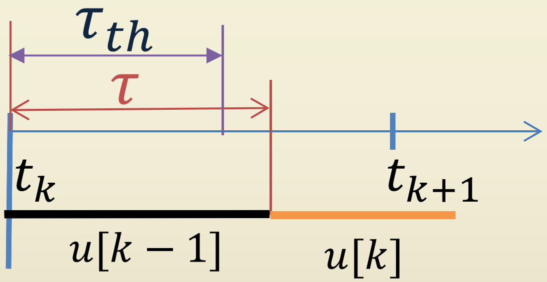
3. i Drops

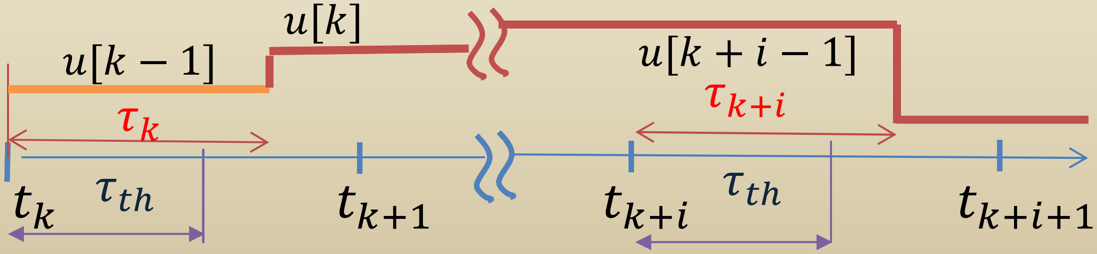
$$\begin{cases} x[k+1] = Ax[k] + (B_1{+}B_2)u[k-i] & \text{if} \quad \boldsymbol{\tau > \tau_{th}} \\ u[k] = u[k-i] & \end{cases}$$

Nominal



Drop

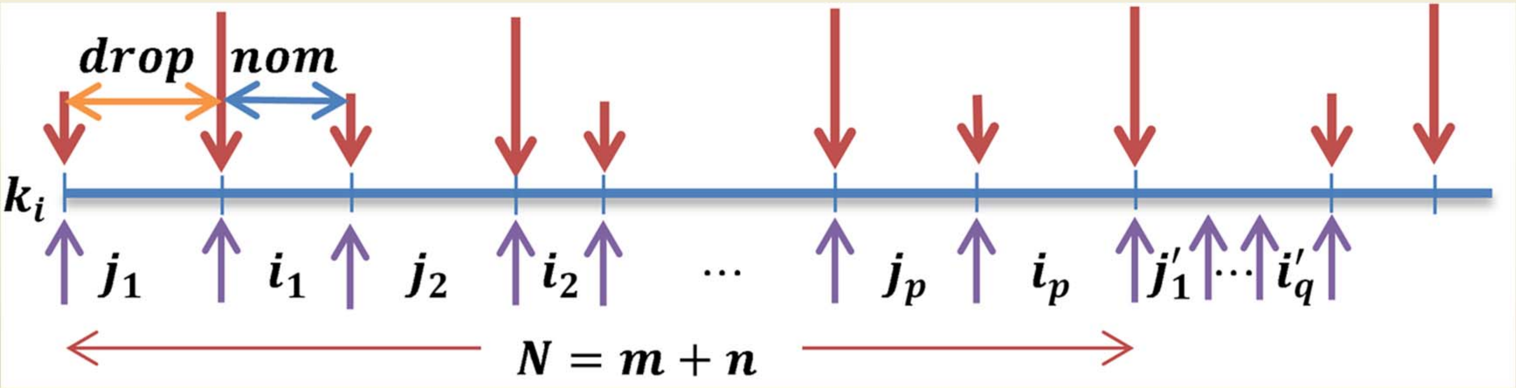




Overall switched system

$$\begin{cases} X[k+i_1] = A_m^{(j_1)} X[k] & \text{(Dropped Mode)} \\ X[k+N] = A_n^{i_p} A_m^{(j_p)} \cdots A_n^{i_2} A_m^{(j_2)} A_n^{i_1} X[k+i_1] & \text{(Stable Mode)} \end{cases}$$

$$A_m^{(j_1)} := \begin{bmatrix} A^{j+1} + A^j B_1 K + \sum_{l=0}^{j-1} A^l B K & A^j B_2 K \\ I & 0 \end{bmatrix}, \qquad A_n^{i_1} := \begin{bmatrix} A + B_1 K & B_2 K \\ I & 0 \end{bmatrix}^{i_1}$$



Control messages with nominal (i's) and drops (j's)

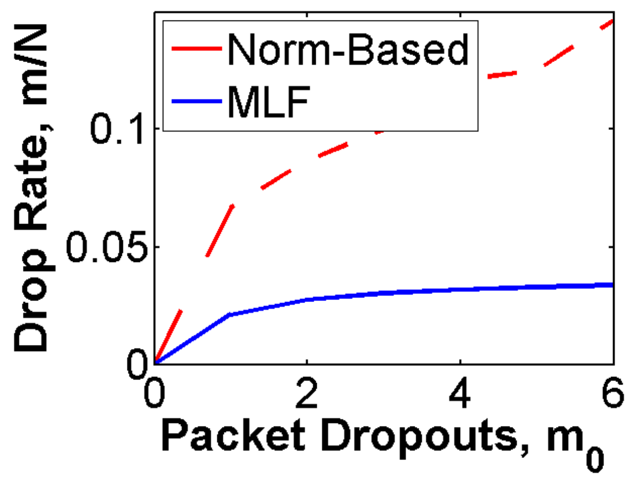
Stability with at most m_0 drops

$$X[k+N] = A_n^{i_p} A_m^{(j_p)} \cdots A_n^{i_2} A_m^{(j_2)} A_n^{i_1} A_m^{(j_1)} X[k]$$

Theorem 1:

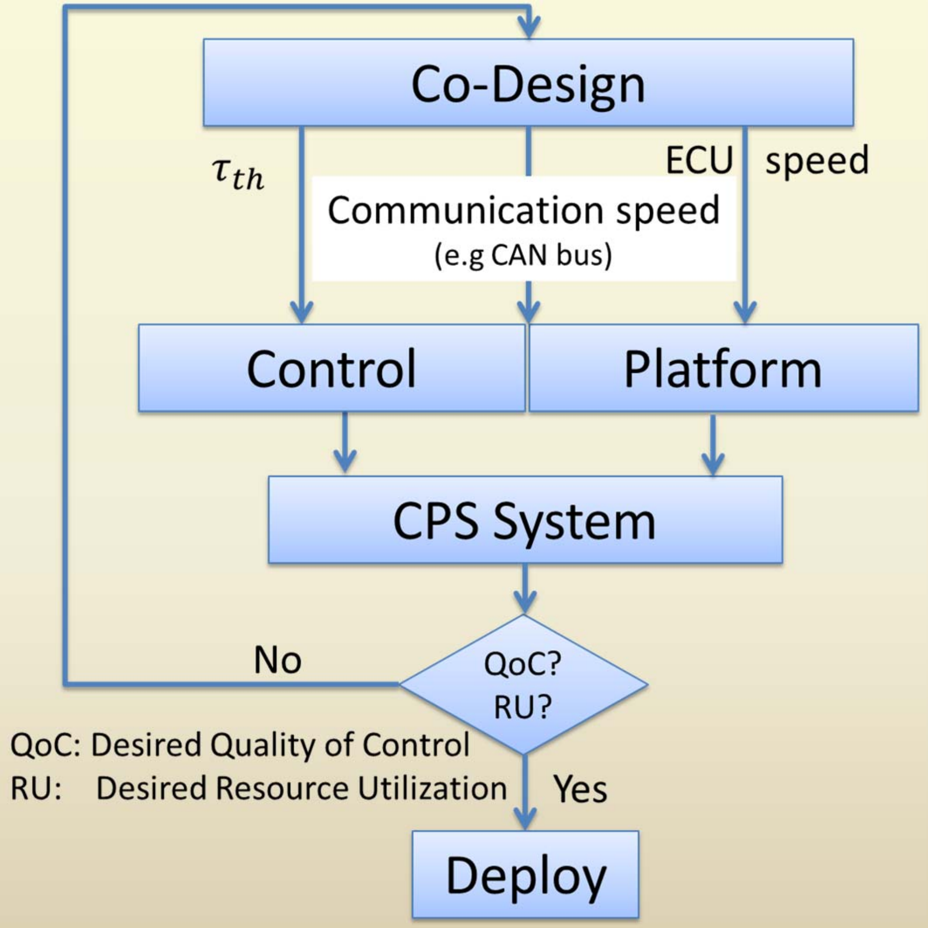
Over every interval N, there exists n_0 such that if there are at most $N - n_0$ drops, then the system is stable.

- LMI-based analysis
  - Multiple Lyapunov Functions(MLF)
    - Stable mode
    - Dropped mode
  - Benefits
    - Less conservative than norm-based approach
    - Drops can be non-consecutive



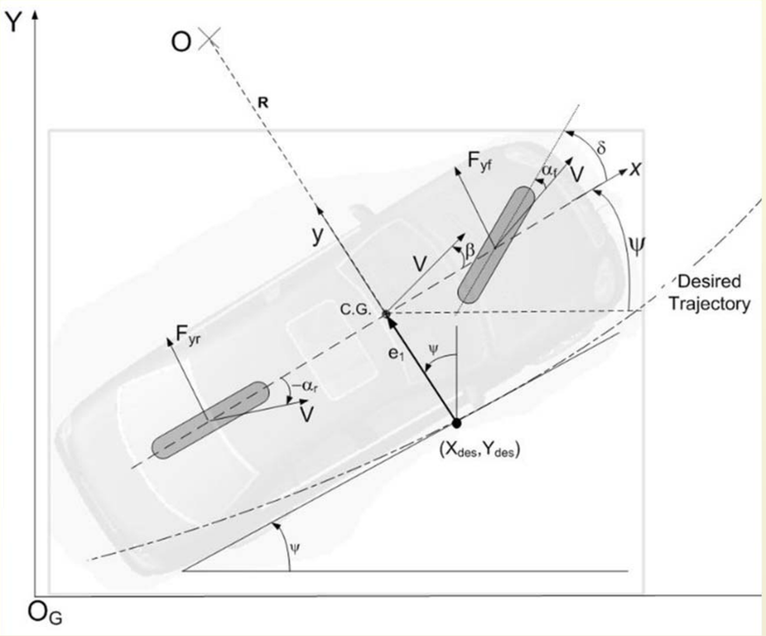
$m_0 = N - n_0$

Overall Co-Design



QoC: Desired Quality of Control
RU: Desired Resource Utilization

Case Study – Lane Keeping System

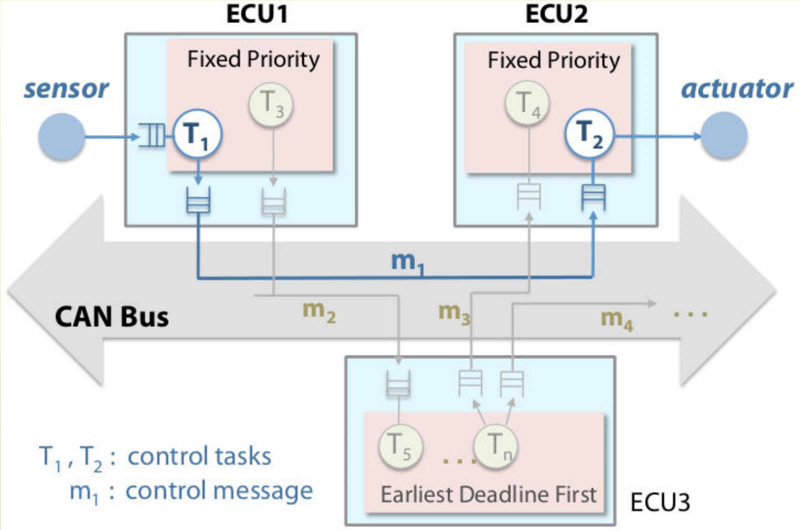


e_1: position error

e_2: yaw angle error

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = A_c \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + B_c\delta + G\psi_{des}$$

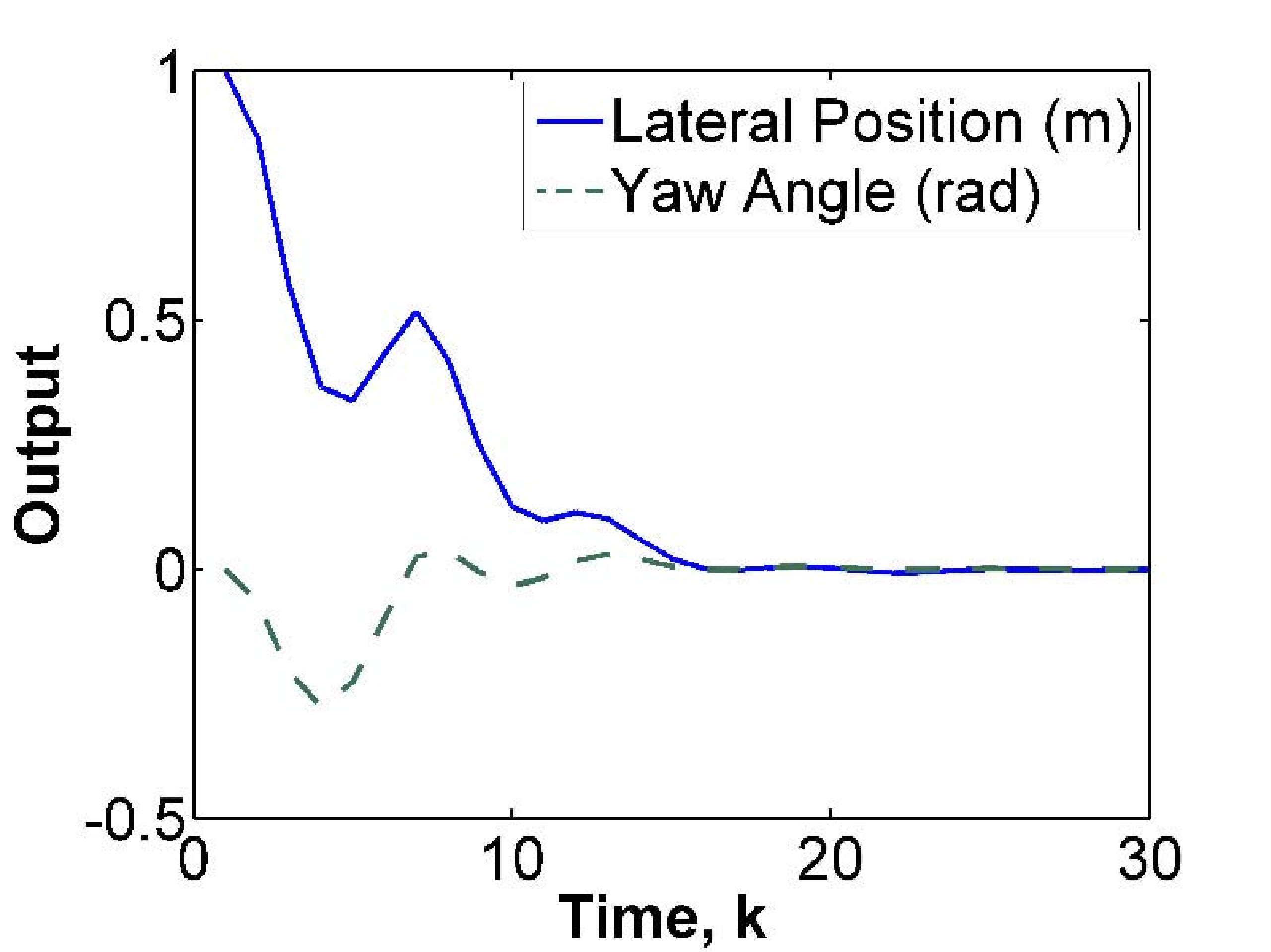
Goal: Help drivers to avoid unintended lane departure



T_1, T_2 : control tasks
m_1 : control message

- Higher priority tasks in ECU1 and ECU2 – can preempt control task
- ECU3 can place additional load on the CAN bus

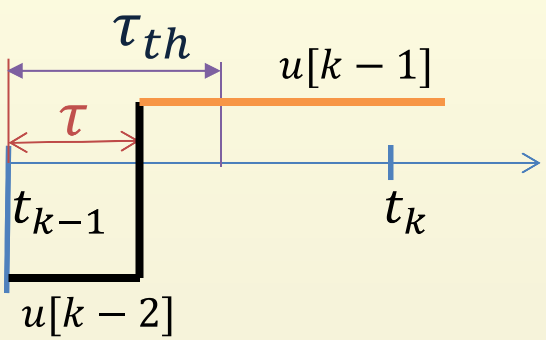
Results – Lane Keeping



N=40; m_0=7

Co-design Strategy 2

$$\begin{cases} \text{Compute nominal u[k]} & \text{if } \boldsymbol{\tau \le \tau_{th}} \\ \text{Compute drop–based u[k]} & \text{if } \boldsymbol{\tau > \tau_{th}} \end{cases}$$



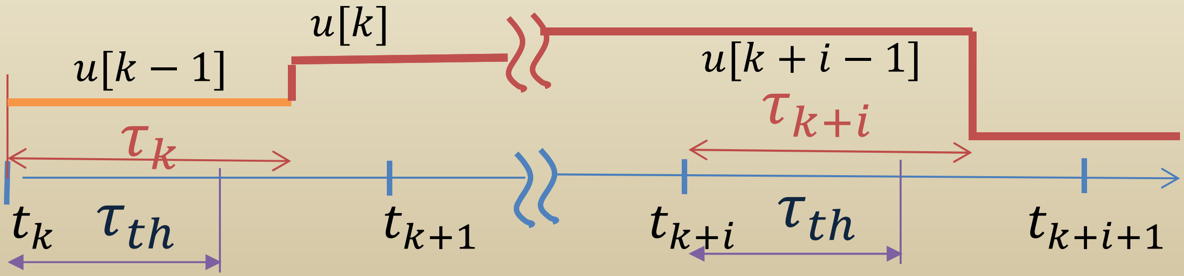
Example: nominal at t_{k-1}, i drops at $t_k, \dots, t_{k+i-1}$

$$\begin{cases} x[k] = Ax[k-1] + B_1u[k-1] + B_2u[k-2] \\ \quad u[k-1] = K_0\, x[k-1] + G_0u[k-2] \end{cases} \quad \text{if} \quad \boldsymbol{\tau \le \tau_{th}} \quad \textit{Nominal}$$

$$\begin{cases} x[k+1] = A_1x[k-1] + B_1u[k] + B_2u[k-1] \\ \qquad u[k] = K_1\, x[k-1] + G_1u[k-2] \end{cases} \quad \text{if} \quad \boldsymbol{\tau \le \tau_{th}} \quad 1\, \textit{Drop}$$

$$\begin{cases} x[k+i] = A_ix[k-i] + B_1u[k+i-1] + B_{2,i}u[k-2] \\ \quad u[k+i-1] = K_ix[k-1] + G_iu[k-2], \end{cases} \quad \text{if} \quad \boldsymbol{\tau \le \tau_{th}} \quad i\, \textit{Drops}$$

$$A_i = f(A, B_1, B_2, i), \qquad B_{2,i} = g(A, B_1, B_2, i) \qquad [K_i, G_i] = h(A, B_1, B_2, i)$$



Drop Compensation Control

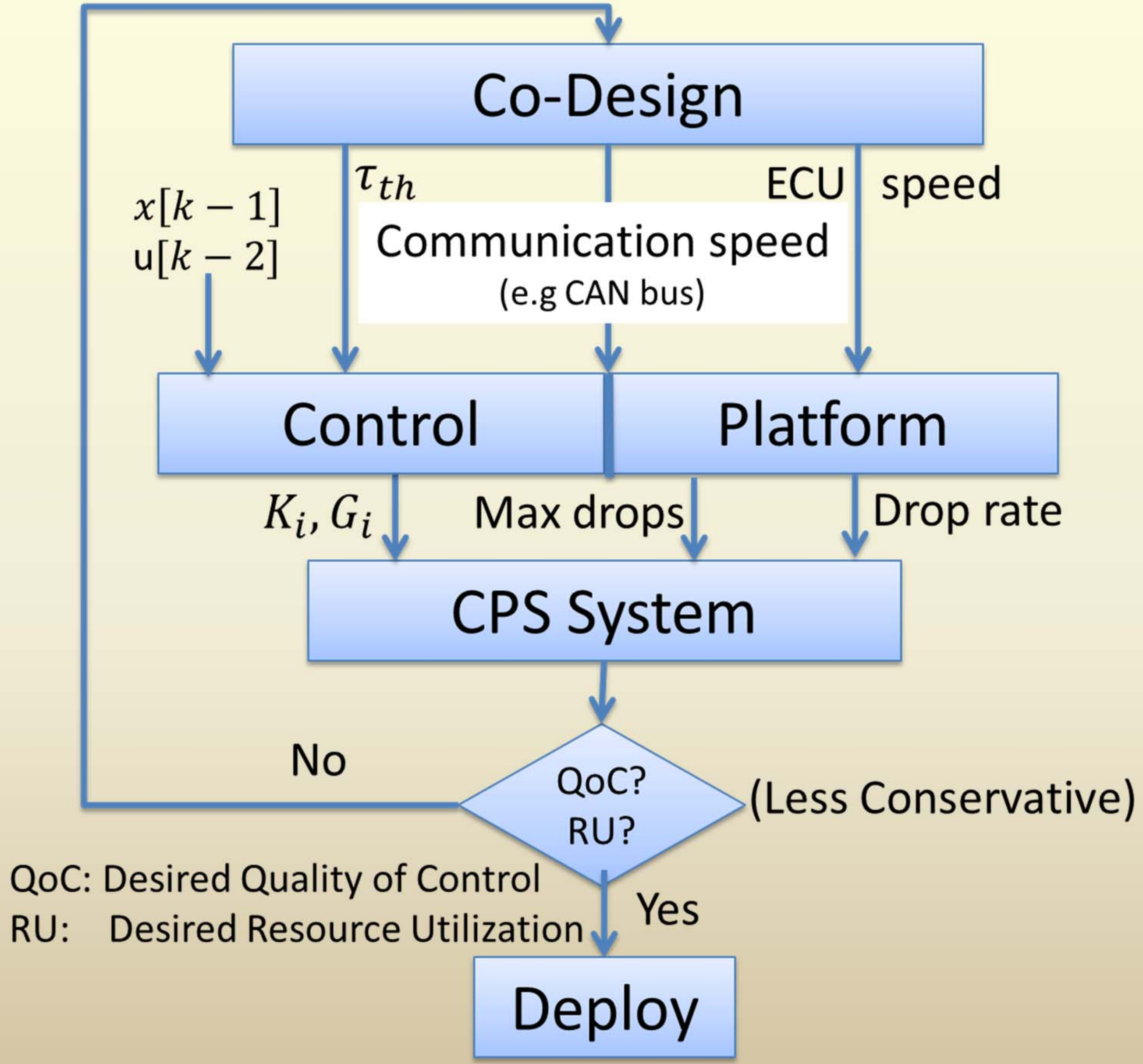
Stability with at most m_0 Consecutive Drops

Theorem 2:

The system with at most m_0 consecutive drops is stable, if there exists a Common Quadratic Lyapunov Function (CQLF) for the nominal and dropped modes of the system.

- LMI-based analysis
  - Common Quadratic Lyapunov Function (CQLF)
    - Nominal mode (K_0, G_0)
    - Dropped mode $(K_1, G_1, K_2\ , G_2, \cdots, K_i, G_i)$
  - Benefits
    - Increased robustness and stability
    - Guaranteed performance if K_i, G_i exist
- If drop rate $\frac{m_0}{N}$ is known, tighter design with guaranteed decay rate

Overall co-design



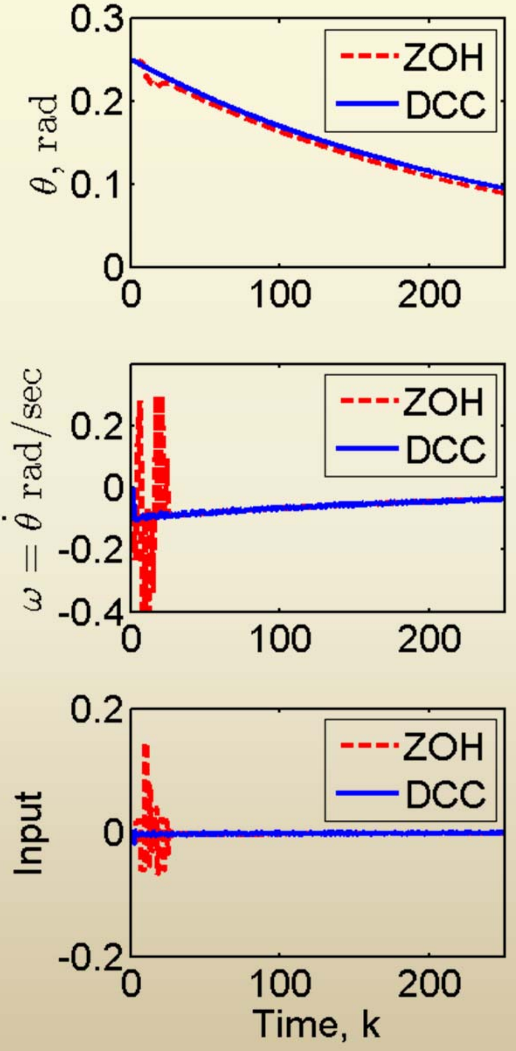
QoC: Desired Quality of Control
RU: Desired Resource Utilization

Results

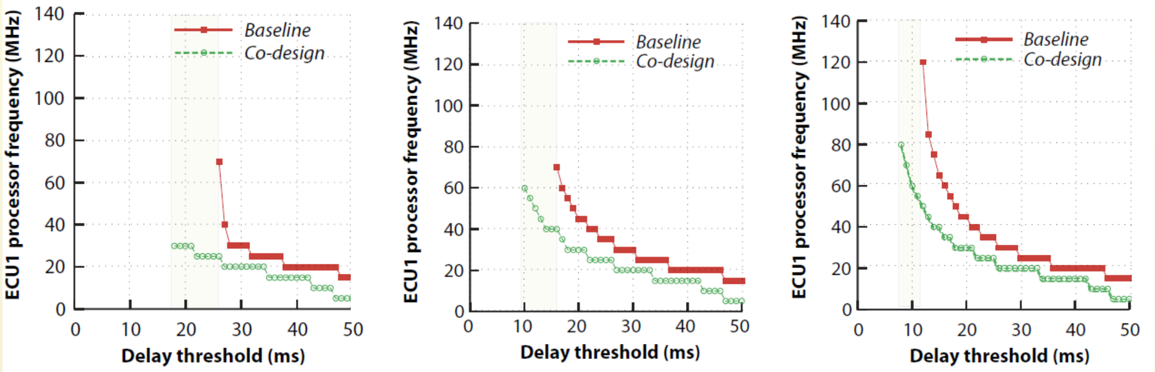
Example:

- Inverse Pendulum
- $A_c = \begin{bmatrix} 0 & 1 \\ 6.31 & -15.48 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1000 \end{bmatrix}$
- Sampling time: 10ms
- Delay threshold: $\tau_{th} = 3$ms
- Maximum 5 consecutive drops
- DCC: Drop Compensation Control
- ZOH: Regular Zero Order Hold
- Drop pattern:

SFSFFSFFSSFSSFSFS…

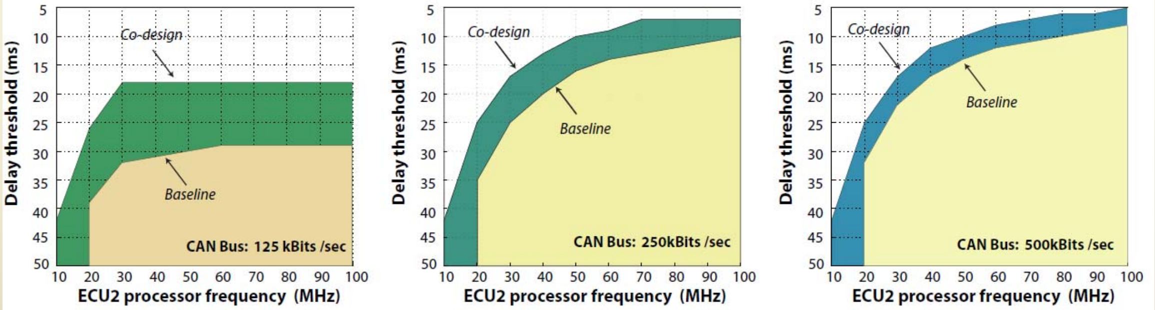


Co-design results (Strategy 1)



(a) ECU2 frequency = 30 MHz (b) ECU2 frequency = 60 MHz (c) ECU2 frequency = 90 MHz

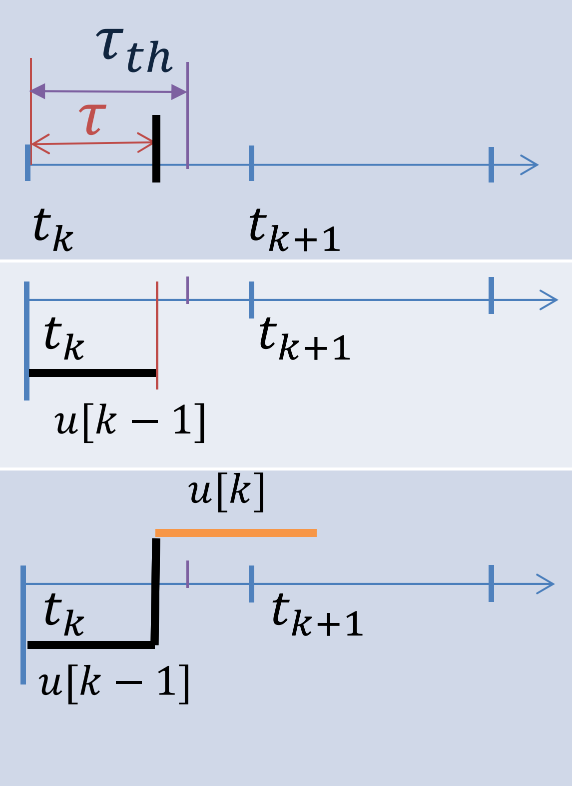
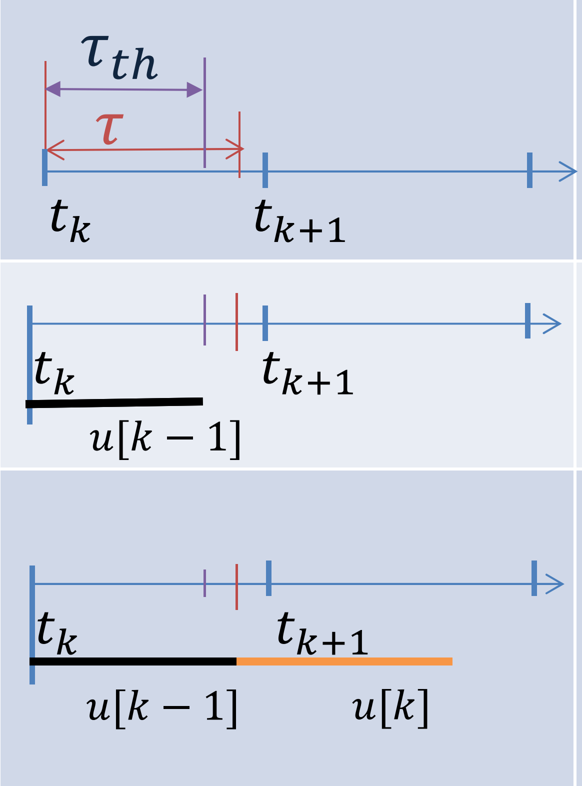
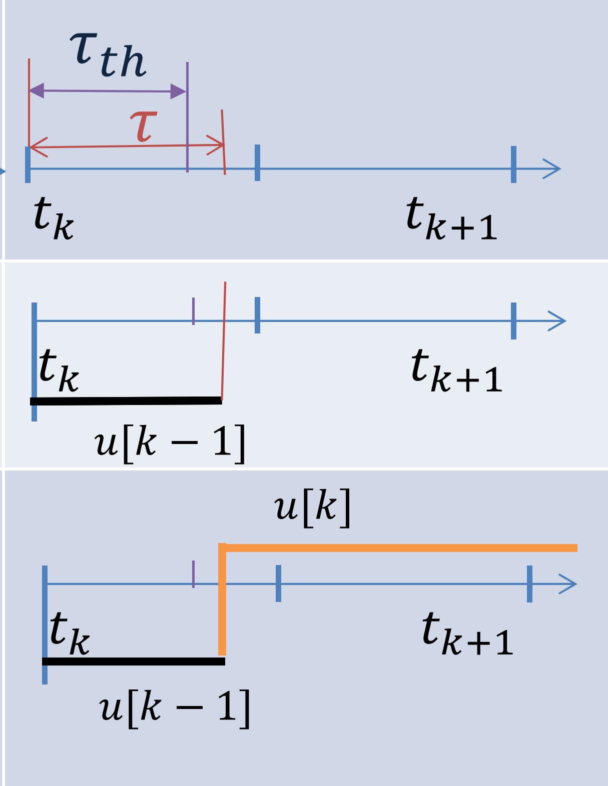
Figure 15: ECU1 processor frequency for different delay thresholds.



(a) Low speed bus (125 kBits/s) (b) Medium speed bus (250 kBits/s) (c) High speed bus (500 kBits/s)

- Our approach enables efficient design space exploration
- Co-design always outperforms the baseline approach
- Resource savings increase on more constrained platforms
- Co-design provides a larger feasible design space

Further Refinements

Nominal	Abort	Skip
		
Control: $u[k] = K_{LQR}x[k]$	$u[k] = u[k-1]$ (Abort Computations of $u[k]$)	$u[k+1] = u[k]$ (Skip computations of $u[k+1]$)

With Skip instead of Abort, better performance can be realized.

Summary

- Design of DES – an important substrate for CPS
- Co-design proposed for Distributed Embedded Systems
- Key ingredient: Arbitration
- Combined use of tools from real-time systems and control theory
- Efficient resource utilization
- Desired Quality of Control (ex. stability)

Selected Publications

1. Annaswamy A.M., Soudbakhsh D., Schneider R., Goswami D., Chakraborty S., “Arbitrated Network Control Systems: A co-design of control and platform for cyber-physical systems,” Control of Cyber-Physical Systems, Lecture Notes in Control and Information Sciences, Vol. 449, Ed: D.C. Tarraf, Springer Verlag, 2013.
2. Soudbakhsh D., Annaswamy A., “Parallelized model predictive control,” American Control Conference, Washington, DC, 2013.
3. Soudbakhsh D., Phan L.X, Sokolsky O., Lee I., and Annaswamy A., “Co-design of control and platform with dropped signals,” The 4th ACM/IEEE International Conference on Cyber-Physical Systems [ICCPS’13], Philadelphia, PA, 2013.
4. Masrur A., Goswami D., Chakraborty S., Chen J., Annaswamy A., Banerjee A., “Timing Analysis of Cyber-Physical Applications for Hybrid Communication Protocols”, Design, Automation, and Test in Europe (DATE2012), Dresden, Germany, March 2012
5. Voit H., Annaswamy A., Schneider R., Goswami D., Chakraborty S., “Adaptive Switching Controllers for Systems with Hybrid Communication Protocols”, American Control Conference (ACC 2012), June 2012.
6. P. Kumar, D. Goswami, S. Chakraborty, A. Annaswamy, K. Lampka, and L.Thiele, “A hybrid approach to cyber-physical systems verification”, in 49th Design Automation Conference, 2012.
7. Annaswamy A., Chakraborty S., Soudbakhsh D., Goswami D., “The Arbitrated Networked Control Systems Approach to Designing Cyber-Physical Systems”, NecSys 2012.
8. H. Voit, A. Annaswamy, R. Schneider, D. Goswami, S. Chakraborty , “Adaptive Switching Controllers for Tracking with Hybrid Communication Protocols”., Proceedings of the Conference on Decision and Control, Maui, HI, USA , 2012.