Beyond Stability: Performance as Efficiency and Disturbance Management in Smart Networked Systems

Chengda Ji, H. Giray Oral, Enrique Mallada and Dennice F. Gayme

Johns Hopkins University (CNS 1544771)

Project Motivation and Goals

Develop new techniques to characterize, predict and control cyber-physical networks in a manner that not only ensures stability but also optimizes performance by managing disturbances and improving efficiency. Application areas include power system, robotic and transportation networks

Power Systems

Goal: *Investigate the effect of heterogeneous droop gains on performance in inverter-based microgrids*

Approach:

- Use an output norm to quantify the system performance in terms of
  - Transient resistive losses
  - Transient deviations from synchrony

Microgrid Model

- frequency and voltage dynamics, accounting for active and reactive power flows.

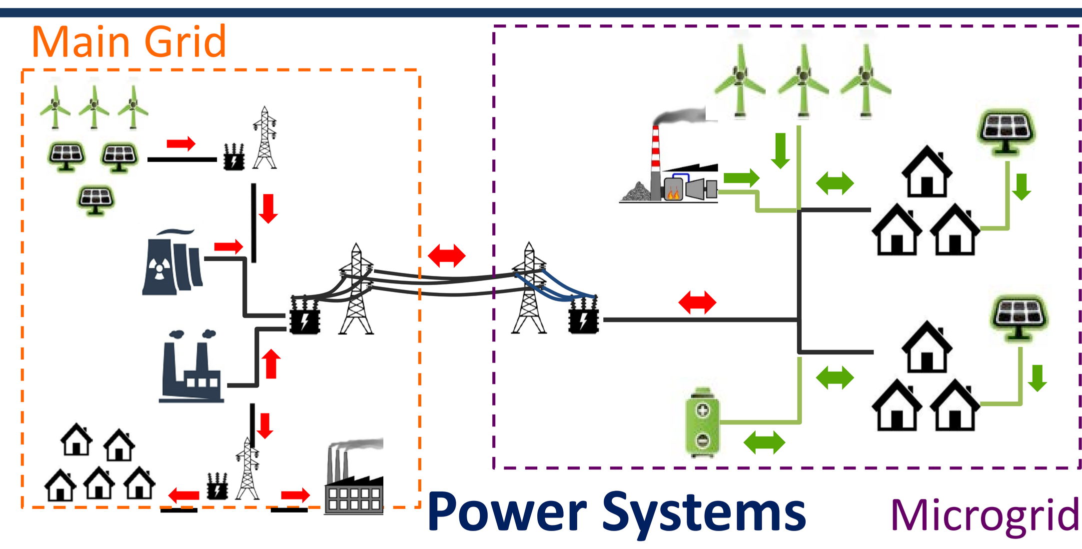
$$\dot{\theta}_i = \omega_i, \quad \tau_{P_i}\dot{\omega}_i = -\omega_i + \omega^* - k_{P_i}(P_i - P_i^*),$$

$$\tau_{Q_i}\dot{V}_i = -V_i + V_i^* - k_{Q_i}(Q_i - Q_i^*).$$

θ_i : phase angle $\quad \omega_i$: frequency $\quad V_i$: voltage magnitude

k_{P_i} k_{Q_i} : active and reactive power gains

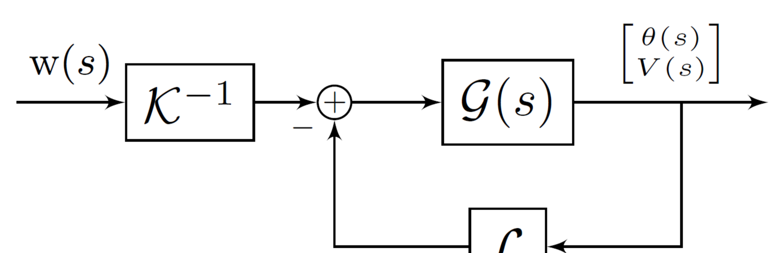
P_i Q_i : active and reactive power injected



$\mathrm{w} = \delta(t)\mathrm{w}_0$,: exogenous disturbance input

$$\dot{x} = Ax + B\mathrm{w},$$

$$x := [\theta^T \quad \omega^T \quad V^T]^T, \; \mathrm{w} := [(\mathrm{w}^P)^T \quad (\mathrm{w}^Q)^T]^T$$



$$\mathcal{L} := \begin{bmatrix} L_B & \\ & 2\bar{b}I + L_B \end{bmatrix} \quad \mathcal{K} := \begin{bmatrix} K_P & \\ & K_Q \end{bmatrix}$$

Assumption: decoupled frequency and voltage dynamics

Transient Resistive Loss Measure

$$\Pi_{loss} \approx \int_0^\infty \left[V(t)^T L_G V(t) + \theta(t)^T L_G \theta(t)\right] dt$$

If disturbances are uniform, i.e. $E\{\mathrm{w}_0\mathrm{w}_0^T\} = I$

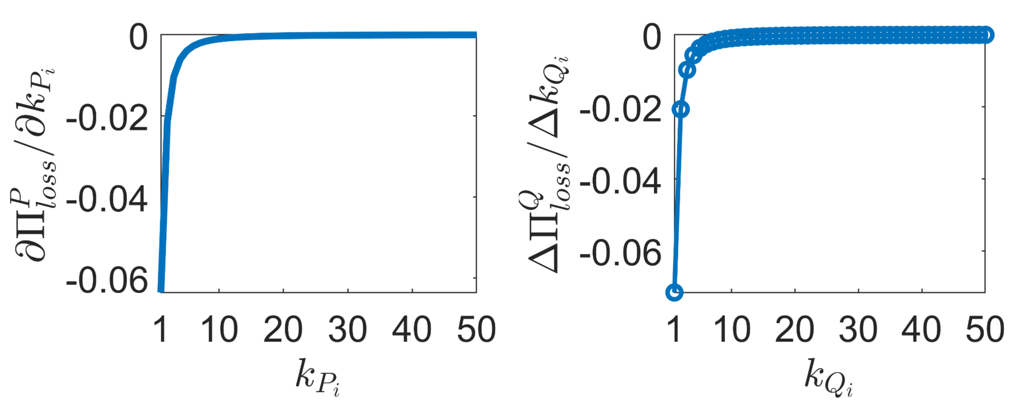
$$\Pi_{loss} = \frac{\alpha}{2}\left(\sum_{i=1}^N k_{P_i}^{-1} - \frac{\sum_{i=1}^N k_{P_i}^{-2}}{\sum_{i=1}^N k_{P_i}^{-1}}\right) + \frac{\alpha}{2\tau_Q}\sum_{i=1}^N \frac{u_i^T F_Q^{-1} u_i}{k_Q + \frac{1}{\lambda_i^Q}} - \Sigma^Q$$

Π_{loss}^P : independent of network topology

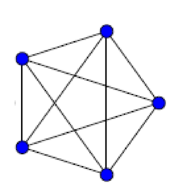
Π_{loss}^Q : function of gains and network topology

$$\frac{\partial}{\partial k_{P_l}}\left[\frac{\alpha}{2}\left(\sum_{i=1}^N k_{P_i}^{-1} - \frac{\sum_{i=1}^N k_{P_i}^{-2}}{\sum_{i=1}^N k_{P_i}^{-1}}\right)\right] < 0, \quad l = 1, \ldots, N$$

Π_{loss}^P monotonically decreasing in active power droop gains



Complete graph with unit edge weights



Transient Deviations from Synchrony

$$\omega(t) = \bar{\omega}(t)\mathbf{1} + \tilde{\omega}(t) \qquad \bar{\omega}(t) = \frac{\sum_{i=1}^N k_{P_i}^{-1}\omega_i(t)}{\sum_{i=1}^N k_{P_i}^{-1}}$$

synchronous frequency; deviations from synchrony; $\mathbf{1}_n \in \mathbb{R}^n$

$$\Pi_{sync} = \int_0^\infty \|\tilde{\omega}(t)\|_2^2 dt + \int_0^\infty \|V(t)\|_2^2 dt$$

$k_{P_i} \to \infty$ and $k_{Q_i} \to \infty$

$$\Pi_{sync} \;\to\; \frac{\alpha}{2\tau_P}\sum_{i=1}^{N-1} \phi_{ii}^P (z_{0i}^P)^2$$

- Voltage deviations vanish in the large gain limit
- Further mitigation of frequency deviations requires inertia

$k_{P_i} \to 0$ and $k_{Q_i} \to 0$

$$\Pi_{sync} \;\to\; \frac{\alpha}{2\tau_P}\sum_{i,j=1}^{N-1} \phi_{ij}^P z_{0i}^P z_{0j}^P + \frac{\alpha}{2\tau_Q}\sum_{i,j=1}^{N} \phi_{ij}^Q z_{0i}^Q z_{0j}^Q$$

1. H. G. Oral and D. F. Gayme, "Performance of droop-controlled microgrids with heterogeneous inverter ratings," submitted to European Control Conference 2019.
2. E. Tegling, H. G. Oral, D. F. Gayme, and H. Sandberg, "Performance Metrics and Trade-Offs in Frequency and Voltage Droop Control of Microgrids," in preparation

Noisy Consensus Networks

Goal: *Characterize robustness in coupled oscillator systems (e.g. robotic, power and transportation networks) with measurement error and explore trade-offs between relative and absolute feedback in systems connected over directed graphs.*

Approach:

- Define a family of controllers that continuously trade between relative and absolute feedback interconnection structures, i.e.

$$u_i = -\alpha \sum_{(i,j)\in\mathcal{E}_\alpha} a_{ij}(\hat{x}_i - \hat{x}_j) - (1-\alpha)a\hat{x}_i$$
$$-\beta \sum_{(i,j)\in\mathcal{E}_\beta} b_{ij}(\hat{v}_i - \hat{v}_j) - (1-\beta)b\hat{v}_i.$$

- $\alpha, \beta = 1$ indicates purely relative feedback
- $\hat{x}_i = x_i + e_i^x, \hat{v}_i = v_i + e_i^v,$
- e^x, e^v model measurement noise.

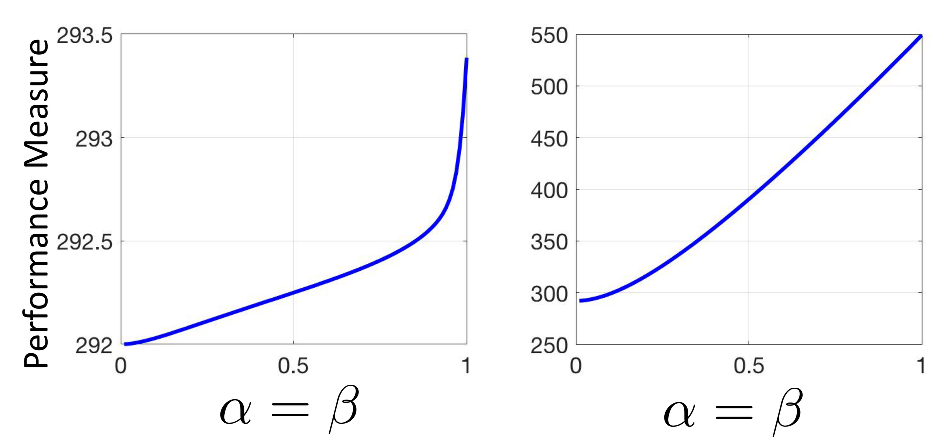
- The controlled dynamics are then

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}0 & I\\ G & F\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} + \begin{bmatrix}0 & I\\ G & F\end{bmatrix}\begin{bmatrix}e^x\\ e^v\end{bmatrix}$$

F and *G* are weighted Laplacians describing the directed interconnection graphs of x and v.

Results:

- Similar to systems with symmetric feedback the absence of absolute control, the systems performance is unbounded in any direction except for the directions orthogonal to vector $[\mathbf{1}_n^T,\ 0^T]^T$, i.e. the system average.

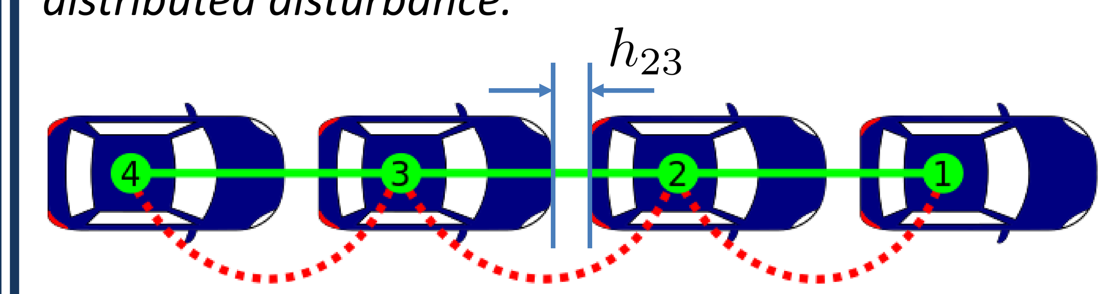


The performance degrades smoothly as the proportion of relative feedback is increased, i.e. as $\alpha, \beta \to 1$.

C. Ji, E. Mallada and D. F. Gayme, "Evaluating Robustness of Consensus Algorithms Under Measurement Error over Digraphs," in *57th IEEE Conf. on Decision and Control (to appear)*.

Vehicle Platoons

Goal: *Evaluate the collision potential of any pair of nodes in vehicle or robotic network subject to distributed disturbance.*



A vehicular network with line graph structure

Approach:

- System model:

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}0 & I\\ G & F\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} w$$

$$y = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}$$

where w is the disturbance and the output measures the distance between nodes *i* and *j*, e.g.

$$C = [0 \cdots 1 \cdots -1 \cdots 0]$$

- We then evaluate the system performance P_{ij} in terms of an $\mathcal{L}_2$ to $\mathcal{L}_\infty$ induced norm.

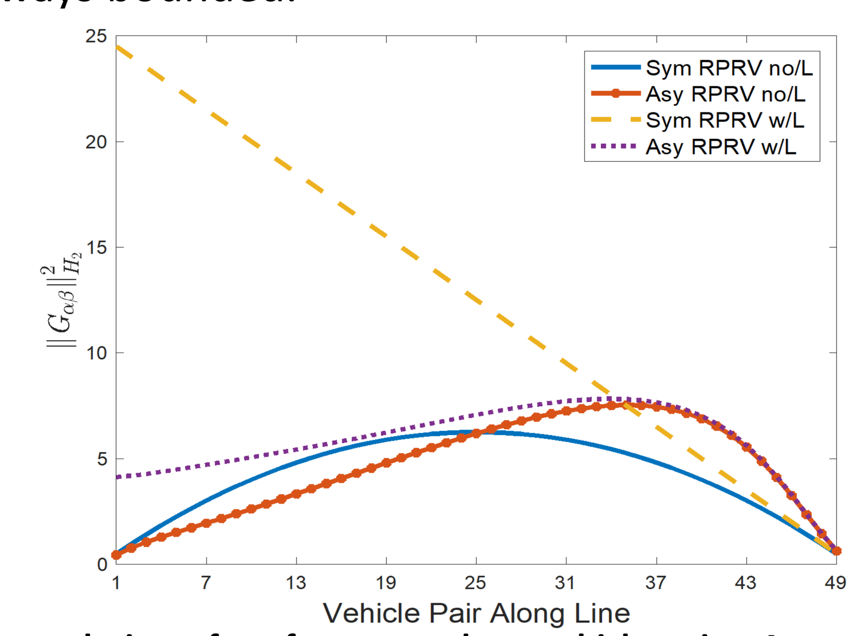
Results

- The maximum disturbance tolerance of the system

$$\|w\|_{\mathcal{L}_2} < \min_{i,j\in\mathcal{E}_c} \frac{h_{ij}}{\sqrt{P_{ij}}}$$

where $\mathcal{E}_c$ is the collision graph.

- Our results also show that for strongly connected graphs, the zero eigenvalues associated with the Laplacian matrix will not contribute to the system performance, therefore the system performance is always bounded.



The evolution of performance along vehicle pairs. Asymmetry (cases Asy.) degrades the average performance (collision potential) of vehicle pairs in platoons with and without Leaders (respectively labeled no/L and w/L)

C. Ji and D. F. Gayme, "Collision Potential Analysis in First and Second Order Integrator Networks Over Strongly Connected Digraphs," ," Proc. Of *57th IEEE Conf. on Decision and Control (to appear)*.