Design of in-line controllers for continuously operating networks with structural uncertainty (CAREER #2000302)

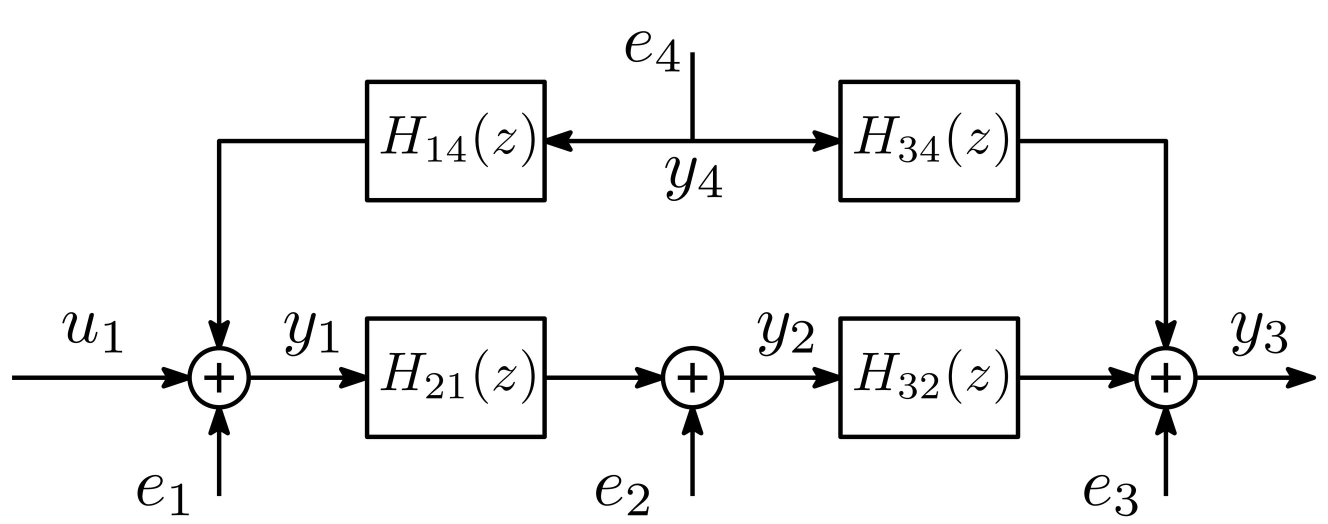
P.I.: Donatello Materassi, Computer and Electrical Engineering Department, University of Minnesota

Linear Dynamic Influence Networks: an I/O model for noninvasively sensed systems

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} + \begin{pmatrix} H_{11}(z) & \dots & H_{1n}(z) \\ \vdots & \ddots & \vdots \\ H_{n1}(z) & \dots & H_{nn}(z) \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

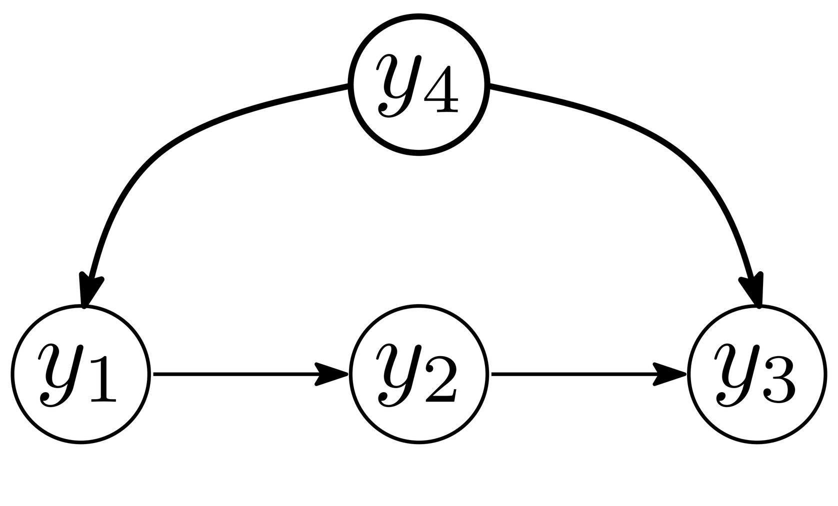
- The inputs y_i are observable
- The inputs u_i are unknown
- u_i and u_j are independent, for $i \neq j$

A graphical model interpretation



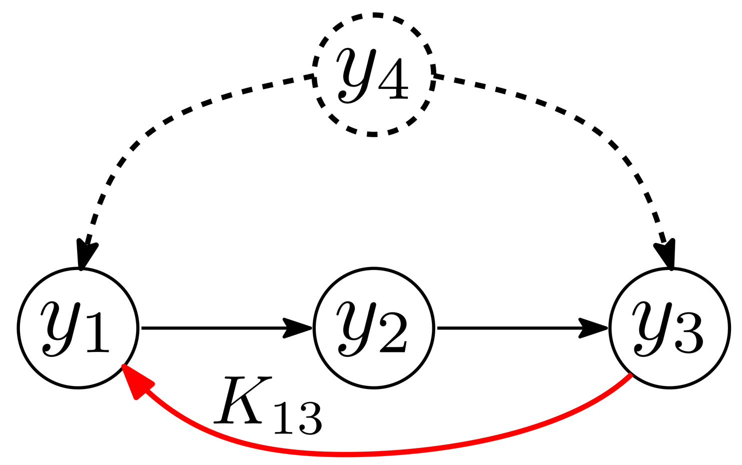
Given a block diagram

- the outputs are nodes
- if $H_{ji}(z) \neq 0$ draw, a link from y_i to y_j.



This representation extends graphical models of random variables to describe stochastic processes

Design Control Challenge I



- Transfer functions are unknown
- Graph is partially known (with unknown structure)
- y_4 is not measurable (and not known if it is present)
- Known Power Spectral Densities $\Phi_{y_i y_j}$, $i, j = 1, 2, 3$

Find the closed loop behavior

In the absence of y_4 the solution is easy

$$H_{32}(z)H_{21}(z) = \frac{\Phi_{y_3 y_2}}{\Phi_{y_2}} \frac{\Phi_{y_2 y_1}}{\Phi_{y_1}}$$

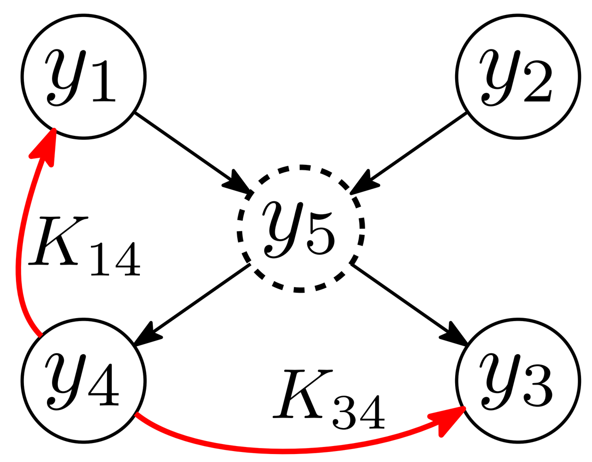
$$G_{y_1 \to y_3} = \frac{H_{32}H_{21}}{1 - H_{32}H_{21}} = \frac{\frac{\Phi_{y_3 y_2}}{\Phi_{y_2}}\frac{\Phi_{y_2 y_1}}{\Phi_{y_1}}}{1 - K_{13}\frac{\Phi_{y_3 y_2}}{\Phi_{y_2}}\frac{\Phi_{y_2 y_1}}{\Phi_{y_1}}} = \frac{\frac{\Phi_{y_3 y_1}}{\Phi_{y_1}}}{1 - K_{13}\frac{\Phi_{y_3 y_1}}{\Phi_{y_1}}}$$

In the presence (or absence) of y_4 (see [1])

$$G_{y_1 \to y_3} = \frac{\frac{\Phi_{y_2 y_1}}{\Phi_{y_1}} \begin{pmatrix} \Phi_{y_3 y_1} & \Phi_{y_3 y_2} \end{pmatrix} \begin{pmatrix} \Phi_{y_1} & \Phi_{y_1 y_2} \\ \Phi_{y_2 y_1} & \Phi_{y_2} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{1 - K_{13}\frac{\Phi_{y_2 y_1}}{\Phi_{y_1}} \begin{pmatrix} \Phi_{y_3 y_1} & \Phi_{y_3 y_2} \end{pmatrix} \begin{pmatrix} \Phi_{y_1} & \Phi_{y_1 y_2} \\ \Phi_{y_2 y_1} & \Phi_{y_2} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

This formula is robust with respect to uncertainties in the structure: a novel notion of robustness!

Design Control Challenge II



- Transfer functions are unknown
- Graph is a tree (with unknown structure)
- y_5 is not measurable and it is not known if it is present
- Known Power Spectral Densities $\Phi_{y_i y_j}$, $i, j = 1, 2, 3, 4$

Find the closed loop behavior

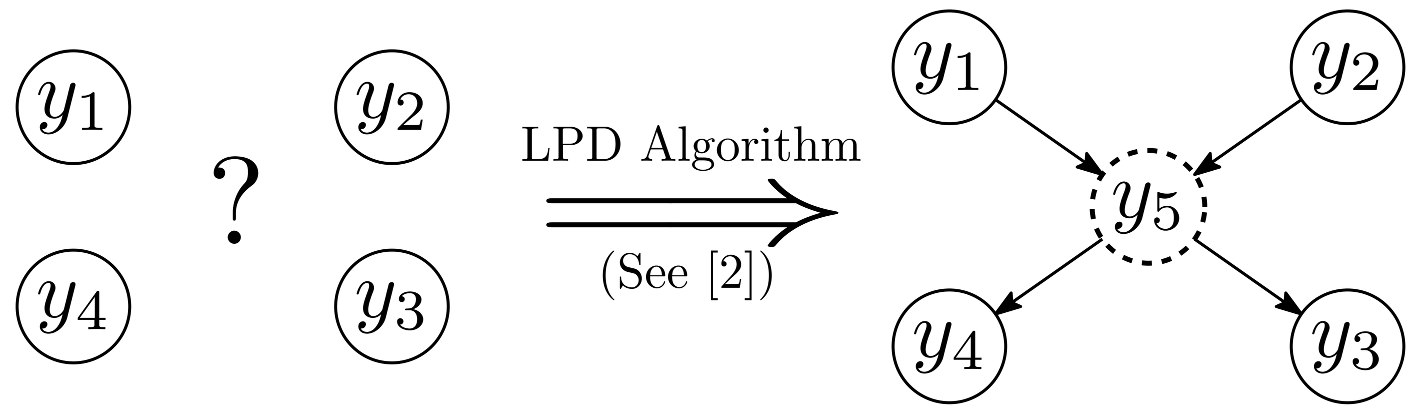
A distance that is additive along paths of a rooted tree, such as

$$d(y_i, y_j) = -\int_0^{2\pi} \log \left(\frac{|\Phi_{e_i e_j}|^2}{|\Phi_{e_i}||\Phi_{e_j}|} \right) d\omega,$$

enables the detection of hidden nodes (see [2])

$$y_5 \text{ is present} \Leftrightarrow d(y_1, y_4) - d(y_4, y_3) \neq d(y_1, y_3)$$

enables the inference of skeleton and link orientations (see [2])



$$G_{y_1 \to y_4} = \frac{\Phi_{y_4 y_1}}{\Phi_{y_1} - K_{14}\Phi_{y_4 y_1}}$$
$$G_{y_3 \to y_4} = 0$$

References

[1] D. Materassi, M. V. Salapaka, *Identification of network components in presence of unobserved nodes*, Conference on Decision and Control, 2015

[2] F. Sepeher, D. Materassi, *Inferring the structure of polytree networks of dynamic systems with hidden nodes*, Conference on Decision and Control, 2016