Embracing Complexity: A Fractal Calculus Approach to the Modeling and Optimization of Medical Cyber-Physical Systems

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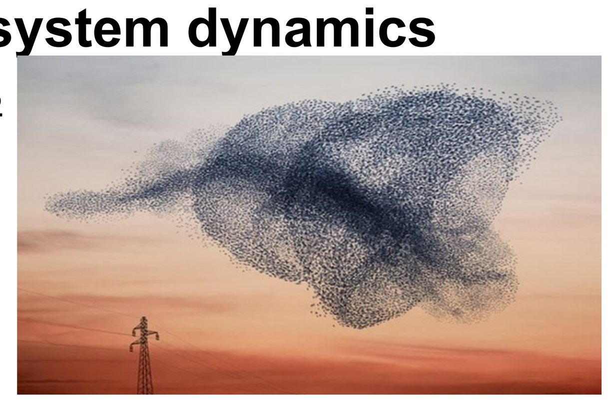
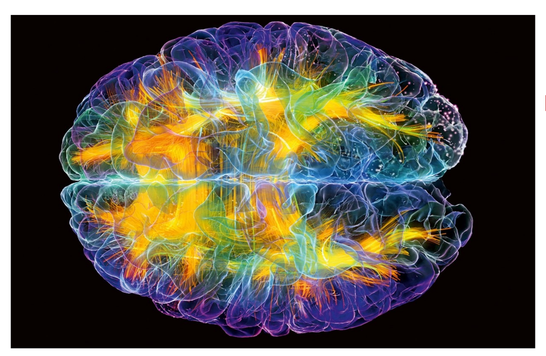
Embrace Complex Systems

- ***No precise relation*** **between the** ***input*** **and** ***output***
- ***Non-linear*** **and** ***unpredictable*** **system dynamics**
- **Collectively intercoupled dynamics**
  - ***Impossible*** **to analyze individually**
  - **Recognizable** ***collective behavior***
  - **High degree of** ***self-organization, emergence, collective perception***

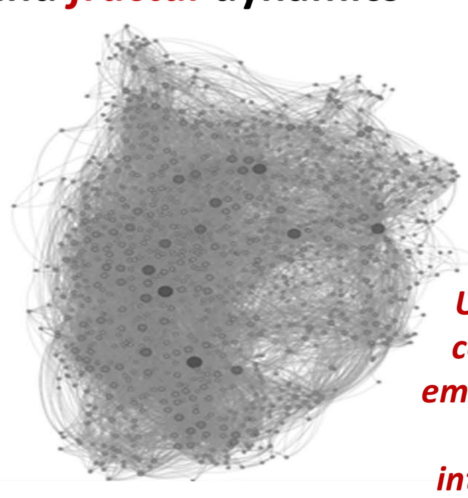


Collective behavioral dynamics



Complex biological and physiological coupling

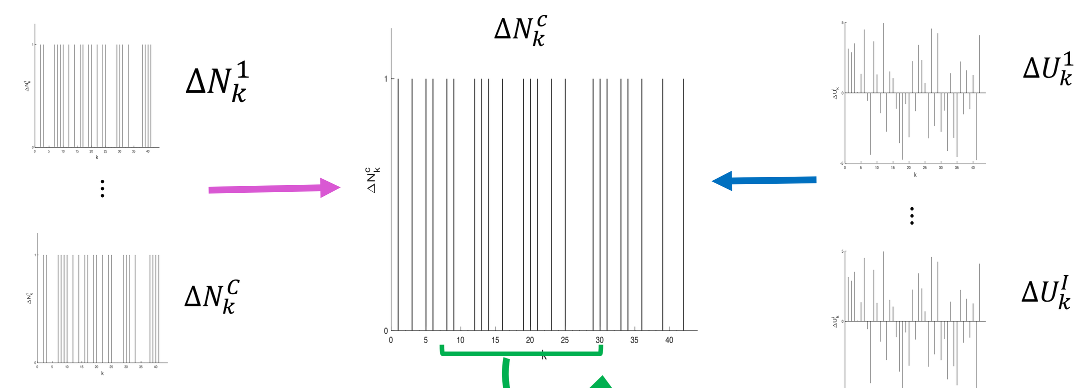
- **Complex spatio-temporal dependencies**
  - ***Non-Markovian*** **stochasticity**
  - ***Long-term*** **memory and dependencies**
  - ***Non-linear*** **and** ***fractal*** **dynamics**
- **Intricate complexity of target system**
  - ***Heterogeneous*** **interactions among** ***exorbitantly large*** **set of system components**
  - ***Under-explored*** **underlying action patterns**
  - ***Difficult*** **for interferential analysis**



Untangled complexity embedded in system interactions

Computational Laws & Neural Architectures

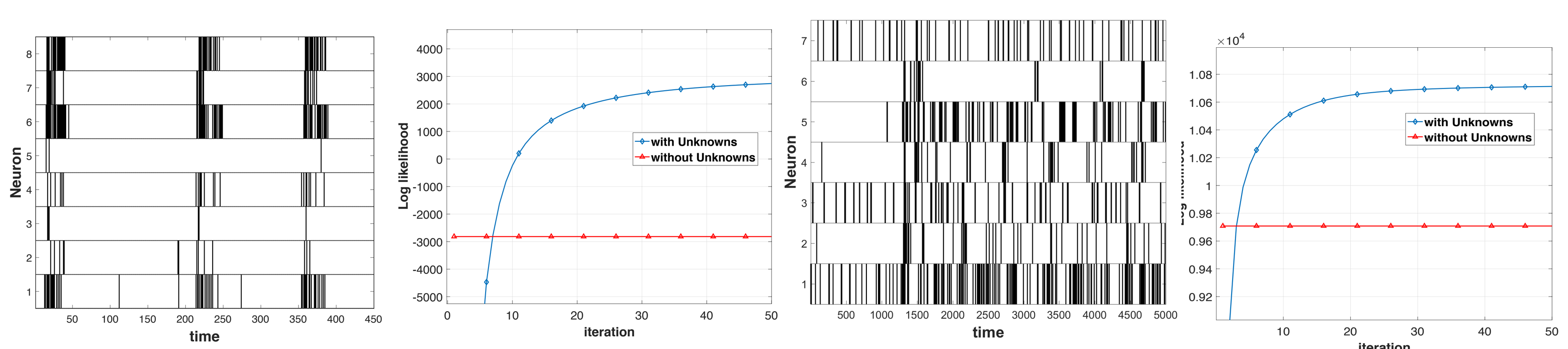
- **Decipher neural activities and model neurons interaction from sparse data and with unknown unknowns (UUs)**
  - Neural spikes available as binary data $\Delta N \in \{0,1\}$ for neurons across time
  - Spiking behavior modeled using generalized intensity function with covariates
  - Generalized linear model (GLM) framework



- $\log\lambda^c(k|\mathcal{H}_k;\Theta) =$

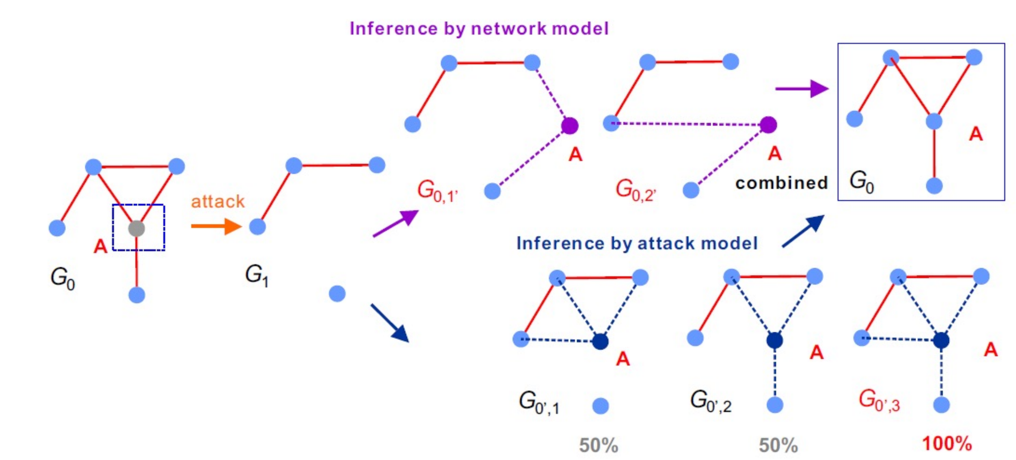
$$\alpha(c) + \underbrace{\sum_{c'=1,c'\neq c}^{C}\sum_{r=1}^{R}\beta_r^{c'}(c)\Delta N_{k-r}^{c'}}_{\text{Extrinsic covariates}} + \underbrace{\sum_{q=1}^{Q}\varepsilon_q(c)\Delta N_{k-q}^{c}}_{\text{Intrinsic covariates}} + \underbrace{\sum_{i=1}^{I}\sum_{m=1}^{M}\gamma_m^i(c)\Delta U_{k-m}^i}_{\text{Unknown covariates}}$$

- Fixed-point iteration with contraction to have fast converging solutions



Causal Inference of CNs / Unknown Unknowns

- **Recover latent CNs under time-varying adversarial interventions**



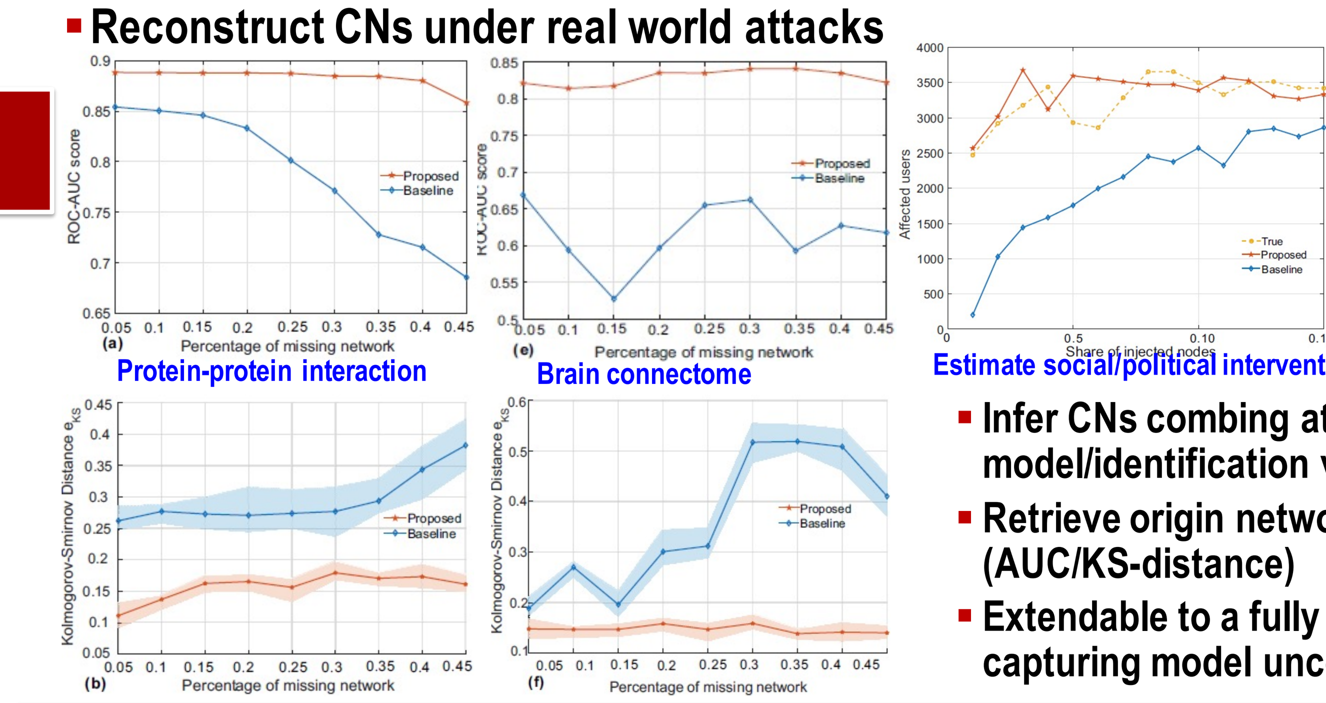
Reconstruction = attack model + network model causal inference

$$\arg\max_{L(t),G(t),\psi} P(\boxed{G(t)},\boxed{L(t)},\psi|M,A)$$

Model inference *Model identification*

- **Given**: CN $G(t)$ **partially observable due to attack** $A = \{A(s)\}$ at time t, $s = \{0,\ldots, t\text{-}1\}$. $G(t)$ obeys a stochastic generative CN model M with **unknown** parameters
- **Find: Latent victim subnetwork** $\boldsymbol{L(t)}$ **and time-stamp of its nodes** $\boldsymbol{\phi(L(t))}$
- Solve inference+identification ***at the same time***

- Machine Learning@CN: ***Causal inference*** + ***Expectation Maximization***
  - Maximum likelihood estimation with a series of maximization of in-complete LL
  - **Make causal Inference** of latent subnetwork $L(t)$ with current belief on M (E-step)
  - Provable and practically good convergence to solutions
  - **Reconstruct CNs under real world attacks**

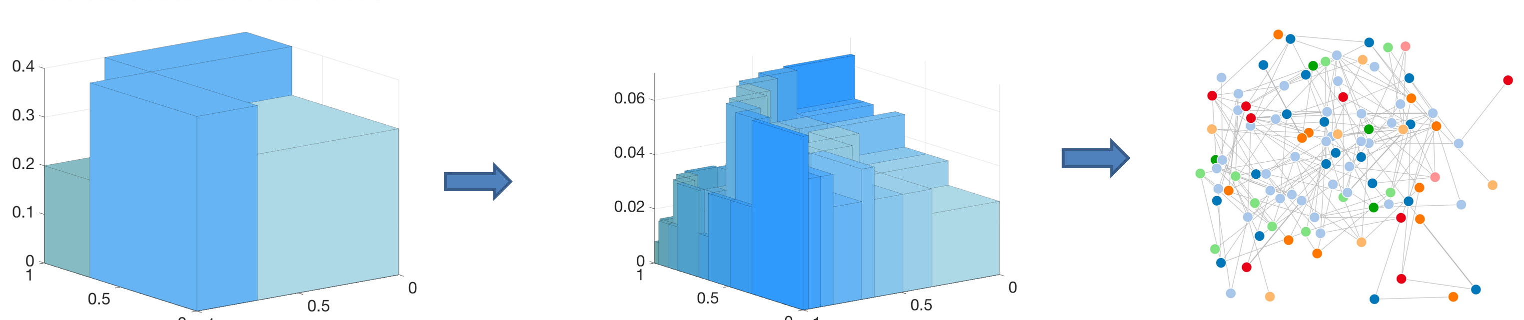


Protein-protein interaction | Brain connectome | Estimate social/political intervention

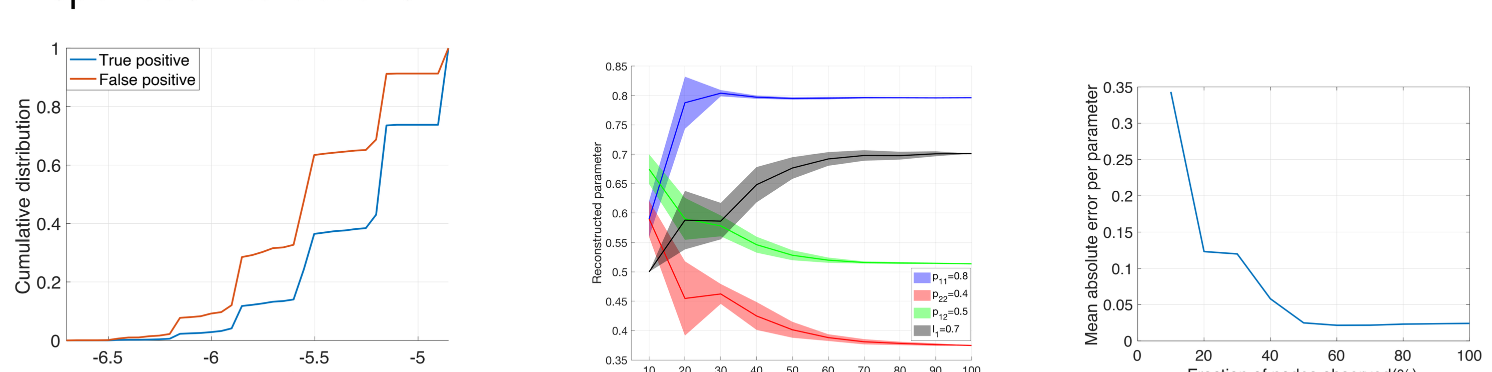
- **Infer CNs combing attack/network model/identification via EM**
- **Retrieve origin network with better fidelity (AUC/KS-distance)**
- **Extendable to a fully Bayesian method capturing model uncertainty**

Inferring & controlling unknown CPS dynamics

- **Weighted multifractal network generator for accurate modeling of complex neuronal networks**



- Capturing heterogeneity & multifractality encoded in the networks at functional level
- Decoding & representing complexity encoded in the interaction weights
- **Uncertainty:** Incomplete observations, noisy system data, detecting missing / spurious interactions

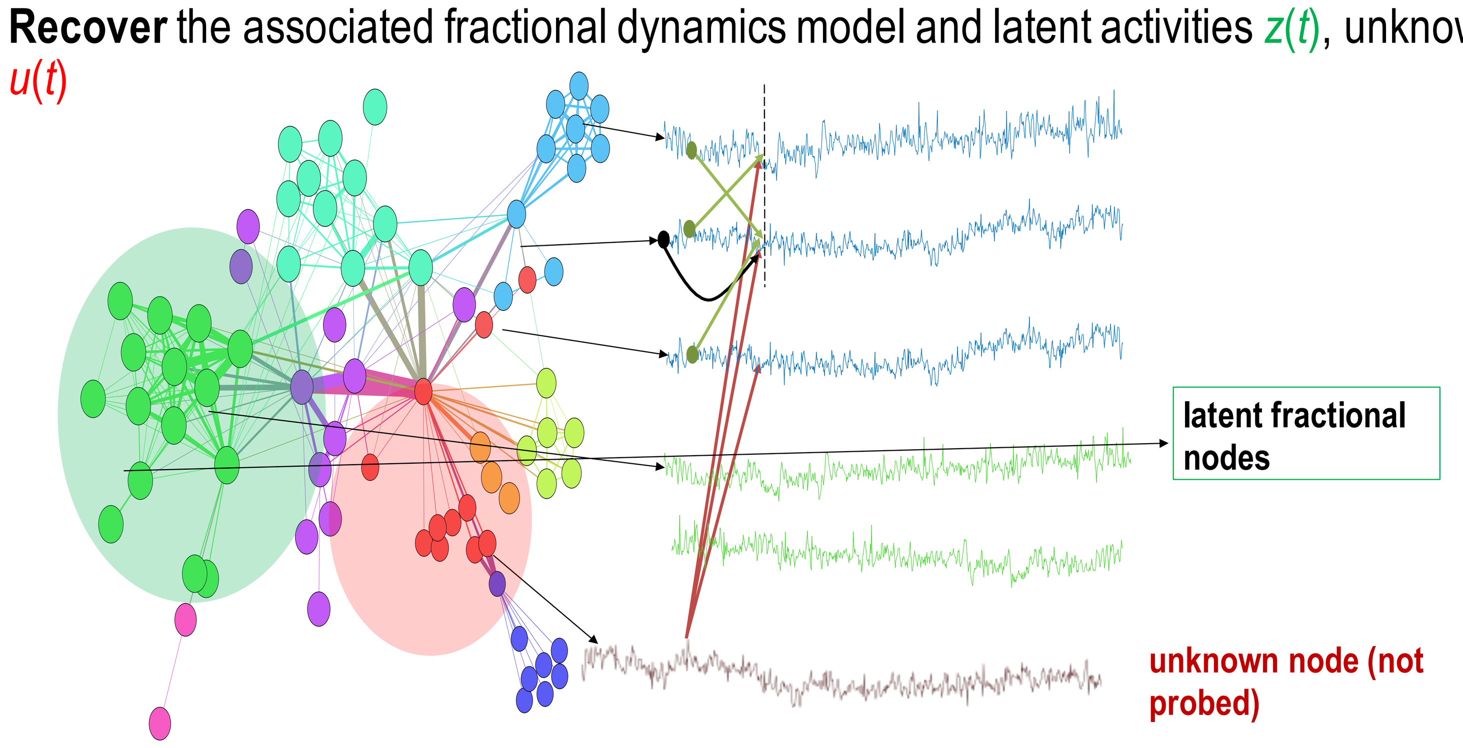


Links +/−	272/200	272/272	549/400	549/549
Recon Acc	100%	99%	100%	97%

- **Incomplete observations:** What is the minimum fraction of node observations required to correctly reconstruct the model?
- varEM converges to 2.5% mean absolute error from the ground truth parameters with only 50% observability

Recovering Latent CN Activities with UUs

- **Recover time-varying CN latent activities & their accurate model**
  - **Given** $x(t)$ partially observed activities, latent nodes fractional orders as α
  - **Recover** the associated fractional dynamics model and latent activities $z(t)$, unknowns $u(t)$

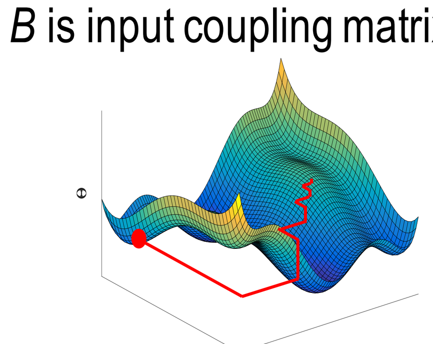


latent fractional nodes

unknown node (not probed)

$$\Delta^\alpha \begin{bmatrix} x[k+1]\\ z[k+1]\end{bmatrix} = \begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}\begin{bmatrix}x[k]\\z[k]\end{bmatrix}+\begin{bmatrix}B_1\\B_2\end{bmatrix}u[k]+\begin{bmatrix}e_1[k]\\e_2[k]\end{bmatrix}$$

- A is the coupling matrix, α is fractional order, B is input coupling matrix
- Fractional Kalman filtering with Bayesian model, and **extended** Expectation Maximization like formulation
- Modeling error reduction by ~40% compared to state-of-the-art models



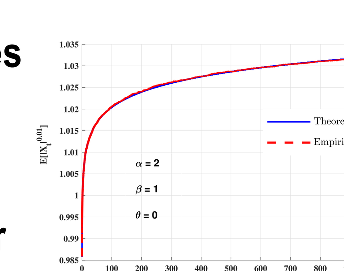
Identifying Fractional Diffusion with few samples

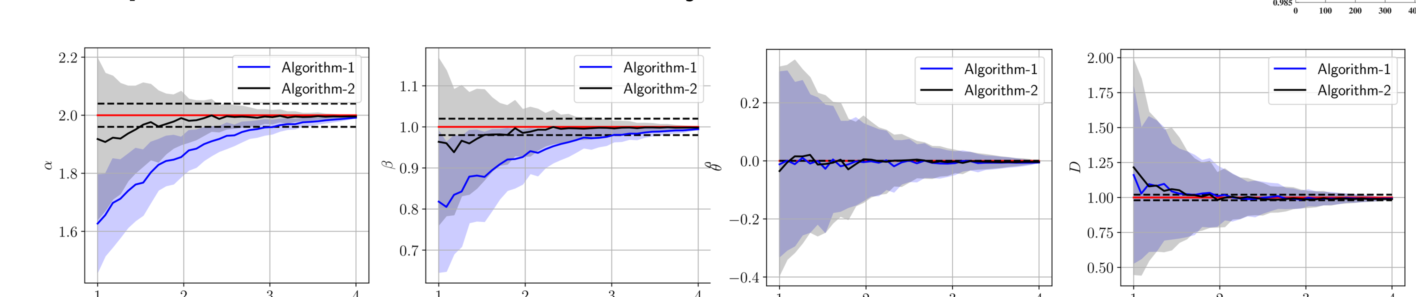
- **Fractional Diffusion process**

  Fractional Reisz-feller derivative of order α and skewness θ $\quad {}_tD_*^\beta u(x,t) = D_x D_\theta^\alpha u(x,t)$

  Fractional Caputo time derivative with order β

- **Identify arguments of fractional differential equation from few trajectories**
- **Moments-based and log Moments based algorithm for quick inference**
- **Theoretical and empirical fractional moments match**
- **Learn parameters from as few as 100 trajectories with less than 2% error**





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